Mechanika Kwantowa dla doktorantów
zestaw 12 i 13 na dzień 18.1.2012 środa godz. 10:00

1. Finish the problem of calculating $\tilde{K}$. We have calculated phase shifts in the large box with the result
$$Tk - \delta_k = \pi n.$$
Let's denote solution to this equation by $\tilde{k}_n$. Similarly, for the Euclidean harmonic oscillator analogous solutions read $k_n = \pi n/T$.

   - The contribution to K' coming from the continuous spectrum, K_{cont}, reads:
$$K_{cont} = \frac{\prod \tilde{\lambda}_n}{\prod \lambda_n} = \prod_{n=1}^{\infty} \frac{1+\tilde{k}_n^2}{1+k_n^2} = \exp\left(\sum_n \ln \frac{1+\tilde{k}_n^2}{1+k_n^2}\right) \approx \exp\left(\sum_n \frac{2k_n(\tilde{k}_n - k_n)}{1+k_n^2}\right).$$
   - To calculate last sum under exponent go to the continuum limit $T \to \infty$ and convert the sum into the integral:
$$\ldots = \exp\left(\frac{1}{\pi}\int_0^{\infty} dk \frac{2\delta_k\, k}{1+k^2}\right) = \frac{1}{9}.$$
Last equality can be obtained by integration by parts and the explicit form of δ_k. Full result for K' is obtained by multiplying K_{cont} by a non-zero λ value from the discrete part.
   - Literature:
S. Coleman, *Aspects of Symmetry*, Cambridge University Press (1988), Section 7, Appendix 1.

A.I. Vainshtein, V.I. Zakharov, V.A. Novikov and M.A. Shifman, *ABC of Instantons*, Sov. Phys. Usp. **24**, 195 (1982) [Usp. Fiz. Nauk **136**, 553 (1982)].

T. Schafer and E.V. Shuryak, *Instantons in QCD*, Rev. Mod. Phys. **70** (1998) 323 [arXiv:hep-ph/9610451].

2. Calculate energy levels for the particle in the potential
$$V(x) = -\frac{V_0}{\cosh^2 \frac{x}{a}}$$
for arbitary V_0 and a. Next, calculate energy levels in the semiclassical (WKB) approximation. Compare with the exact result.