

Mechanika Kwantowa dla doktorantów
zestaw 4 na dzień 2.11.2011 środa

1. Continuation. Find classical action for the harmonic oscillator with external force $F(t)$. Take limit $\omega \rightarrow 0$ to obtain classical action for a particle moving in external force.
2. One slit diffraction.

Consider one dimensional problem of a particle moving from $x = 0$ at $t = 0$ to point (x, t) through a screen which the particle reaches at time $t_1 < t$. The screen has a hole of size $2b$ centered around x_0 . (This process can be understood in the following way: at time t_1 we block all possible trajectories except those passing through a hole). Calculate the propagator for such a motion.

In order to simplify calculations replace the sharp hole by a "Gaussian hole"

$$G(x) = \begin{cases} 0 & \text{for } x < x_0 - b \text{ or } x_0 + b < x \\ 1 & \text{for } x_0 - b \leq x \leq x_0 + b \end{cases} \rightarrow G(x) = e^{-(x-x_0)^2/2b^2}.$$

What is the width of the image of this "Gaussian hole" at time t ? How does it compare with the classical expectation?

Since the propagation before and after the slit is free, we can use the formula for a free particle propagator

$$K_0(b, a) = \sqrt{\frac{m}{2\pi i\hbar(t_b - t_a)}} e^{i \frac{m}{2\hbar} \frac{(x_b - x_a)^2}{t_b - t_a}}$$

and the formula for "folding" the propagators

$$K(b, a) = \int dx_c K(b, c)K(c, a).$$

Here $a = (x_a, t_a)$ etc.

This problem can be found in the book of Feynman and Hibbs "Quantum Mechanics and Path Integrals" chapt. 3.2.