Mechanika Kwantowa dla doktorantów

zestaw 2 na dzień 26.10.2011 środa

1. Find classical action for the harmonic oscillator with external force $F(t)$. Take limit $\omega \to 0$ to obtain classical action for a particle moving in external force.

2. One slit diffraction.

   Consider one dimensional problem of a particle moving from $x = 0$ at $t = 0$ to point (x, t) through a screen which the particle reaches at time $t_1 < t$. The screen has a hole of size $2b$ centered around x_0. (This process can be understood in the following way: at time t_1 we block all possible trajectories except those passing through a hole). Calculate the propagator for such a motion.

   In order to simplify calculactions replace the sharp hole by a "Gaussian hole"

$$G(x) = \begin{cases} 0 & \text{for} \quad x < x_0 - b \text{ or } x_0 + b < x \\ \\ 1 & \text{for} \quad x_0 - b \le x \le x_0 + b \end{cases} \quad \to \; G(x) = e^{-(x-x_0)^2/2b^2}.$$

   What is the width of the image of this "Gaussian hole" at time t? How does it compare with the classical expectation?

   Since the propagation before and after the slit is free, we can use the formula for a free particle propagator

$$K_0(b,a) = \sqrt{\frac{m}{2\pi i\hbar(t_b - t_a)}}\, e^{\frac{i}{\hbar}\frac{m}{2}\frac{(x_b - x_a)^2}{t_b - t_a}}$$

   and the formula for "folding"the propagators

$$K(b,a) = \int dx_c\, K(b,c)K(c,a).$$

   Here $a = (x_a, t_a)$ etc.

   This problem can be found in the book of Feynman and Hibbs "Quantum Mechanics and Path Integrals" chapt. 3.2.

3. Sturm-Liouville operator for the harmonic oscillator is defined as

$$D(t) = m\left(-\frac{d^2}{dt^2} - \omega^2\right).$$

   Find eigenfunctions and eigenvalues of this operator

$$D(t)y_n(t) = \lambda_n y_n(t)$$

   subject to the boundary conditions

$$y_n(0) = y_n(T) = 0.$$

Calculate determinant of D (i.e. $\prod_n \lambda_n$). Strictly speaking one can calculate ω-dependent part of det$D$ up to an undetermined constant that can be fixed in some other way.