

Logika i Teoria Mnogości, zestaw 3

3.1. Proszę udowodnić, że zdania poniżej są tautologiami:

- (a) $\exists x [\phi(x) \vee \psi(x)] \Leftrightarrow \exists x \phi(x) \vee \exists x \psi(x)$
- (b) $\exists x [\phi(x) \wedge \psi(x)] \Rightarrow \exists x \phi(x) \wedge \exists x \psi(x)$
- (c) $\forall x \phi(x) \vee \forall x \psi(x) \Rightarrow \forall x [\phi(x) \vee \psi(x)]$
- (d) $\forall x \phi(x) \wedge \forall x \psi(x) \Leftrightarrow \forall x [\phi(x) \wedge \psi(x)]$
- (e) $\forall x [\phi(x) \Rightarrow \psi(x)] \Rightarrow [\forall x \phi(x) \Rightarrow \forall x \psi(x)]$
- (f) $\forall x [\phi(x) \Leftrightarrow \psi(x)] \Rightarrow [\forall x \phi(x) \Leftrightarrow \forall x \psi(x)]$

3.2. Proszę sprawdzić, czy następujące formuły są tautologiami

- (a) $\forall x \exists y \phi(x, y) \Rightarrow \exists y \forall x \phi(x, y)$
- (b) $\forall x [\phi(x) \vee \psi(x)] \Rightarrow \forall x \phi(x) \vee \forall x \psi(x)$
- (c) $\exists x \phi(x) \wedge \exists x \psi(x) \Rightarrow \exists x [\phi(x) \wedge \psi(x)]$
- (d) $[\forall x \phi(x) \Rightarrow \forall x \psi(x)] \Rightarrow \forall x [\phi(x) \Rightarrow \psi(x)]$
- (e) $[\forall x \phi(x) \Leftrightarrow \forall x \psi(x)] \Rightarrow \forall x [\phi(x) \Leftrightarrow \psi(x)]$

3.3. Proszę udowodnić, że zdania poniżej są tautologiami:

- (a) $\exists x \phi(x) \vee \forall x \psi(x) \Leftrightarrow \exists x \forall y [\phi(x) \vee \psi(y)] \Leftrightarrow \forall y \exists x [\phi(x) \vee \psi(y)]$
- (b) $\exists x \phi(x) \wedge \forall x \psi(x) \Leftrightarrow \exists x \forall y [\phi(x) \wedge \psi(y)] \Leftrightarrow \forall y \exists x [\phi(x) \wedge \psi(y)]$
- (c) $\forall x \{[\forall y \phi(x, y)] \Rightarrow \psi(x)\} \Leftrightarrow \forall x \exists y [\phi(x, y) \Rightarrow \psi(x)]$
- (d) $\forall x \{[\exists y \phi(x, y)] \Rightarrow \psi(x)\} \Leftrightarrow \forall x, y [\phi(x, y) \Rightarrow \psi(x)]$

Leszek Hadasz
hadasz@th.if.uj.edu.pl