Statistics and Data Analysis (HEP at LHC)

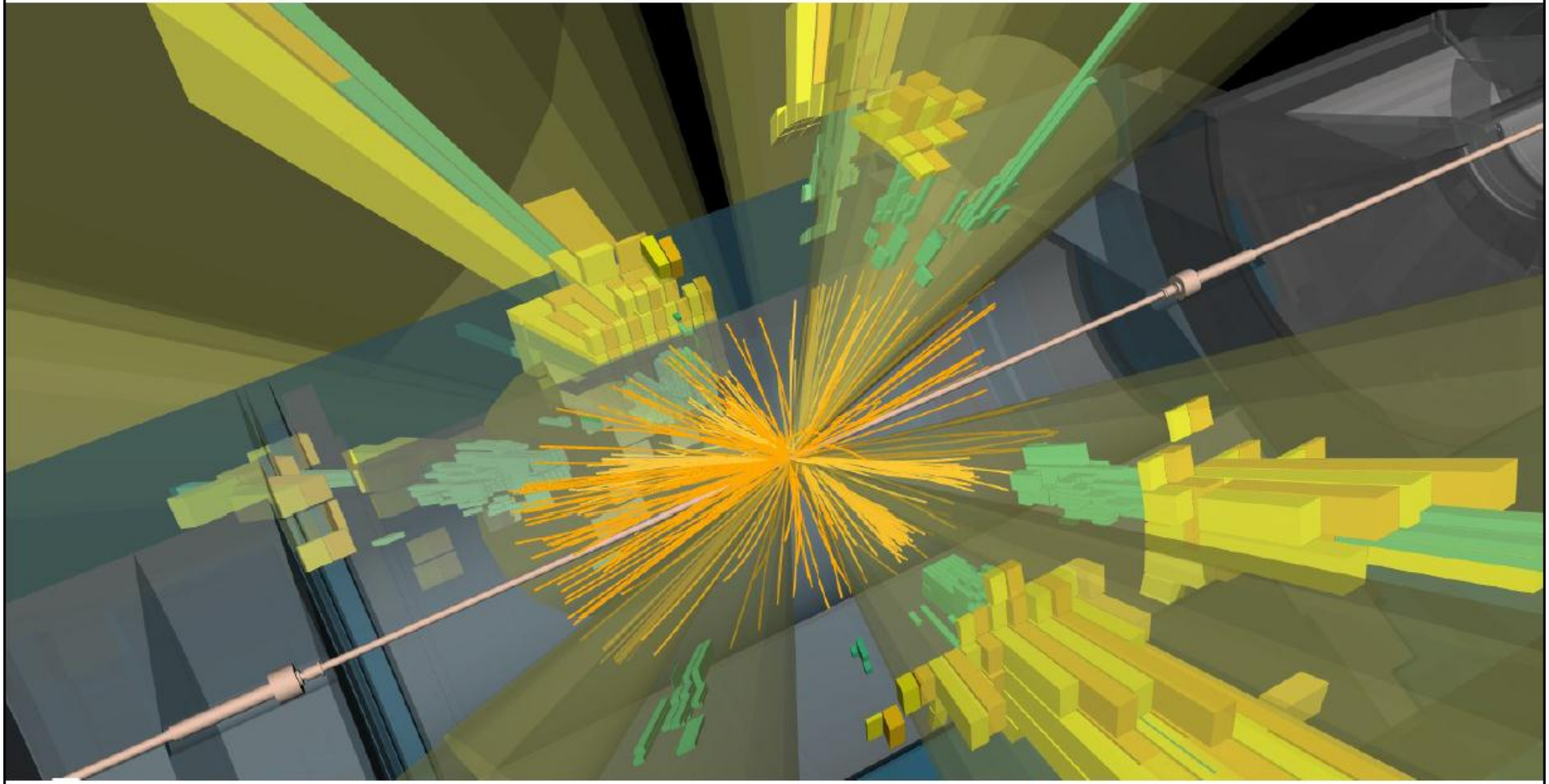
- **Statistical basics for physics**
  - **Random processes**
  - **Probability distributions**
- **Describing physics measurements**
  - **Binned and unbinned data**
  - **Model parameters**

Slides extracted from N. Berger lectures at CERN Summer School 2019

Prof. dr hab. Elżbieta Richter-Wąs

Randomness in High Energy Physics

Experimental data is produced by incredibly complex processes



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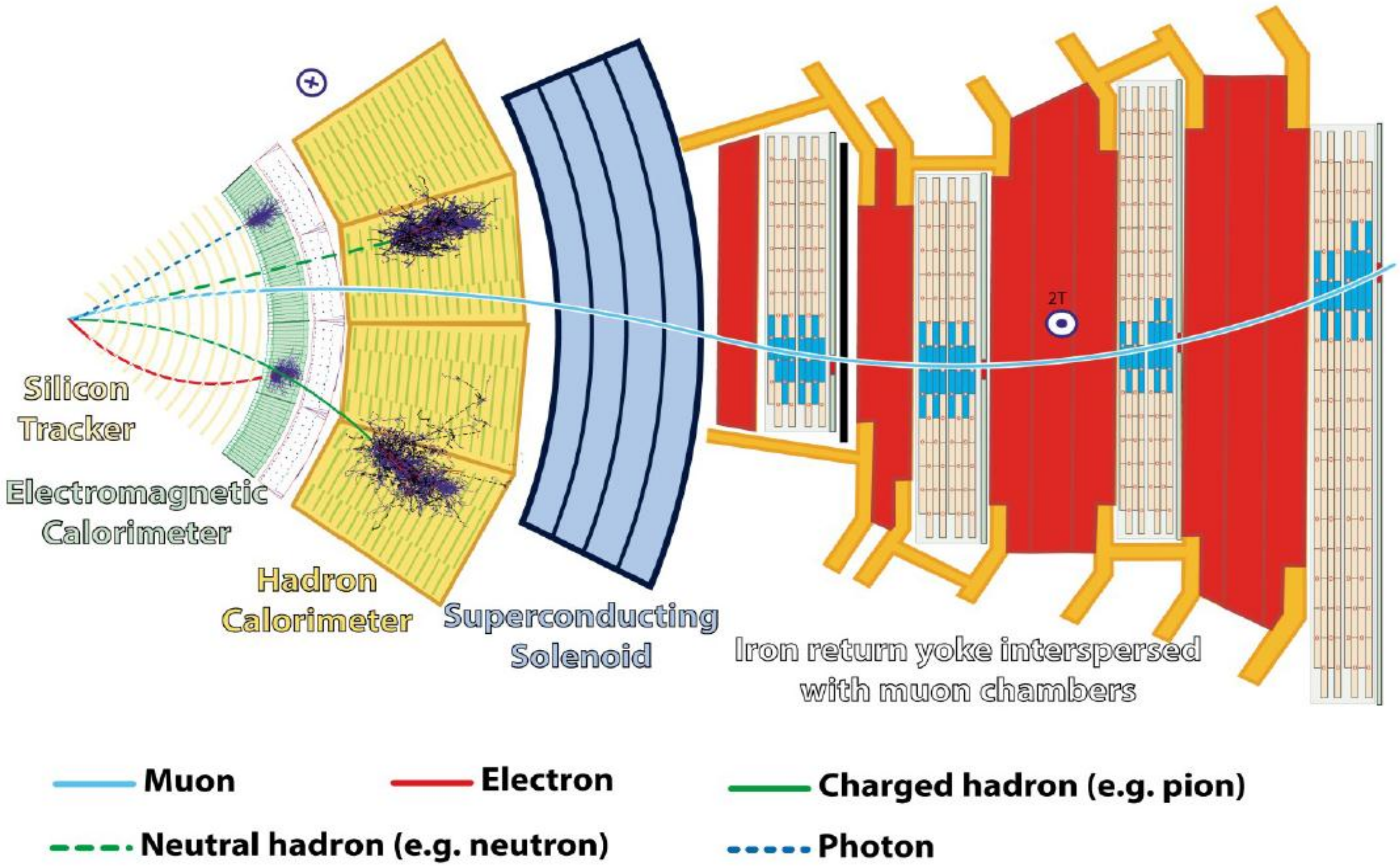

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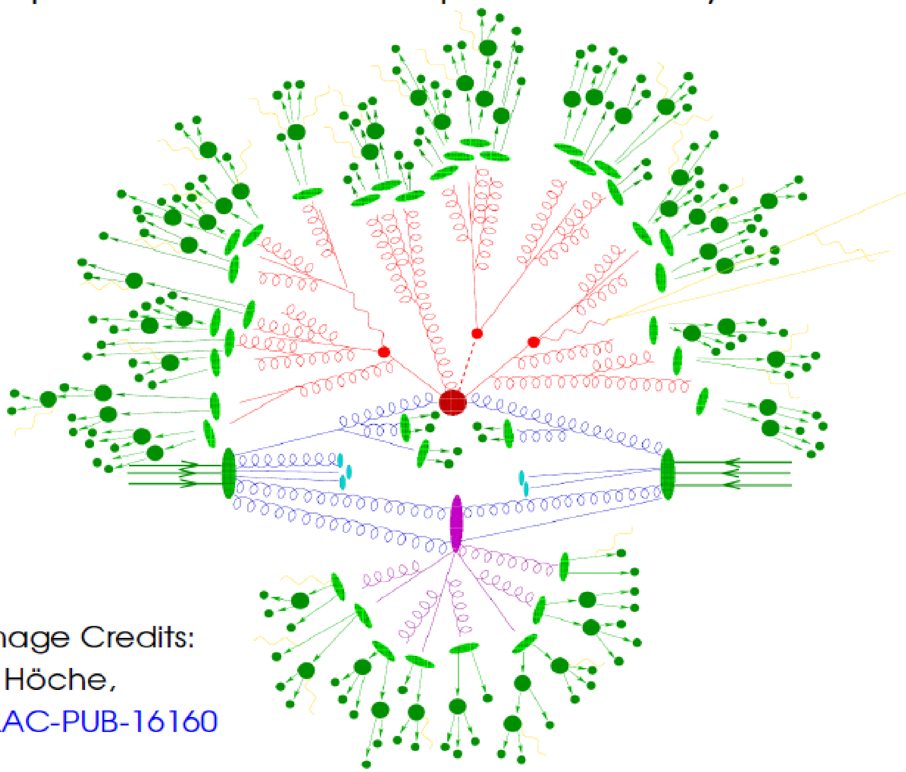


Image Credits:
S. Höche,
SLAC-PUB-16160

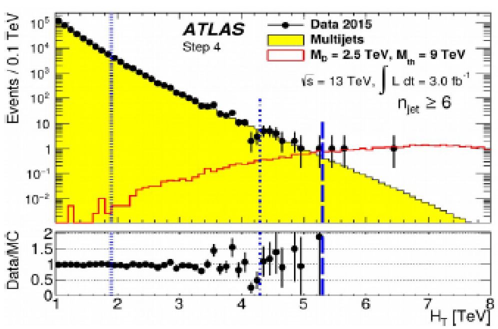
Hard scattering

PDFs, Parton shower, Pileup

Decays

Detector response

Reconstruction



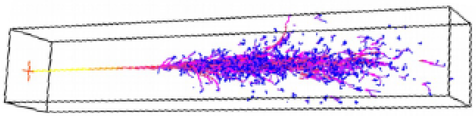
Randomness involved in all stages

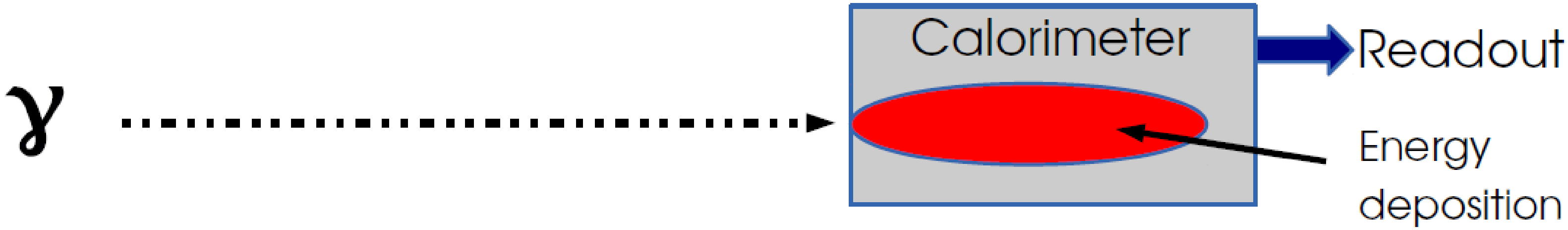
→ **Classical** randomness: detector reponse

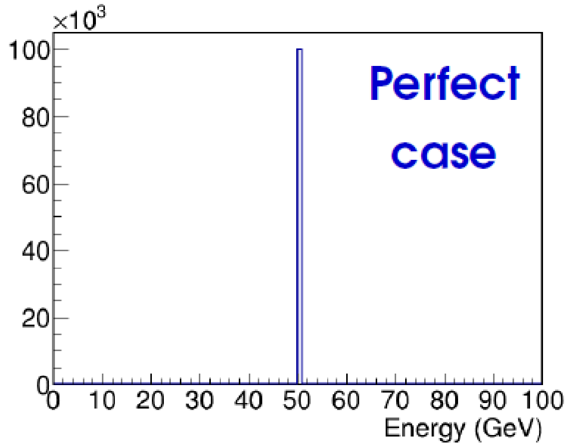
→ **Quantum** effects in particle production, decay

Measurement Errors: Energy measurement

Example: measuring the energy of a photon in a calorimeter







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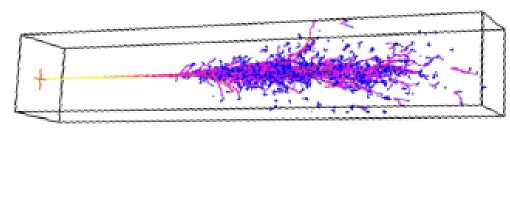

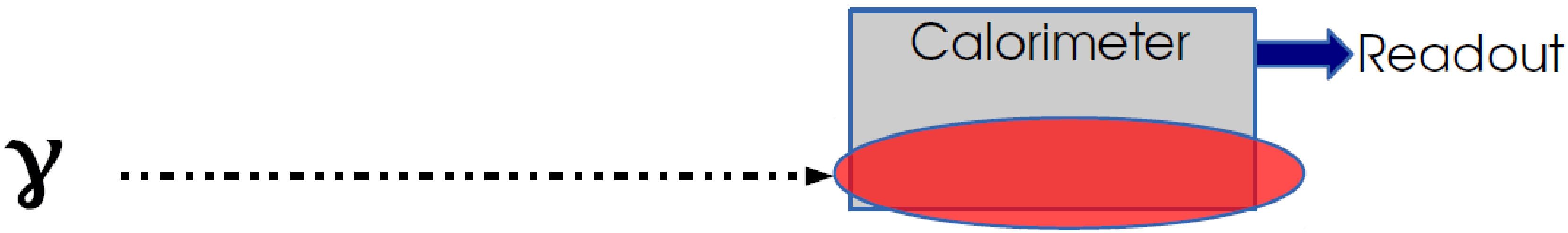

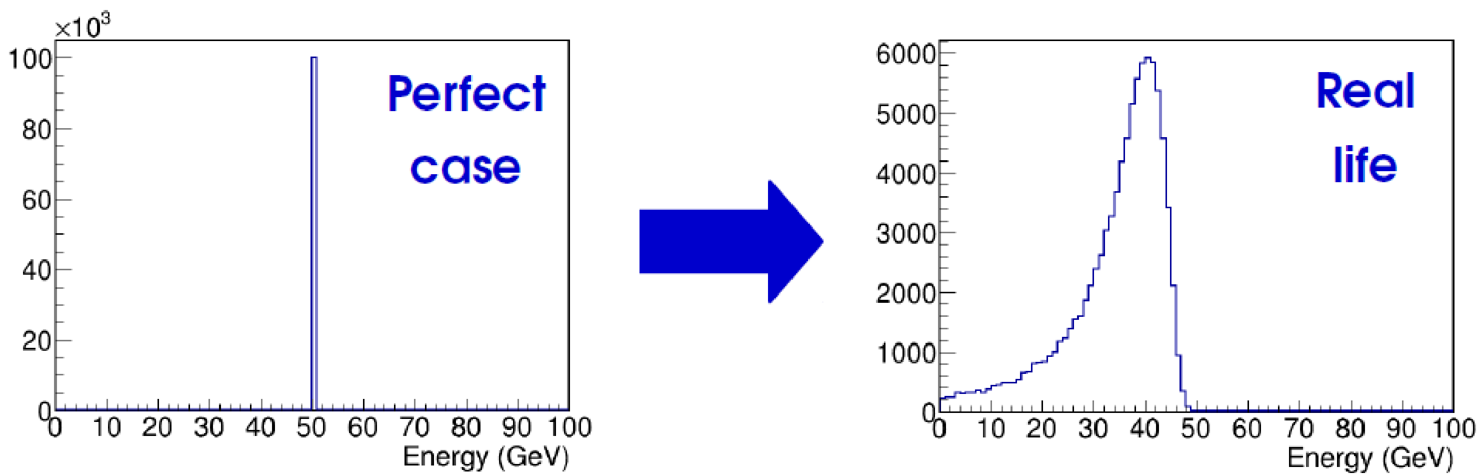

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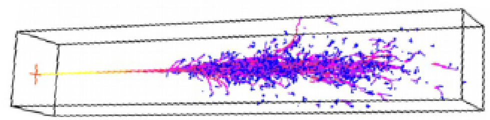
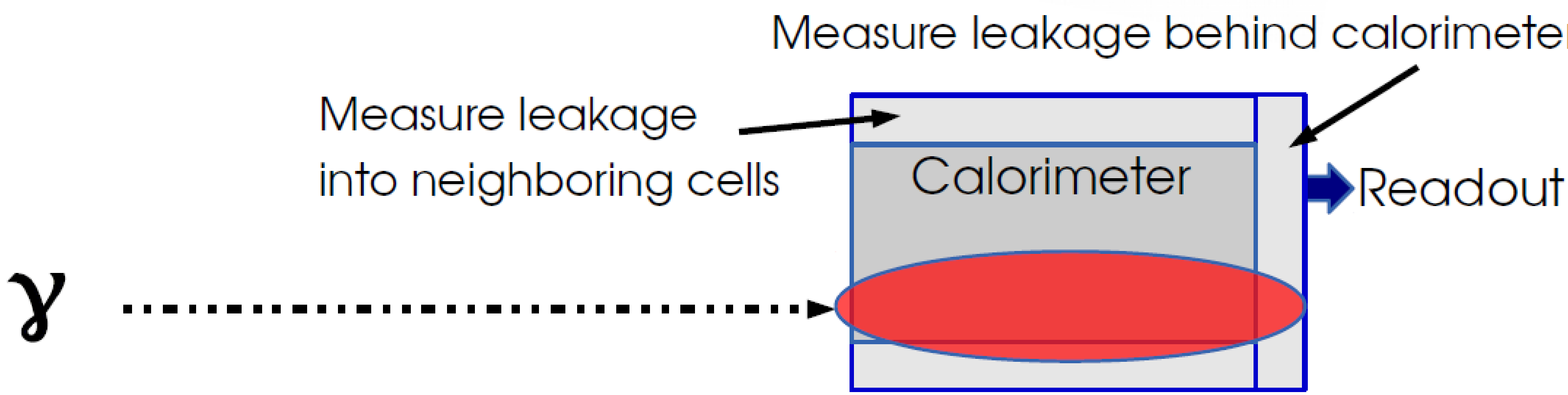
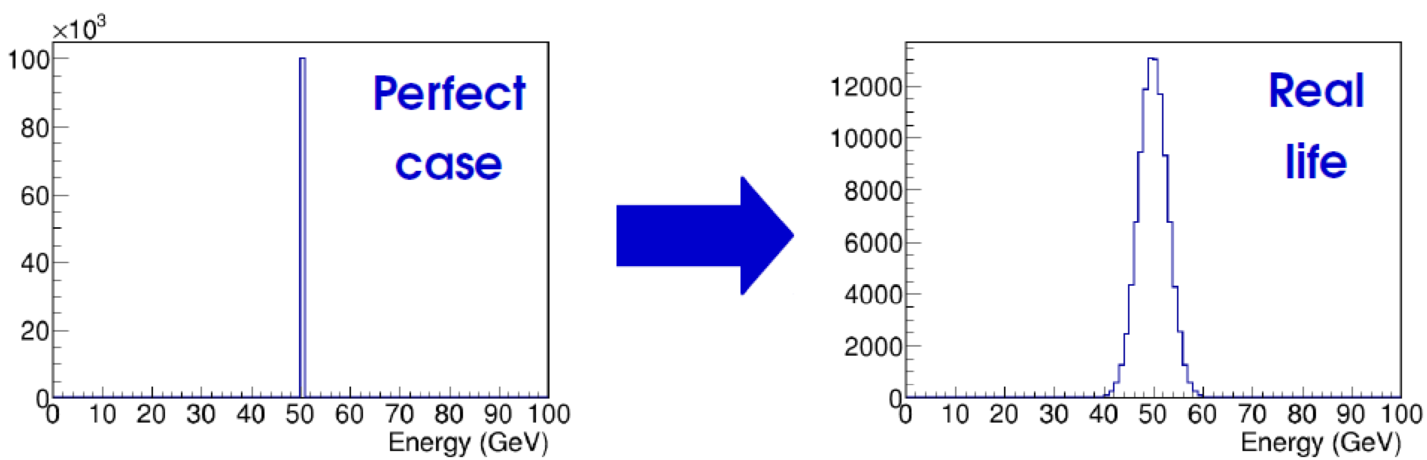


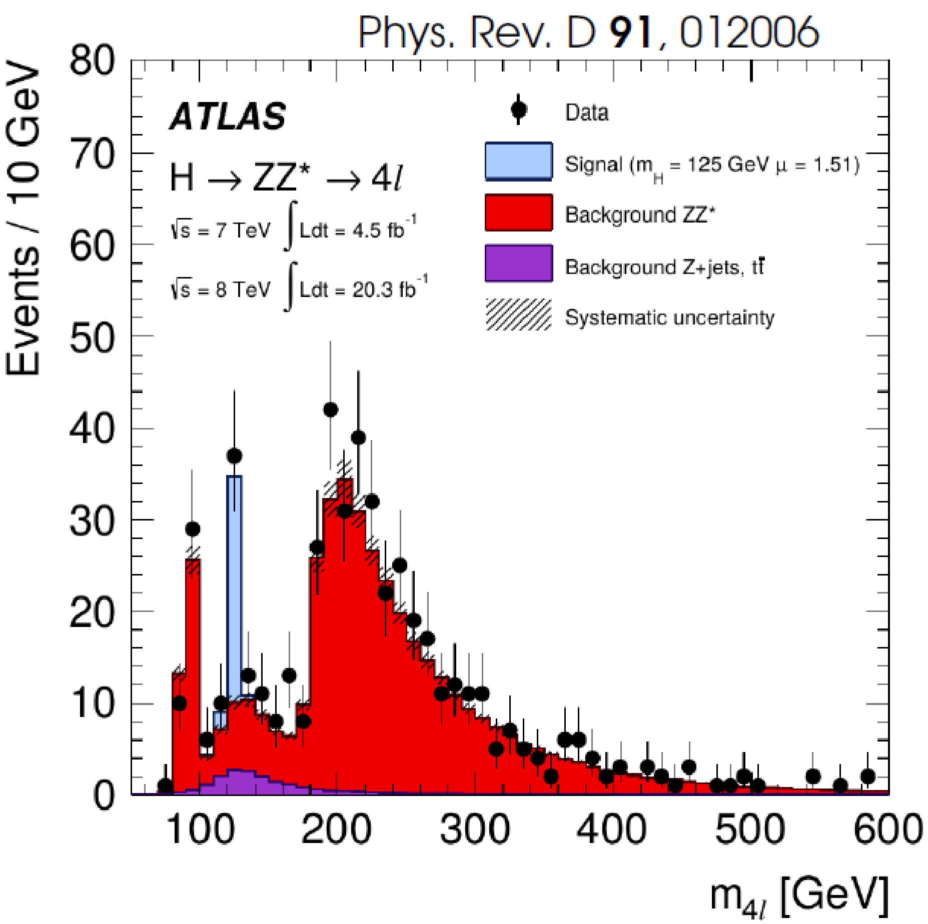




Cannot predict the measured value for a given event

⇒ **Random process** ⇒ **Need a probabilistic description**

Quantum randomness: H->ZZ*-> 4 l



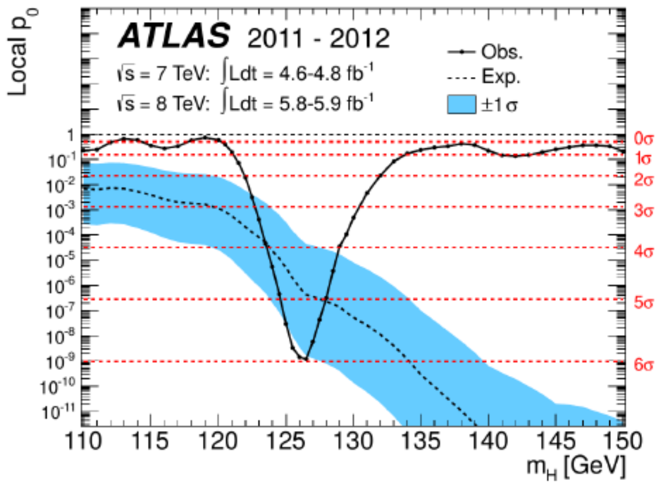
Rare process: Expect 1 signal event every **~6 days**

"Will I get an event today ?" → only **probabilistic** answer

Randomness in Physics

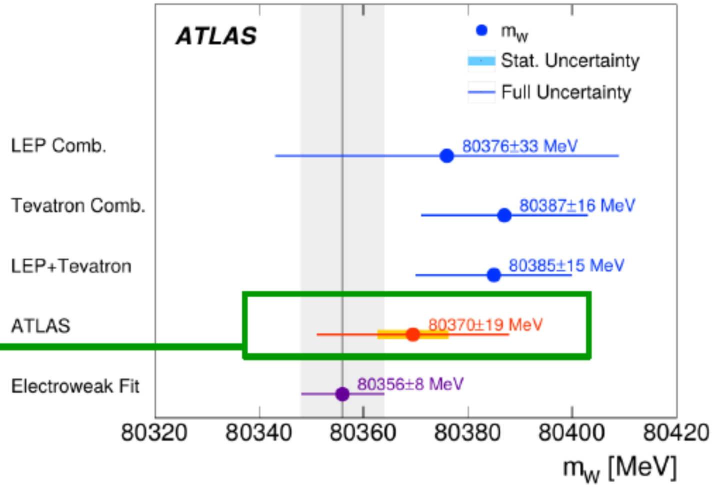
Questions with probabilistic answers:

- **Is my Higgs-like excess just a background fluctuation?**
  → associated with prob ~10^{-9} (by now ~10^{-24})
  ⇒ above the famous (and conventional) ***5σ***



- For measurements: probability that the **true value** of a parameter is within an interval:

68% chance that the true m_W is within the orange interval



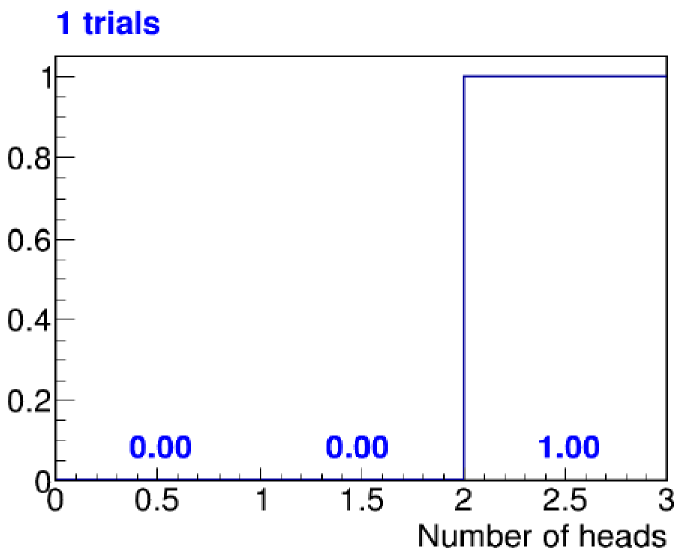
Probability distributions

Probabilistic treatment of possible outcomes

⇒ ***Probability Distribution***

Example: two-coin toss

→ Fractions of events in each bin i converge to a limit p_i



Probability distribution :

{ P_i } for i = 0, 1, 2

Properties

- $P_i > 0$
- $\Sigma P_i = 1$

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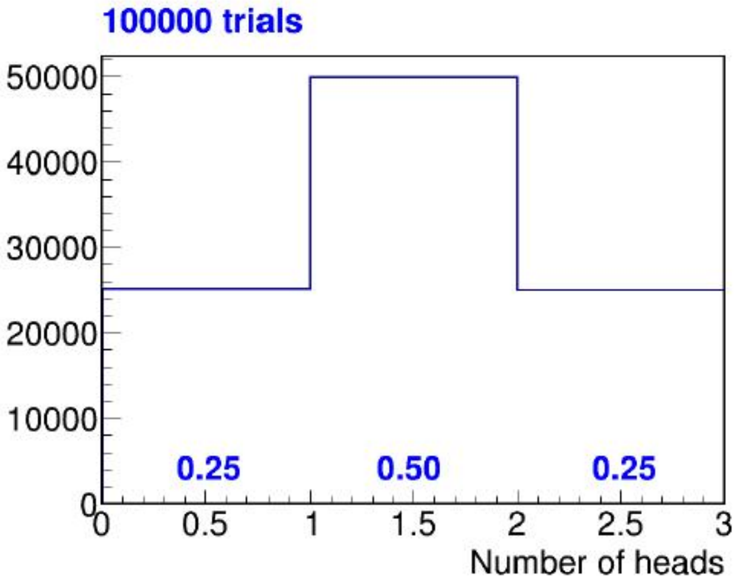

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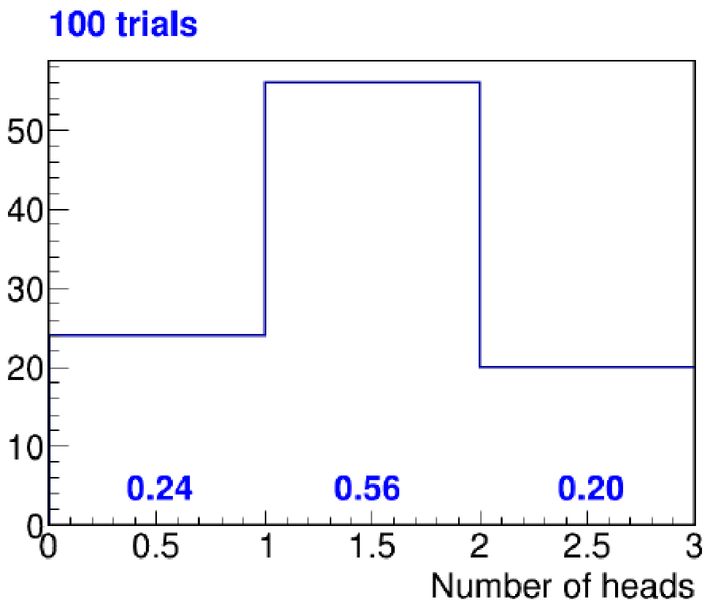

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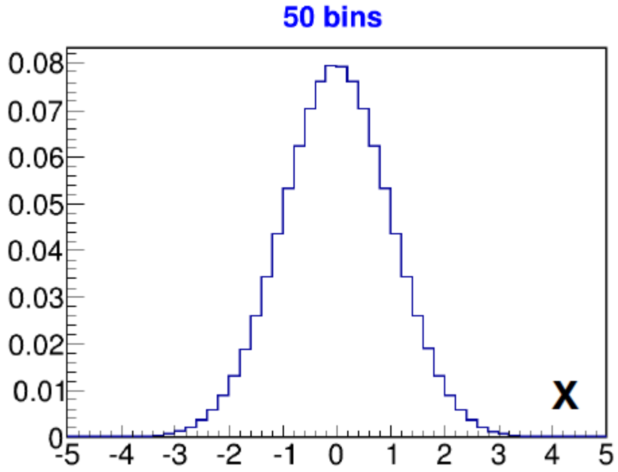
Continuous Variables: PDFs

Continuous variable: can consider **per-bin** probabilities p_i, $i=1..\ n_{bins}$



Bin size → 0 :

Probability distribution function P(x)

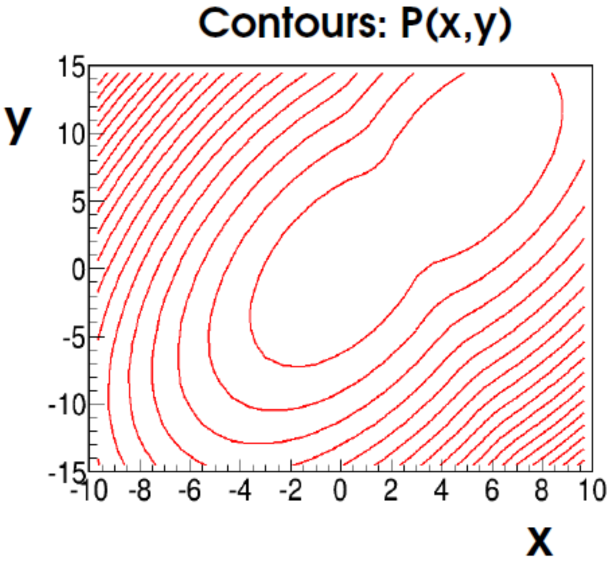
→ $P(x) > 0$, $\int P(x)\,dx = 1$

→ High values ⇔ high chance to get a measurement here

Generalizes to **multiple variables** :

→ $P(x,y) > 0$

→ $\int P(x,y)\,dx\,dy = 1$

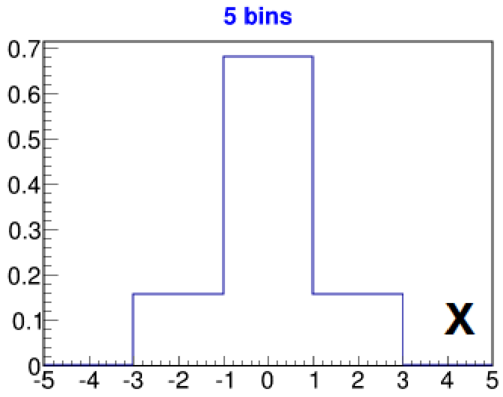


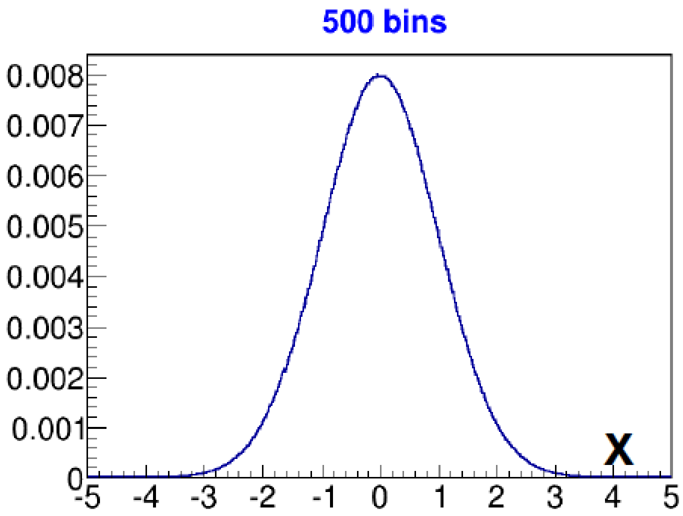
Random Variables

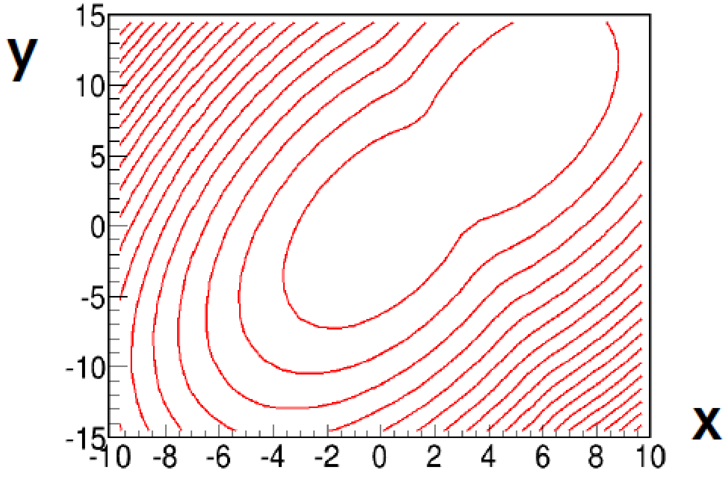
X, Y... are **Random Variables** (continuous or discrete), a.ka. **observables** :
→ X can take any value x, with probability **P(X=x).**

→ P(X) is the **PDF** of X, a.k.a. the **Statistical Model.**

→ The **Observed data** is **one value** x_{obs} of X, drawn from P(X).







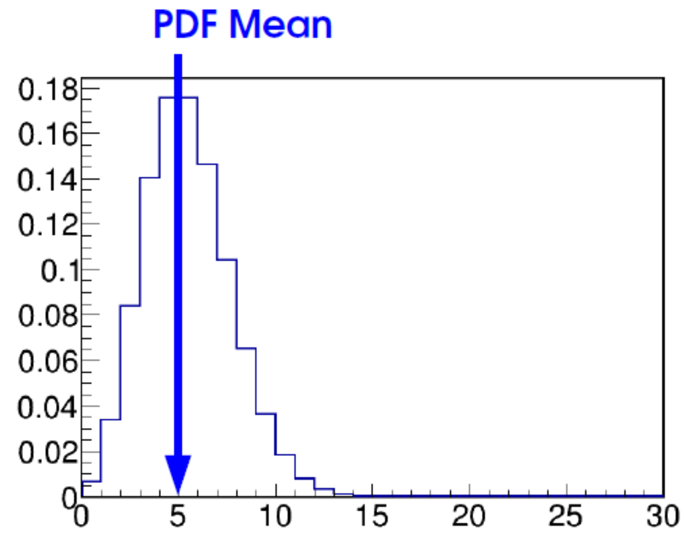
PDF properties: mean

E(X) = <X> : **Mean** of X – expected outcome **on average over many measurements**

$$\langle X \rangle = \sum_i x_i \, P_i \qquad \text{or}$$

$$\langle X \rangle = \int x \, P(x) \, dx$$

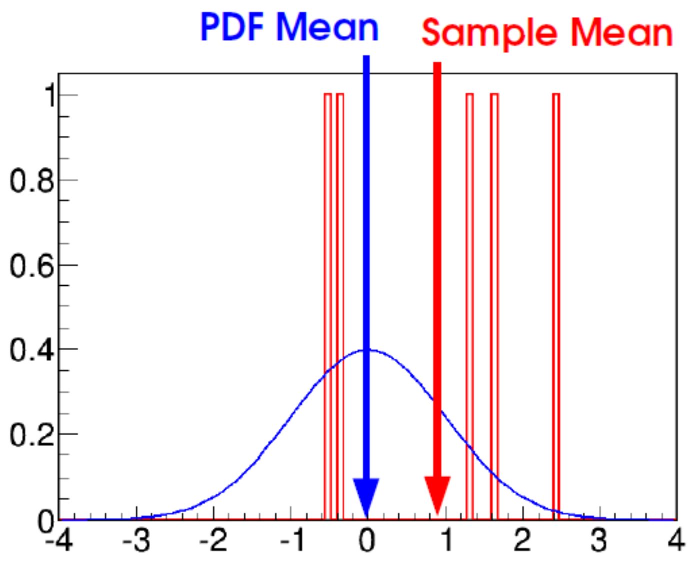
→ Property of the **PDF**



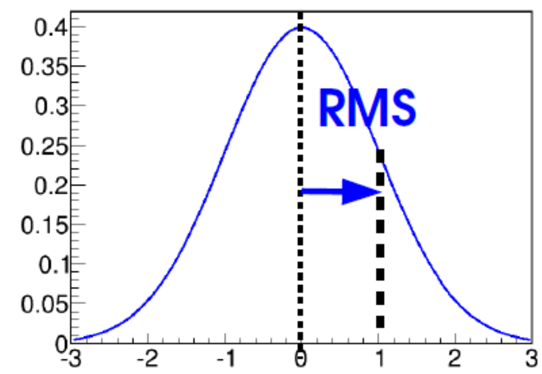
For measurements $x_1 \ldots x_n$,
then can compute the **Sample mean**:

$$\bar{x} = \frac{1}{n} \sum_i x_i$$

→ Property of the **sample**
→ approximates the PDF mean.



PDF properties: (co)variance



Variance of X:

$$\mathrm{Var}(X) = \langle (X - \langle X \rangle)^2 \rangle$$

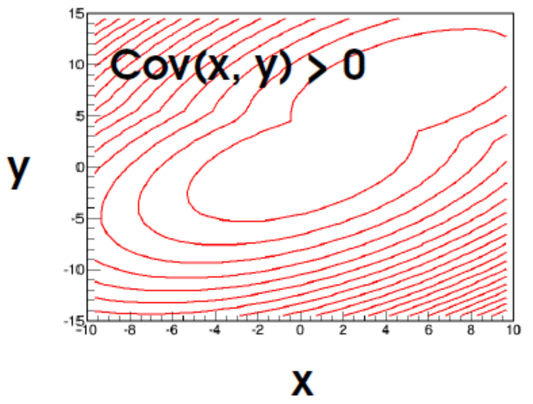
→ Average square of deviation from mean

→ **RMS(X) = √Var(X) = σ_X** *standard deviation*

Can be approximated by **sample variance**:

$$\hat{\sigma}^2 = \frac{1}{n-1} \sum_i (x_i - \bar{x})^2$$

Covariance of X and Y:

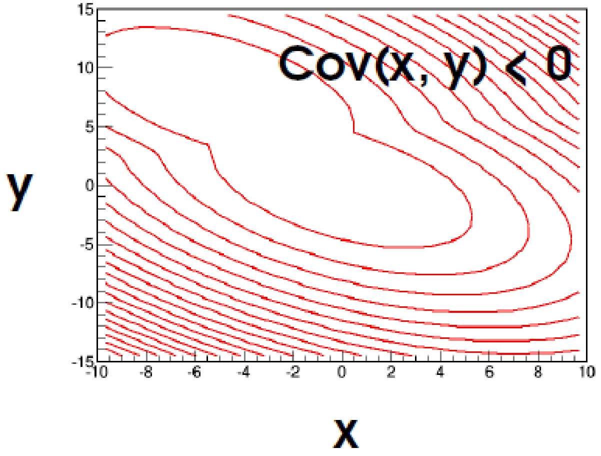


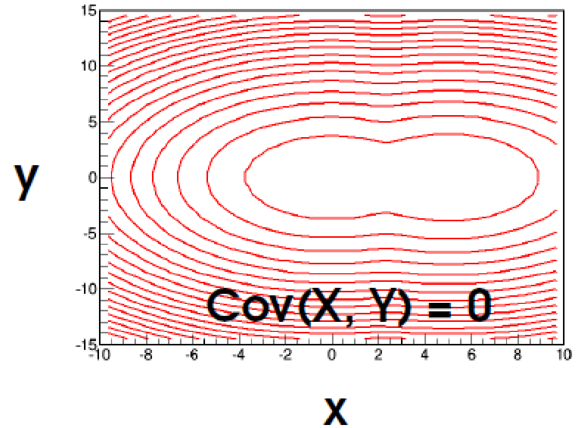
$$\mathrm{Cov}(X,Y) = \langle (X - \langle X \rangle)(Y - \langle Y \rangle) \rangle$$

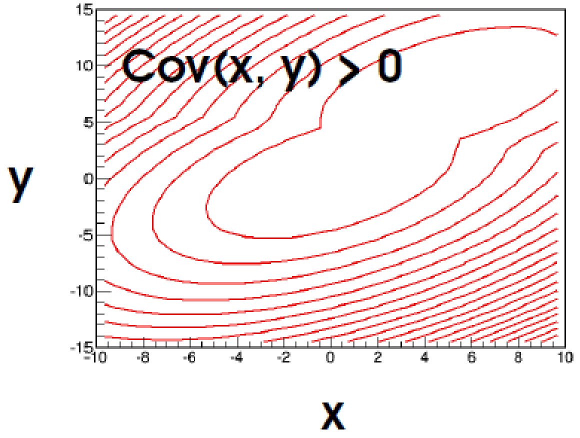
→ Large if variations of X and Y are "synchronized"

Correlation coefficient $\rho = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}}$ $-1 \leq \rho \leq 1$

PDF properties: (co)variance

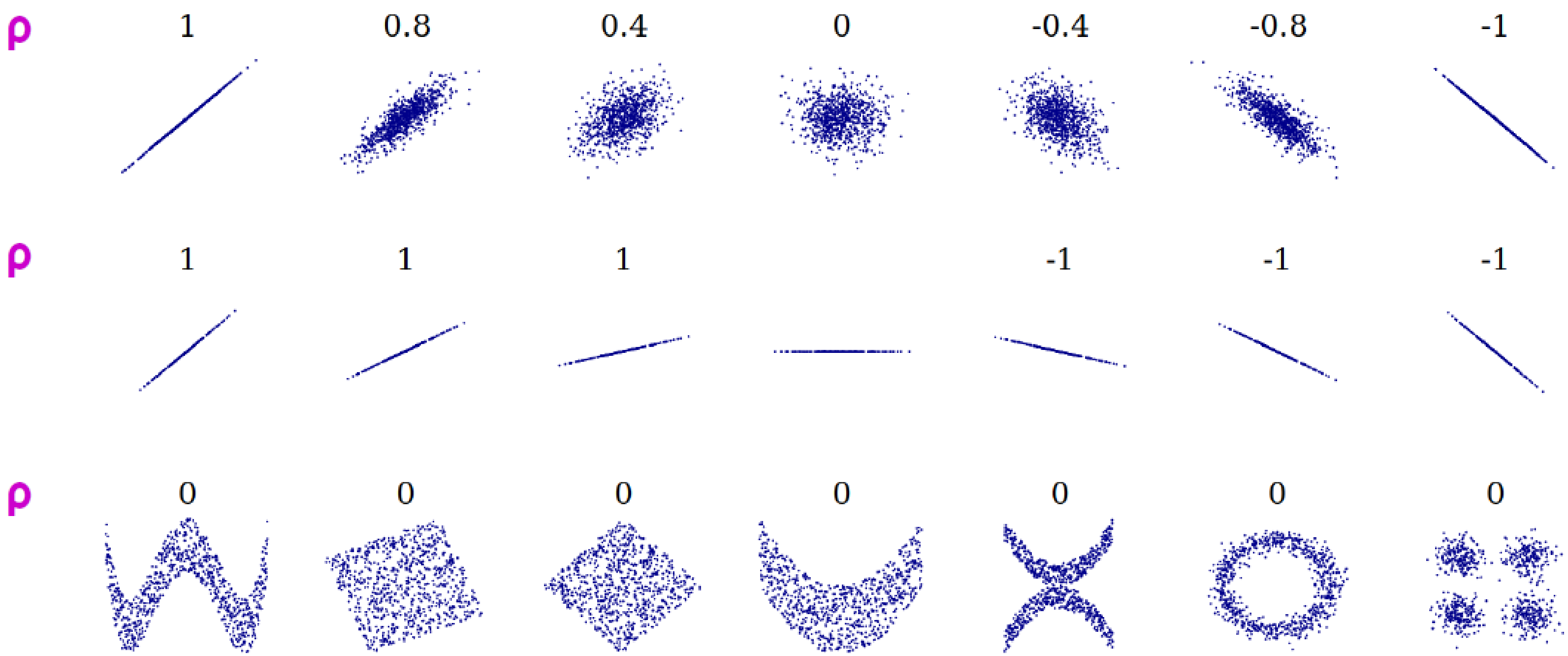






„Linear” vs. „non-linear” correlations

For non-Gaussian cases, the **Correlation coefficient ρ** is not the whole story:



$$\tan 2\alpha = \frac{2\rho\sigma_1\sigma_2}{\sigma_1^2 - \sigma_2^2}$$

Source: Wikipedia

In particular, variables can still be correlated even when ρ=0 : *“Non-linear”* correlations.

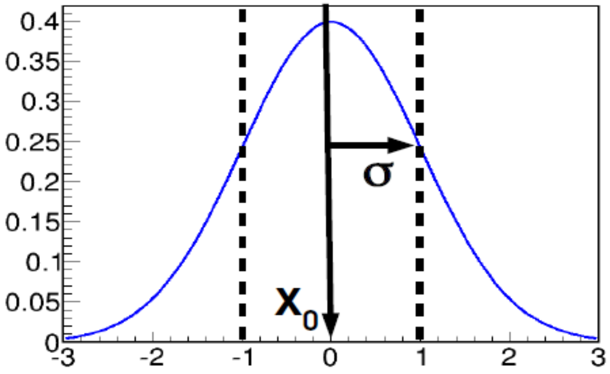
Gaussian PDF

Gaussian distribution:

$$G(x; X_0, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-X_0)^2}{2\sigma^2}}$$

→ **Mean** : X_0

→ **Variance** : σ^2 ($\Rightarrow$ RMS = σ)



Generalize to **N dimensions**:

→ **Mean** : X_0

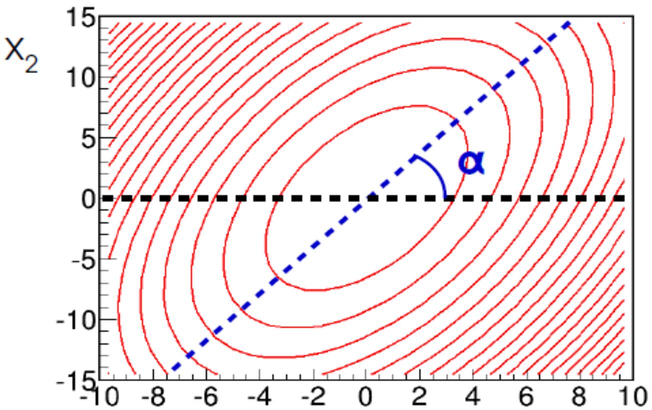
→ **Covariance matrix** :

$$G(x; X_0, C) = \frac{1}{(2\pi|C|)^{N/2}} e^{-\frac{1}{2}(x-X_0)^T C^{-1}(x-X_0)}$$

$$C = \begin{bmatrix} Var(X_1) & Cov(X_1, X_2) \\ Cov(X_2, X_1) & Var(X_2) \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$$

$$\tan 2\alpha = \frac{2\rho\sigma_1\sigma_2}{\sigma_1^2 - \sigma_2^2}$$



Gaussian quantiles

Consider $z = \left(\frac{x - x_0}{\sigma} \right)$ *"pull"* of x

$G(x; x_0, \sigma)$ depends only on $z \sim G(z; 0,1)$

Probability $P(|x - x_0| > Z\sigma)$ to be away from the mean:

Z	$P(\|x - x_0\| > Z\sigma)$
1	**0.317**
2	**0.045**
3	**0.003**
4	**3×10^{-5}**
5	**6×10^{-7}**

Gaussian **Cumulative Distribution Function (CDF)** :

$$\Phi(z) = \int_{-\infty}^{z} G(u; 0,1)\, du$$

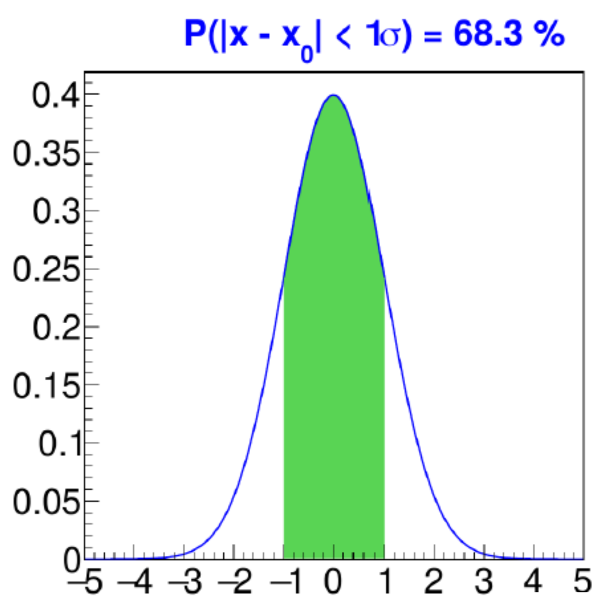
In ROOT,

$\Phi(z)$: `ROOT::Math::gaussian_cdf(z)`

$\Phi^{-1}(p)$: `ROOT::Math::gaussian_quantile(p,1)`

and add "_c" to use 1-Φ instead of Φ

```
root [0] ROOT::Math::gaussian_cdf(1) - ROOT::Math::gaussian_cdf(-1)
(double) 0.68268949
root [1] ROOT::Math::gaussian_quantile_c(0.05/2, 1)
(double) 1.9599640
```



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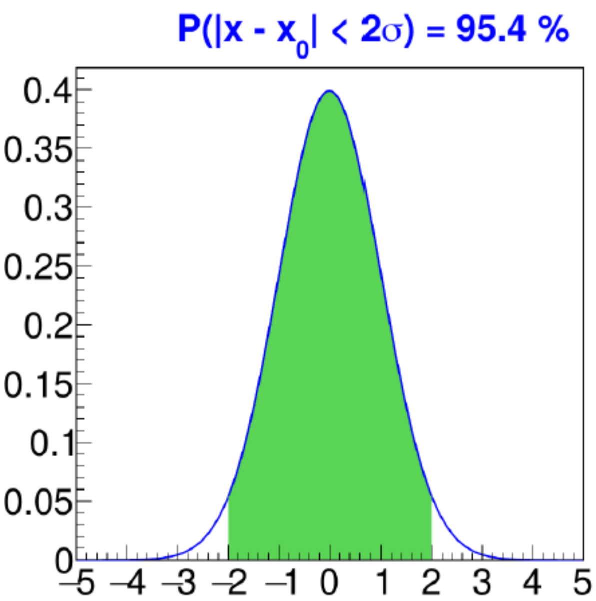

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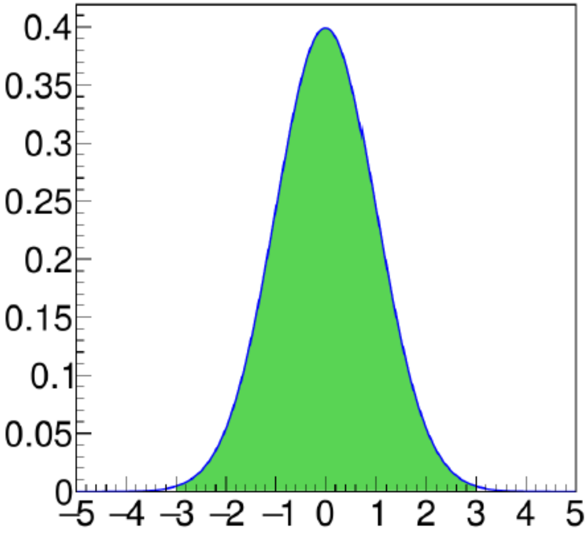
$P(|x - x_0| < 3\sigma) = 99.7\ \%$



Central Limit Theorem

(*) Assuming $\sigma_X < \infty$ and other regularity conditions

For an observable X with **any distribution**, one has(*)

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \overset{n \to \infty}{\sim} G\left(\langle X \rangle, \frac{\sigma_X}{\sqrt{n}}\right)$$

What this means:

- **The average of many measurements is always Gaussian**, whatever the distribution for a single measurement
- The **mean** of the Gaussian is the **average of the single measurements**
- The **RMS** of the Gaussian **decreases as √n** : smaller fluctuations when averaging over many measurements

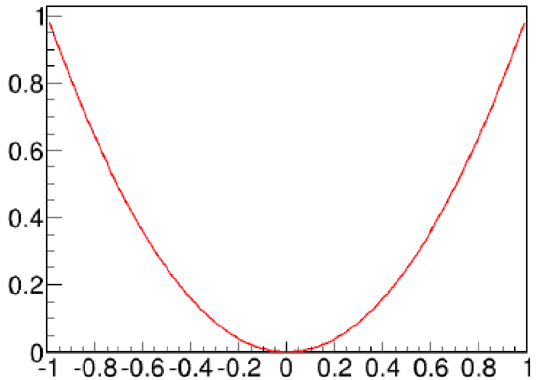
Another version, for the sum:

$$\sum_{i=1}^{n} x_i \overset{n \to \infty}{\sim} G\left(n\langle X \rangle, \sqrt{n}\,\sigma_X\right)$$

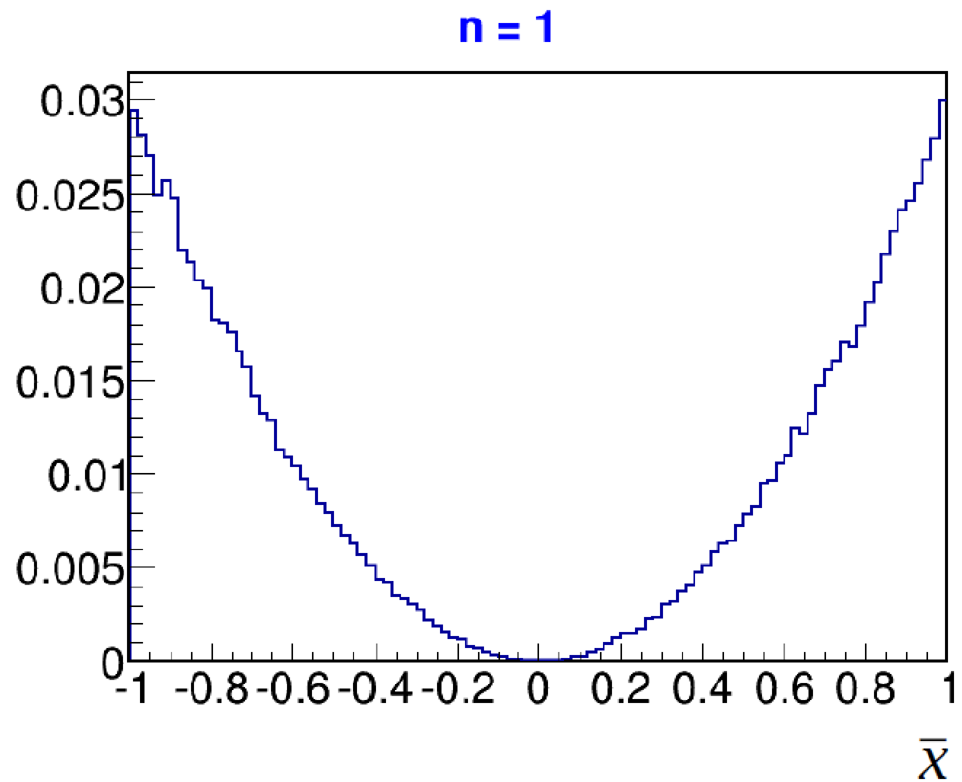
Mean scales like n, but RMS only like √n

Central Limit Theorem in action

Draw events from a parabolic distribution (e.g. decay cos θ*)



$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \rightarrow$$



Distribution becomes Gaussian, although very non-Gaussian originally

Distribution becomes narrower as expected (as 1/√n)

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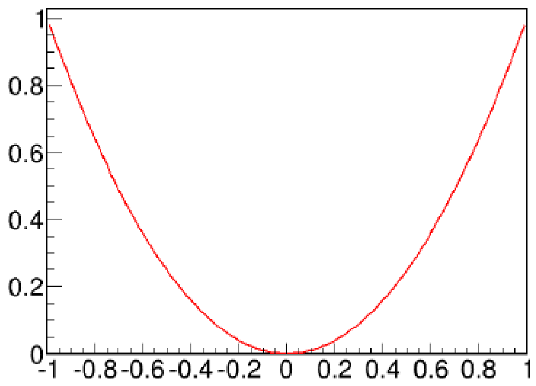

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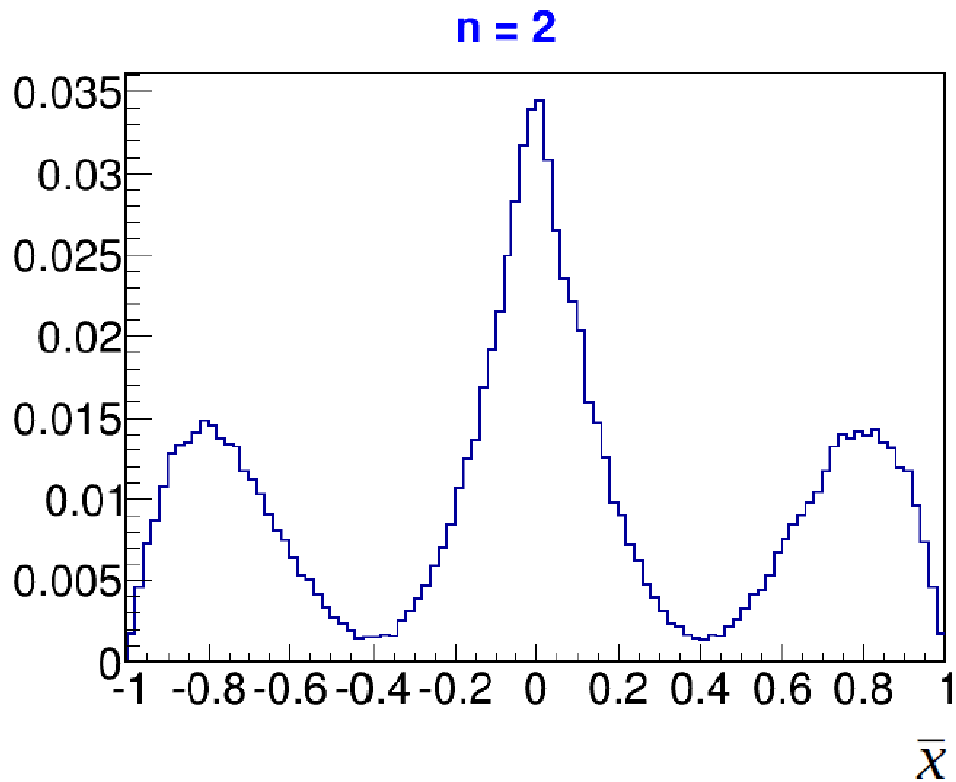

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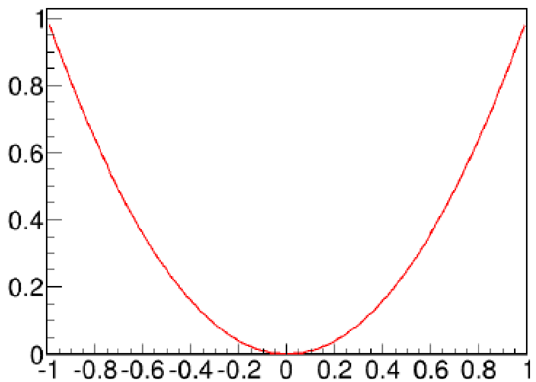

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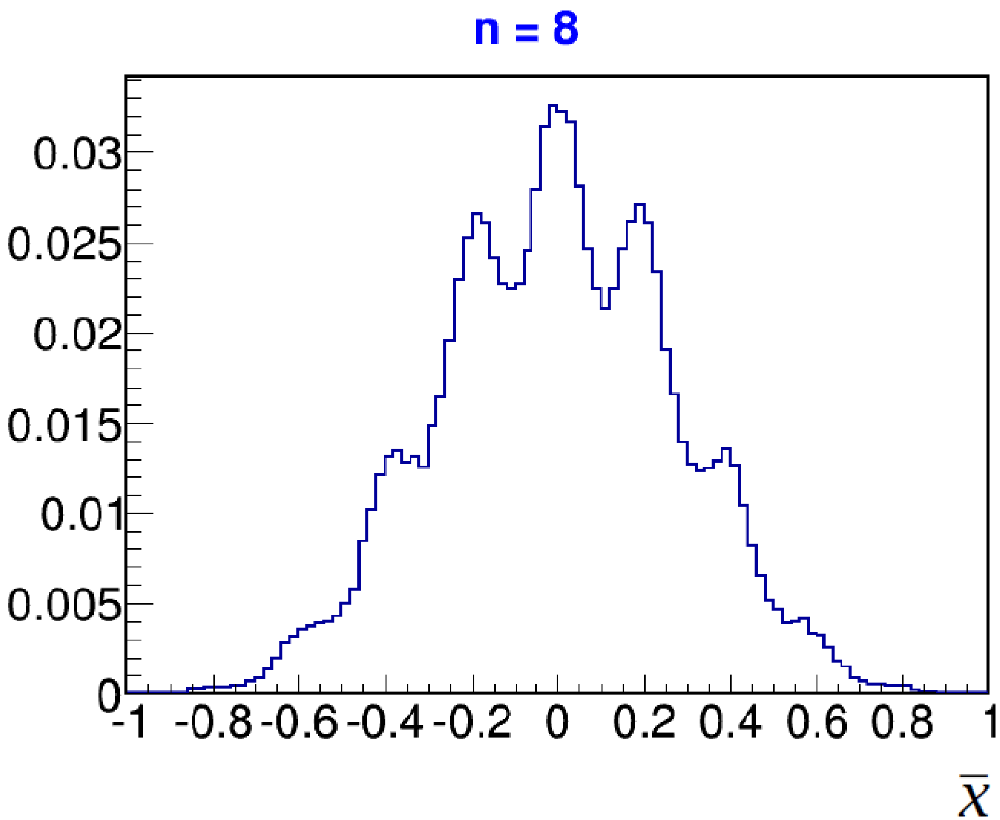

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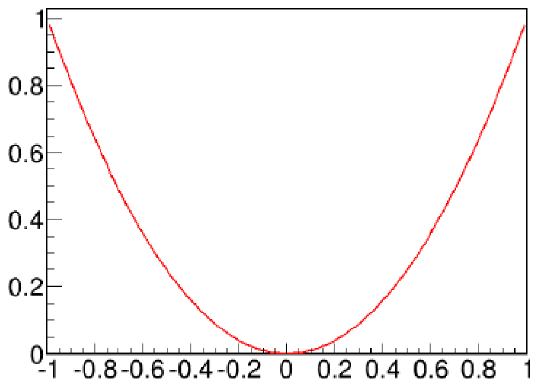

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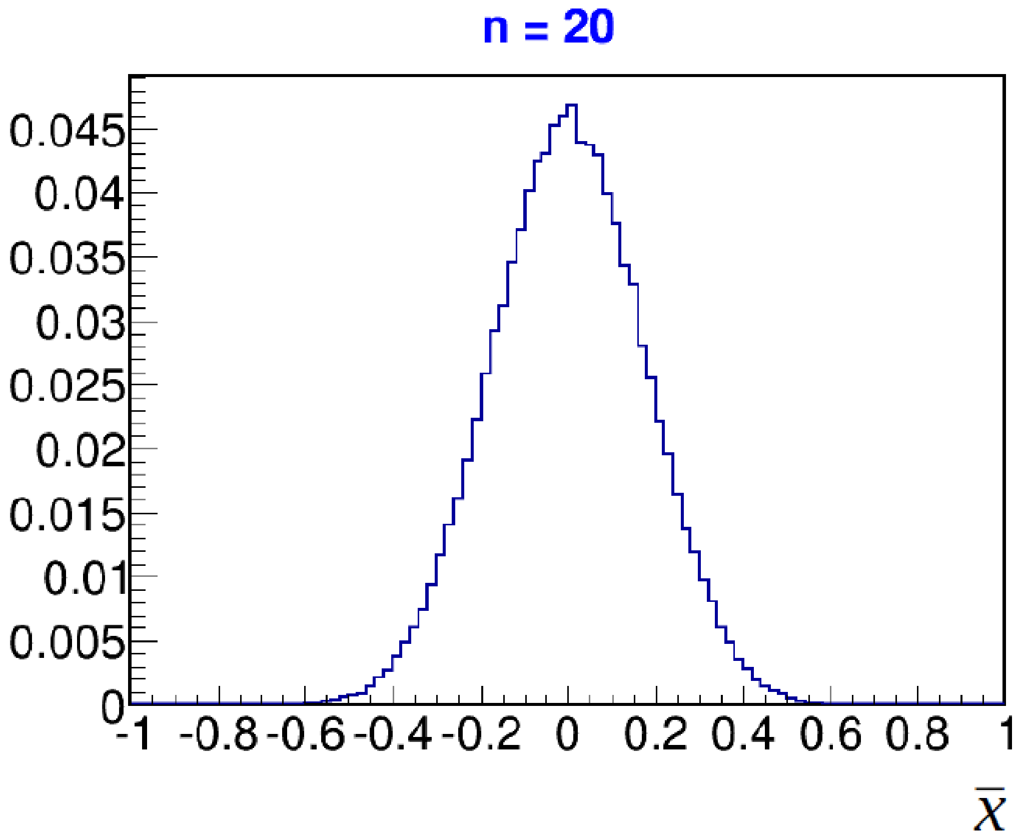

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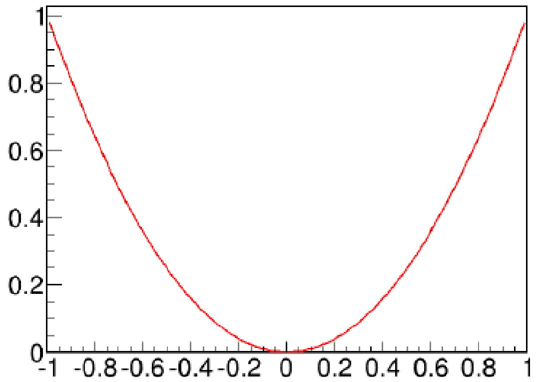
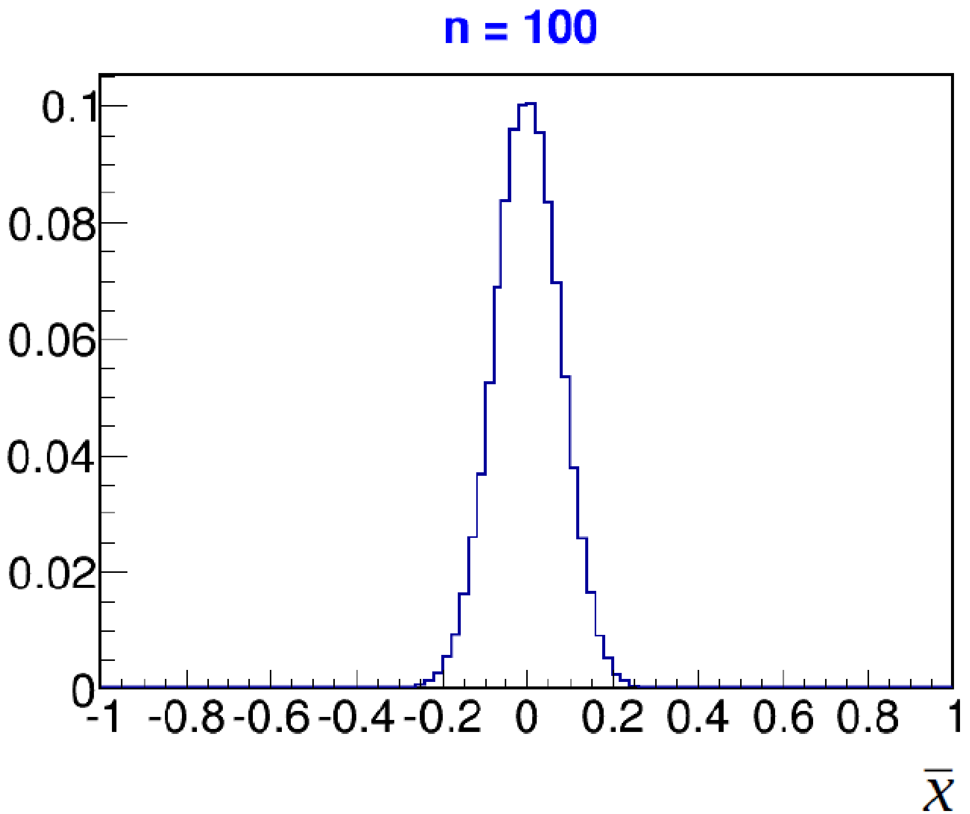
Central Limit Theorem in action

Draw events from a parabolic distribution (e.g. decay cos θ*)



$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$



Distribution becomes Gaussian, although very non-Gaussian originally

Distribution becomes narrower as expected (as 1/√n)

Chi-squared

Chi-squared

Multiple Independent Gaussian variables x_i: Define

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{x_i - x_i^0}{\sigma_i} \right)^2$$

Measures global distance from reference point (x_1^0 …. x_n^0)

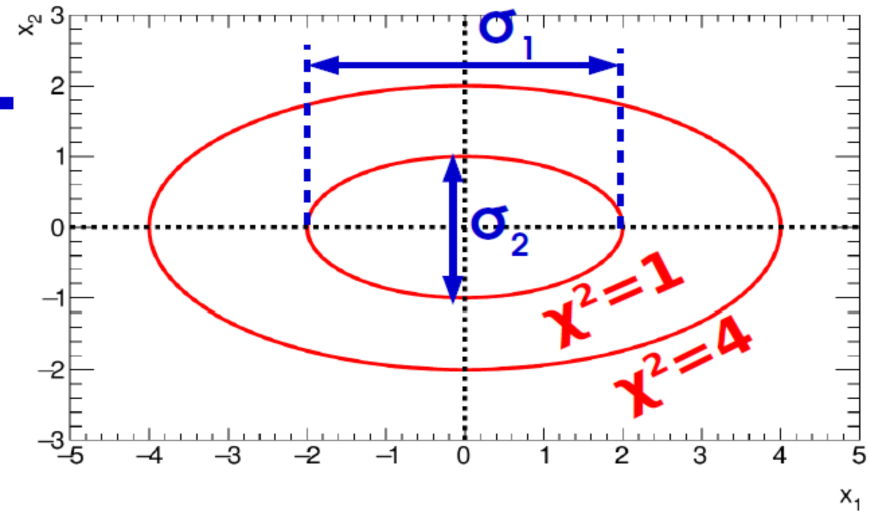
Distribution depends on n :

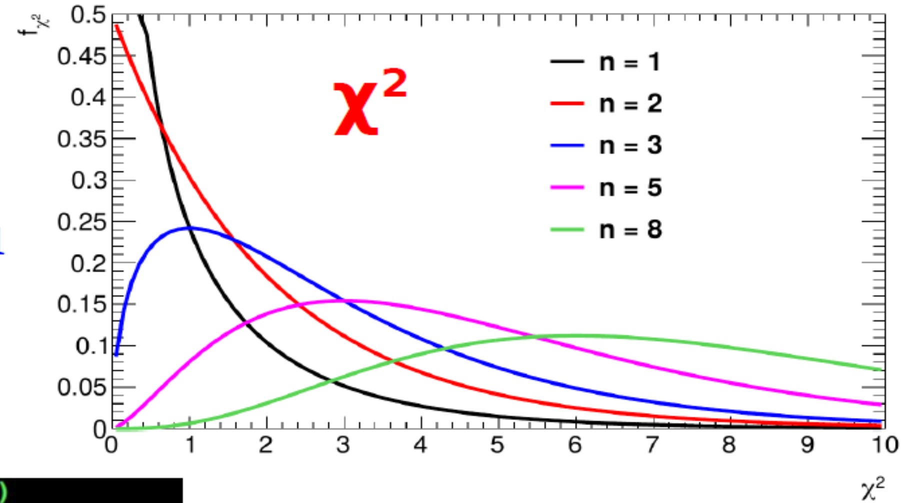
Rule of thumb: χ^2/n **should be** $\lesssim 1$

Exact distributions in ROOT:

```
ROOT::Math::chisquared_pdf(x, n)
ROOT::Math::chisquared_cdf(x, n)
```

```
root [0] ROOT::Math::chisquared_cdf(1, 1)
(double) 0.68268949
root [1] ROOT::Math::chisquared_cdf(4, 1)
(double) 0.95449974
```





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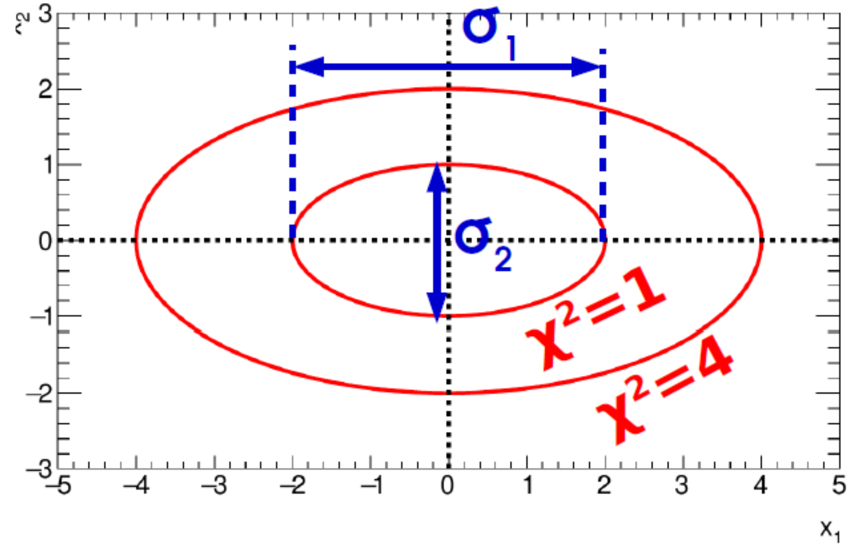
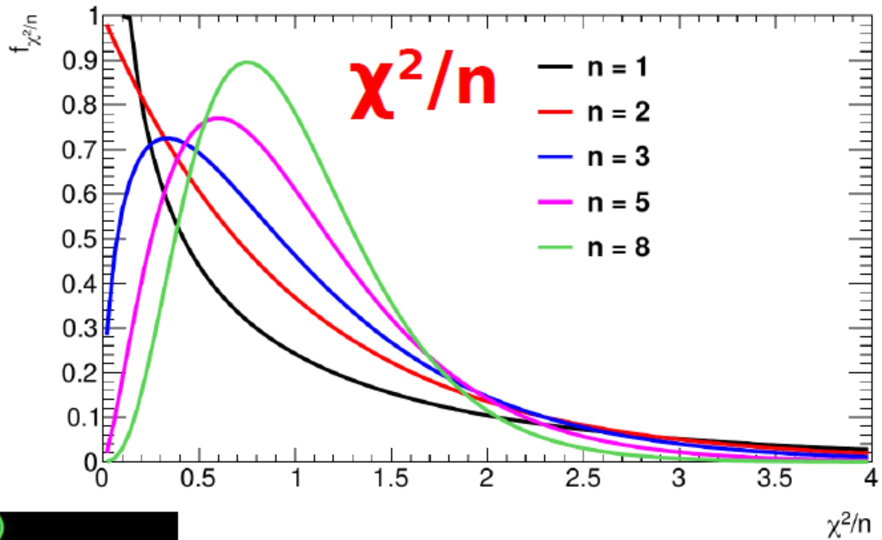
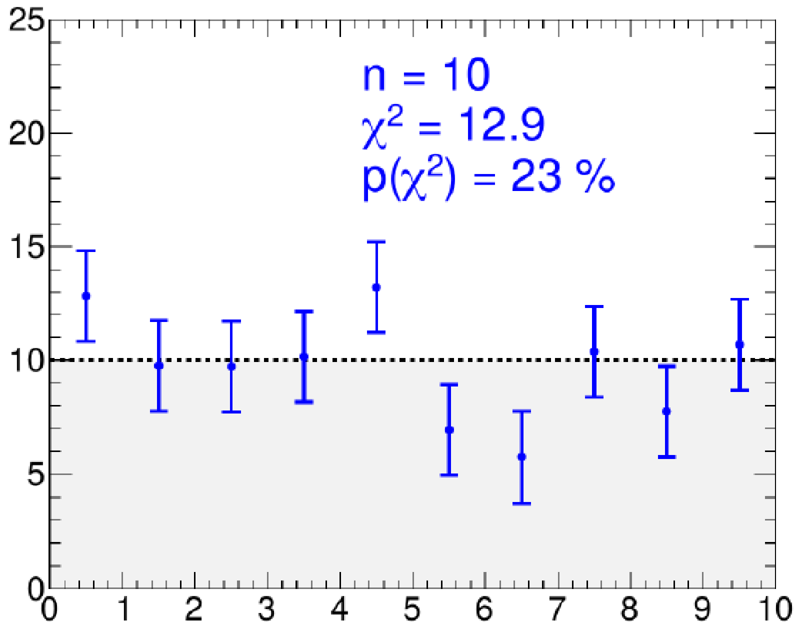




Histogram Chi-squared

Histogram χ2 with respect to a reference shape:

- Assume an independent Gaussian distribution in each bin
- Degrees of freedom = (number of bins) – (number of fit parameters)



BLUE histogram vs. flat reference

$\chi^2 = 12.9$, $p(\chi^2=12.9, n=10) = $ **23%** ✔

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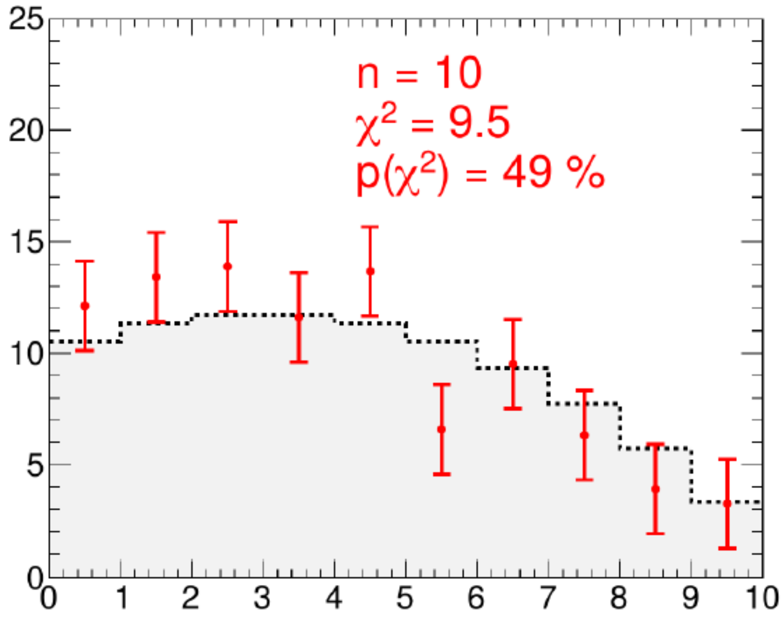


BLUE histogram vs. flat reference

$\chi^2 = 12.9$, $p(\chi^2=12.9, n=10) =$ **23%** ✔

RED histogram vs. flat reference

$\chi^2 = 38.8$, $p(\chi^2=38.8, n=10) =$ **0.003%** ✘

RED histogram vs. correct reference

$\chi^2 = 9.5$, $p(\chi^2=9.5, n=10) =$ **49%** ✔

ROOT commands:

```
root [0] ROOT::Math::chisquared_cdf_c(12.9, 10)
(double) 0.22931681
root [1] ROOT::Math::chisquared_cdf_c(38.8, 10)
(double) 2.7519383e-05
```

Error Bars

Strictly speaking, ***the uncertainty is given by the model :***

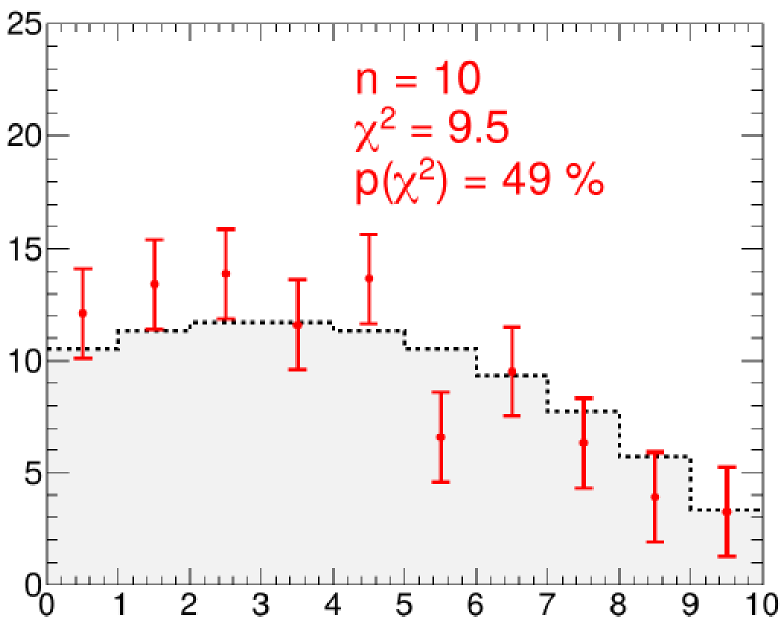
→ ***Bin central value*** ~ *mean of the bin PDF*

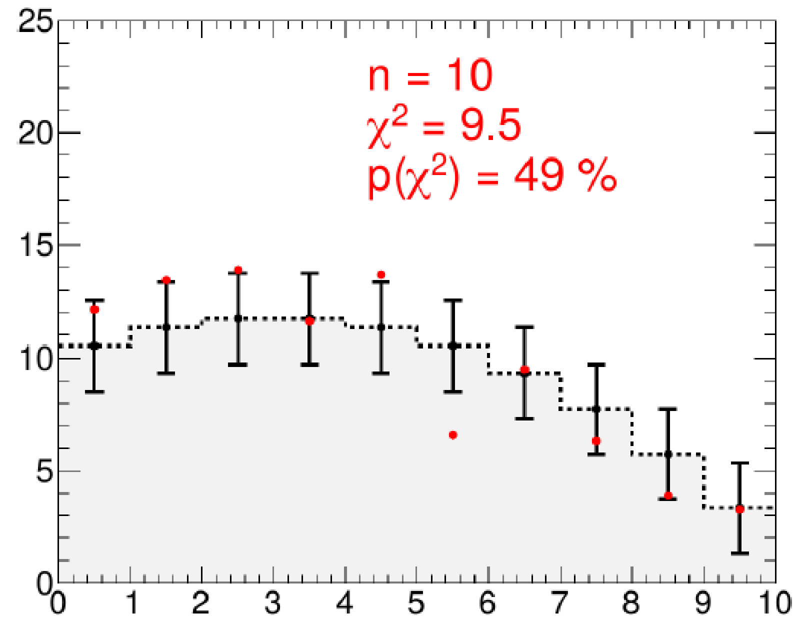
→ ***Bin uncertainty*** ~ *RMS of the bin PDF*

The data is just what it is, a simple observed point.

⇒ One should in principle **show the error bar on the prediction.**

→ In practice, the usual convention is to have **error bars on the data points**.





Example analyses

Example 1: $Z \to ee$ Inclusive σ^{fid}

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Measurement Principle:

Signal events	$34865 \pm 187 \pm 7 \pm 3$
Correction C	$0.552\,^{+0.006}_{-0.005}$
σ^{fid} [nb]	$0.781 \pm 0.004 \pm 0.008 \pm 0.016$

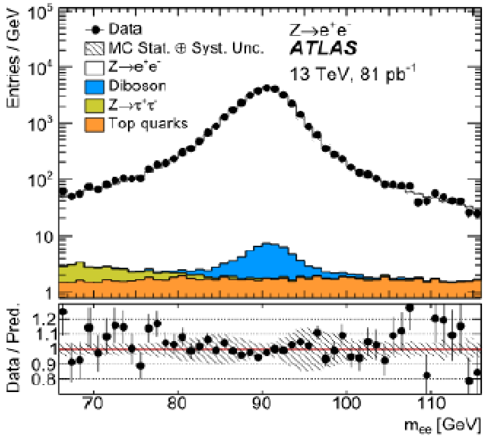
35000 ± 187

175 ± 8

$$\sigma^{fid} = \frac{n_{data} - N_{bkg}}{C_{fid}\, L}$$

(81 ± 2) pb^{-1}

0.552 ± 0.006



Simple uncertainty propagation:

σ^{fid} = 0.781 ± 0.004 (stat) ± 0.008 (syst) ± 0.016 (lumi) nb

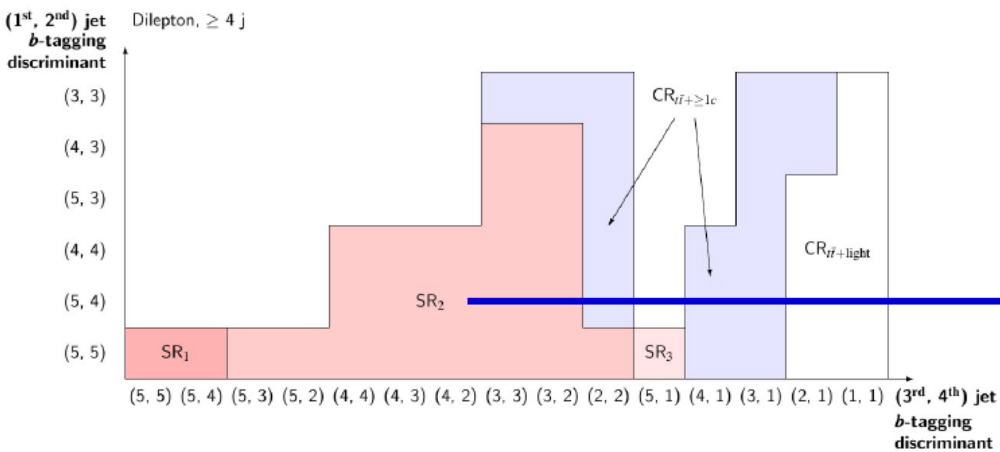
→ **Simplest possible example** in several ways (from the Statistics point of view!)

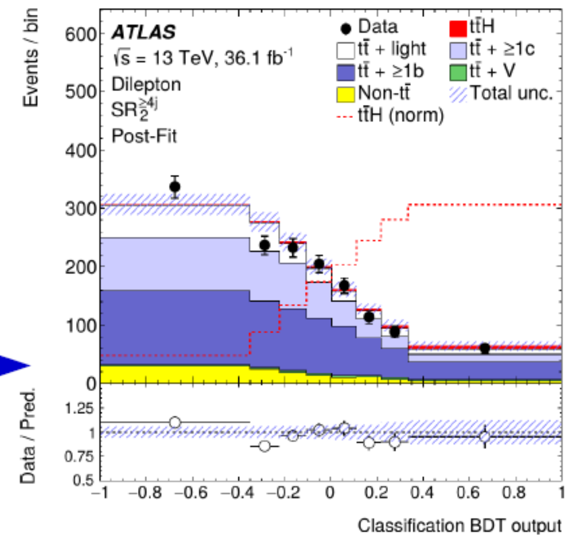
→ **"Single bin counting"** : only data input is $\mathbf{n}_{data}$.

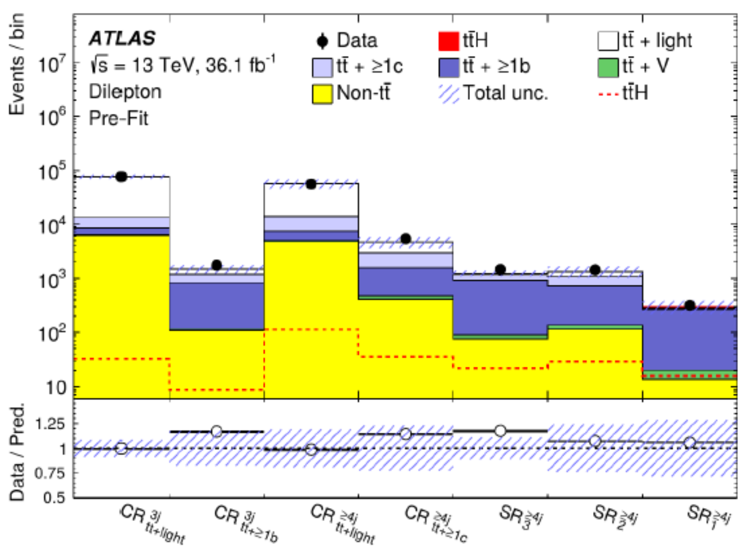
Example analyses

Example 2: ttH→bb

arXiv:1712.008895







Event counting in different regions:

Multiple-bin counting

Lots of information available

→ Potentially higher sensitivity

→ How to make optimal use of it ?

Example analyses

Example 3: Unbinned shape analysis

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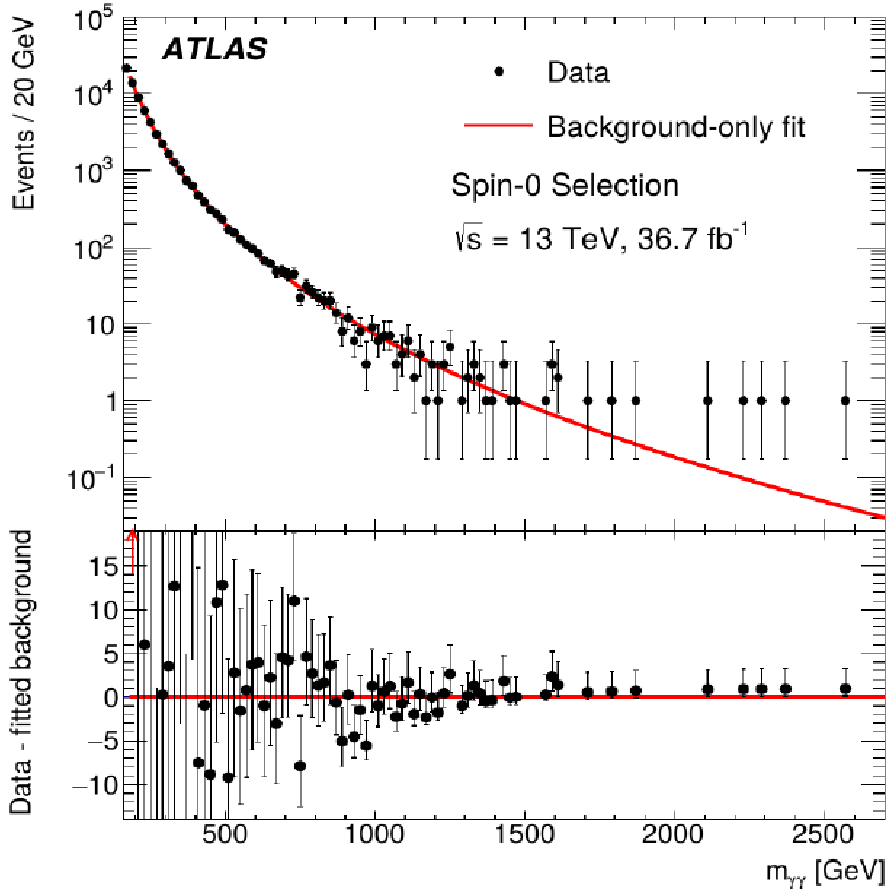


Describe spectrum without discrete binning
→ use smooth functions of a continuous variable.

Unbinned shape analysis

→ No binning effects
→ Use all available information

→ How to describe the shapes ?

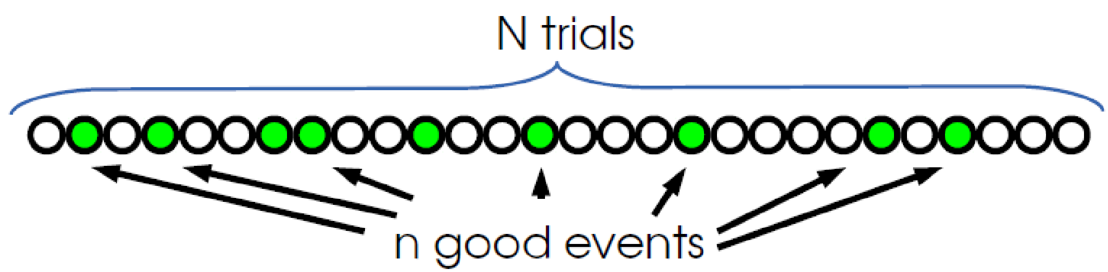
Counting events

Consider N total events, select ***good*** events with probability p.
Probability to get **n good events** ?

Binomial distribution : $P(n;N,p)=C_N^n\, p^n (1-p)^{N-n}$

Mean = **N·p**

Variance = **N·p(1 - p)**



However suppose **p ≪ 1, N ≫ 1**, and let **λ = N·p :**

→ *i.e.* **very rare** process, but **very many trials** so still expect to see good events

Poisson distribution: $P(n;\lambda)=e^{-\lambda}\dfrac{\lambda^n}{n!}$

$(1-p)^{N-n} \overset{n\ll N}{\simeq} \left(1-\dfrac{\lambda}{N}\right)^N \overset{N\gg 1}{\simeq} e^{-\lambda}$

Mean = **λ**

Variance = **λ** ⇒ **RMS = √λ**

For n expected events, the uncertainty is √n

Rare processes ?

HEP : almost always use Poisson distributions. Why ?

ATLAS :

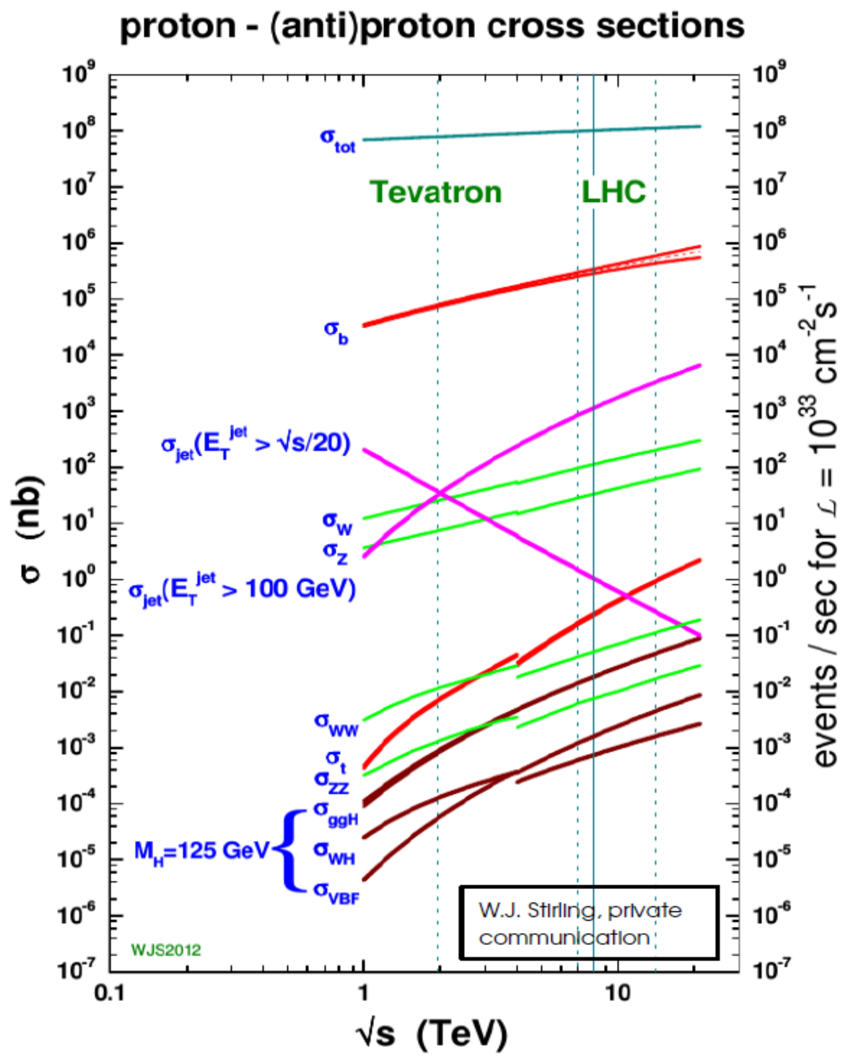
- **Event rate ~ 1 GHz**
  ($L \sim 10^{34}\ cm^{-2}s^{-1} \sim 10\ nb^{-1}/s$, $\sigma_{tot} \sim 10^{8}$ nb,)
- **Trigger rate ~ 1 kHz**
  (Higgs rate ~ **0.1 Hz**)

$\Rightarrow p \sim 10^{-6} \ll 1$ ($p_{H\to\gamma\gamma} \sim 10^{-13}$)

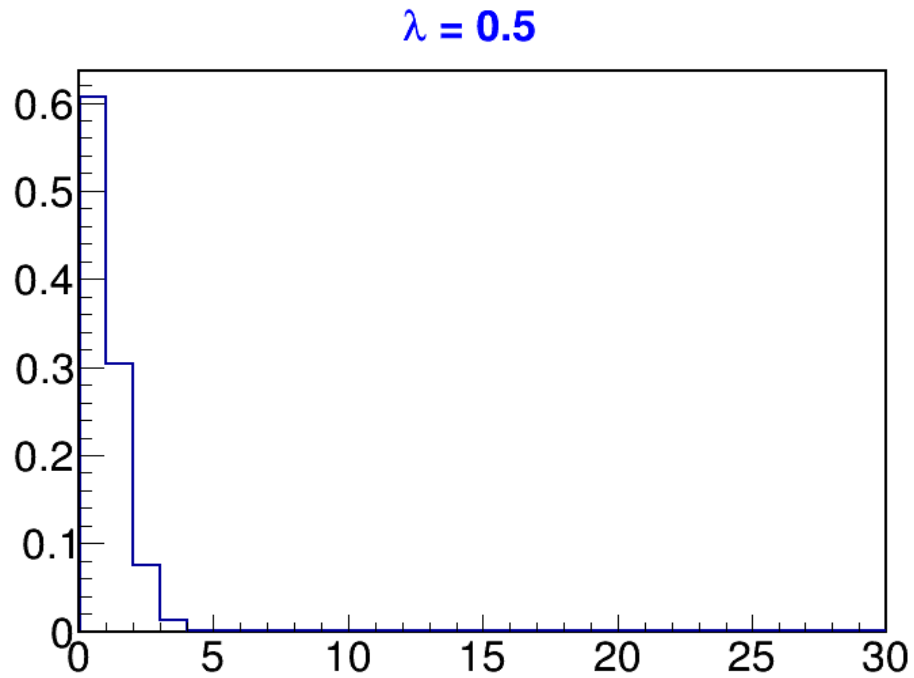
A day of data: $N \sim 10^{14} \gg 1$

⇒ **Poisson regime!**

(Large N = design requirement, to get not-too-small λ=Np...)



Poisson distributions



$$P(n;\lambda) = e^{-\lambda} \frac{\lambda^n}{n!}$$

λ : expected number of events

Mean = λ

Variance = λ

σ = √λ

- **Discrete distribution** (positive integers only), **asymmetric for small λ**
- Typical variation (RMS) of n events is **√n**
- Central limit theorem : becomes **Gaussian for large λ** :

$$P(\lambda) \xrightarrow{\lambda \to \infty} G(\lambda, \sqrt{\lambda})$$

Poisson distributions

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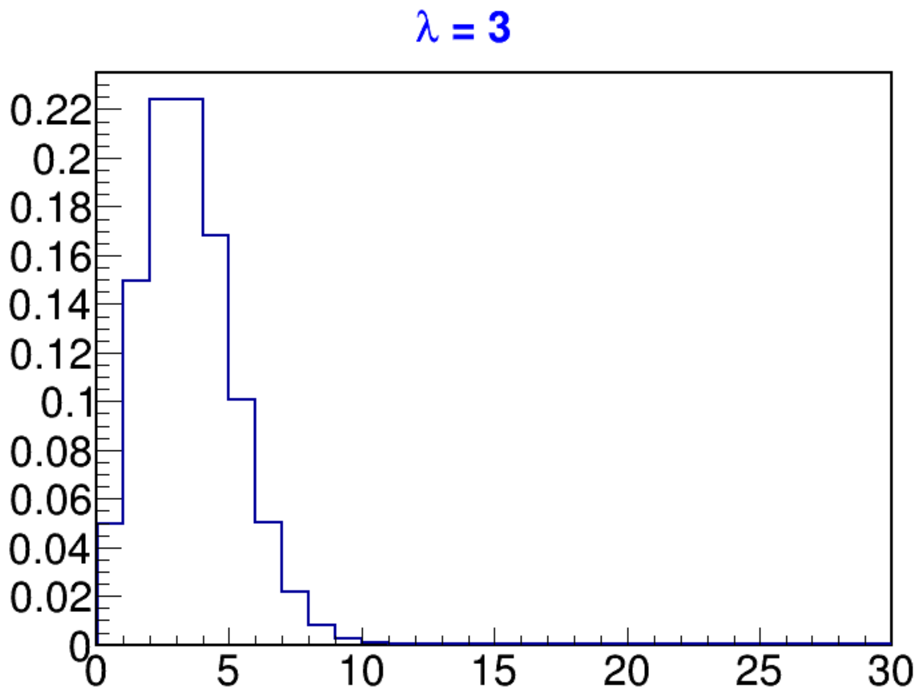

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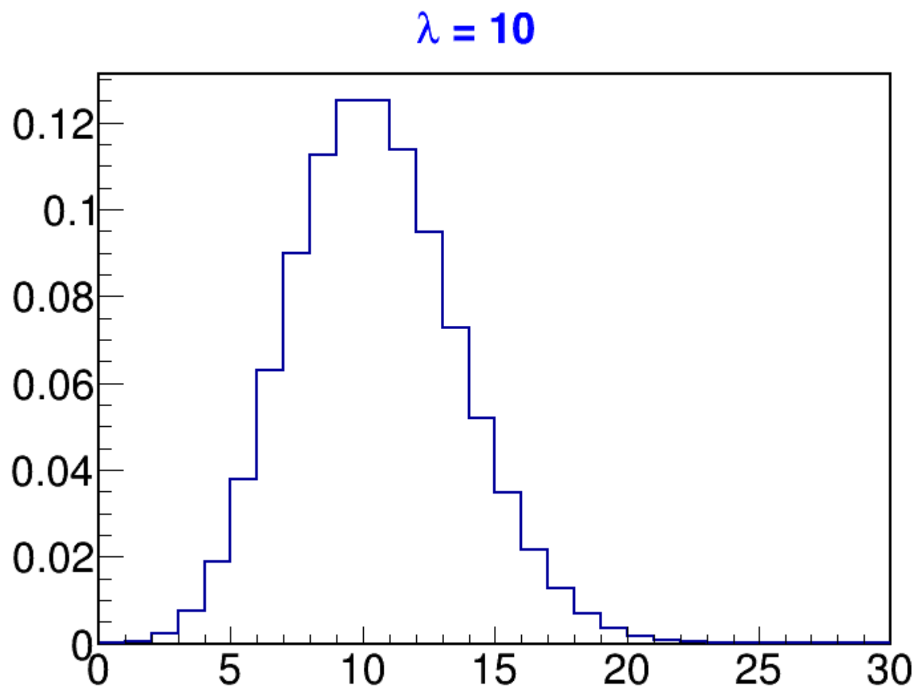

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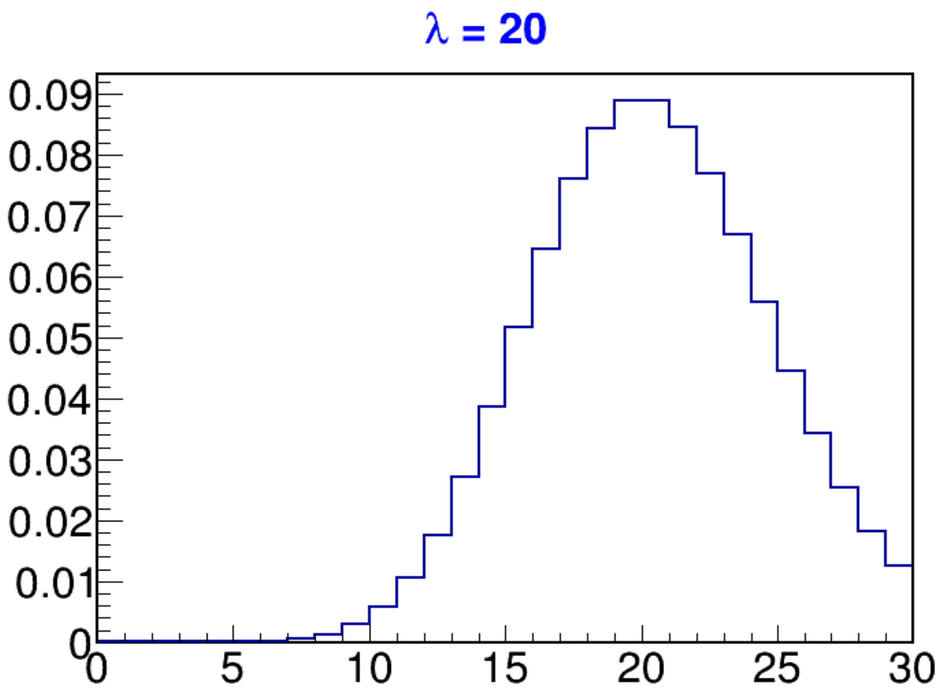

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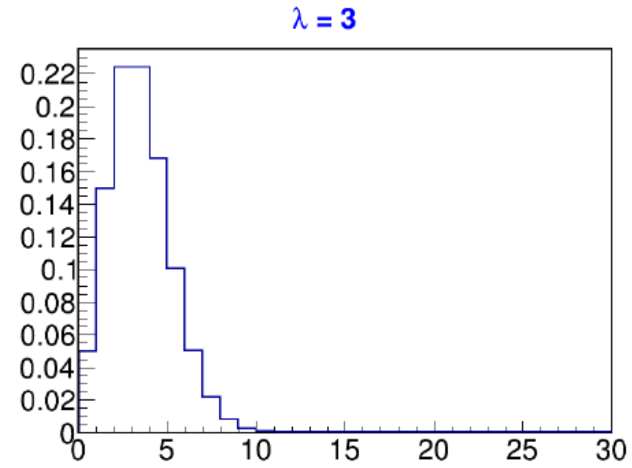
Statistical model for counting



Counting experiment:

Observable: number of events n

→ describe by a **Poisson distribution**

$$P(n;\lambda) = e^{-\lambda}\frac{\lambda^n}{n!}$$

Typically both signal and background expected:

$$P(n;S,B)=e^{-(S+B)}\frac{(S+B)^n}{n!}$$

S : # of events from signal process

B : # of events from bkg. process(es)

We have **assumed** a Poisson distribution for n : This is our model, based on physics knowledge (but usually a very safe one).

Model has **parameters S** and **B.** B can be known a priori or not (S usually not...)

→ Example: can **assume B is known,** use the **measured n** to find out about the **parameter S.**

└→ usually up to uncertainties → **systematics**

Z->ee inclusive σ^{fid}

Measurement Principle:

35000 ± (√35000 = 187)

175 ± 8

$$\sigma^{fid} = \frac{n_{data} - N_{bkg}}{C_{fid}\, L}$$

(81 ± 2) pb^{-1}

0.552 ± 0.006

Signal events	34865 ± 187 ± 7 ± 3
Correction C	$0.552^{+0.006}_{-0.005}$
σ^{fid}[nb]	0.781 ± 0.004 ± 0.008 ± 0.016

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Simple uncertainty propagation:

σ^{fid} = 0.781 ± 0.004 (stat) ± 0.008 (syst) ± 0.016 (lumi) nb

Statistical uncertainty: Derived from assumption that n_{data} is ~ Poisson(S+B)

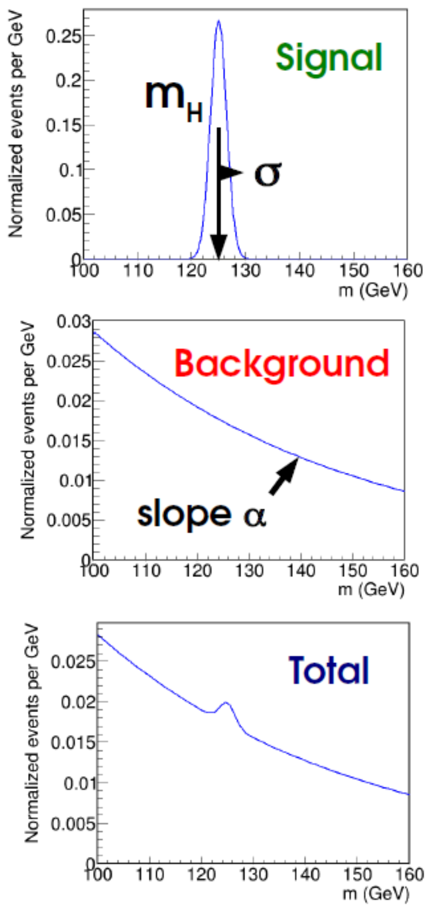
Systematics: more on this in Lecture 3

Unbinned shape analysis

Observable: set of values $m_1 \ldots m_n$, **one per event**

→ Describe shape of the ***distribution of m***

→ Deduce the **probability to observe $m_1 \ldots m_n$**



H→γγ-inspired example:

- **Gaussian signal** $P_{sig}(m) = G(m; m_H, \sigma)$
- **Exponential bkg** $P_{bkg}(m) = \alpha\, e^{-\alpha m}$

⇒ **Total PDF for a single event**:

Expected yields : **S, B**

$$P_{total}(m) = \frac{S}{S+B} G(m; m_H, \sigma) + \frac{B}{S+B} \alpha\, e^{-\alpha m}$$

⇒ **Total PDF for a dataset**

Probability to observe n events

Probability to observe the value m_i

$$P(\{m_i\}_{i=1\ldots n}) = e^{-(S+B)} \frac{(S+B)^n}{n!} \prod_{i=1}^{n} \frac{S}{S+B} G(m_i; m_H, \sigma) + \frac{B}{S+B} \alpha\, e^{-\alpha m_i}$$

Unbinned shape analysis

H→γγ

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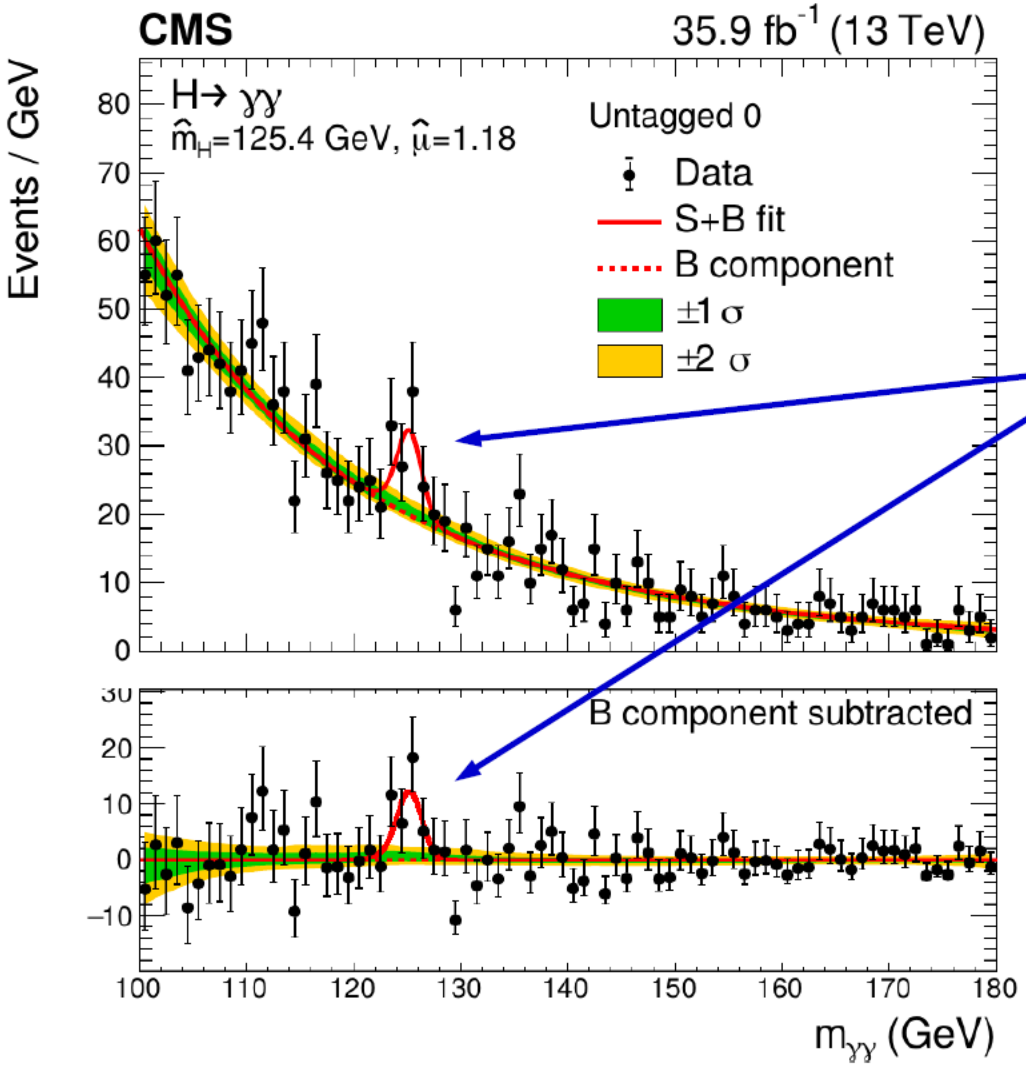
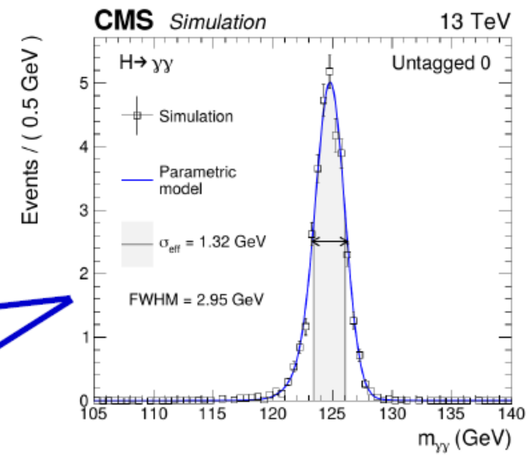




Binned shape analysis

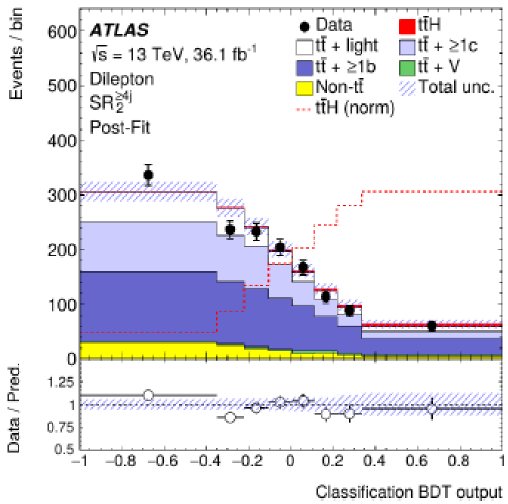
Instead of using $m_1 \ldots m_n$ directly, can build a ***histogram*** $n_1 \ldots n_N$.

→ N_{bins} : number of bins

Per-bin fractions (=shapes) of Signal and Background

$$P(\{n_i\}; S, B) = \prod_{i=1}^{N_{bins}} e^{-(S f_{S,i} + B f_{B,i})} \frac{(S f_{S,i} + B f_{B,i})^{n_i}}{n_i!}$$

Poisson distribution in each bin



N_{bins}=1: Counting analysis

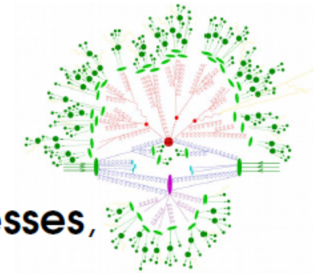
$N_{bins}\rightarrow\infty$: Unbinned shape analysis (the fractions become PDF values)

Shapes specified through $f_{S,i}$, $f_{B,i}$ rather than $P_{signal}(m)$, $P_{bkg}(m)$

⊕ **Obtained directly from MC**, no need to define continuous PDFs.

⊖ **MC stat fluctuations** can create artefacts, especially for S≪B.

How to describe data



Physics measurement data are produced through **random processes**,
Need to be described using a statistical model:

Description	Observable	Likelihood
Counting	n	**Poisson** $P(n;S,B) = e^{-(S+B)}\frac{(S+B)^n}{n!}$
Binned shape analysis	n_i, $i=1..N_{bins}$	**Poisson product** $P(n_i;S,B) = \prod_{i=1}^{N_{bins}} e^{-(S f_i^{sig} + B f_i^{bkg})}\frac{(S f_i^{sig} + B f_i^{bkg})^{n_i}}{n_i!}$
Unbinned shape analysis	m_i, $i=1..n_{evts}$	**Extended Unbinned Likelihood** $P(m_i;S,B) = \frac{e^{-(S+B)}}{n_{evts}!}\prod_{i=1}^{n_{evts}} S\, P_{sig}(m_i) + B\, P_{bkg}(m_i)$

Model can include multiple **categories**, each with a separate description