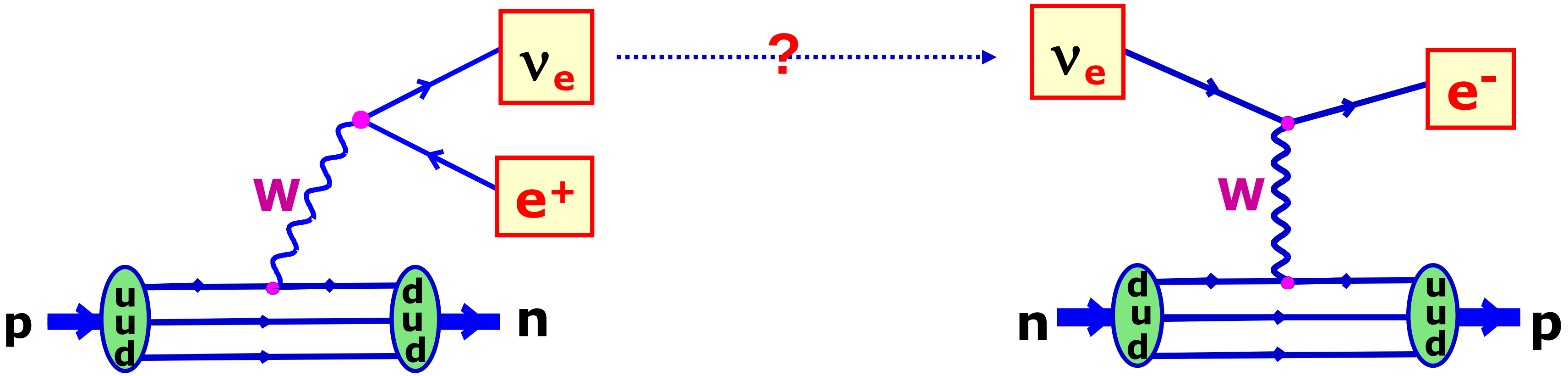
NEUTRINO FLAVOURS REVISITED

★ Never **directly** observe neutrinos – can only detect them by their weak interactions. Hence by **definition** ν_e is the neutrino state produced along with an electron. Similarly, charged current weak interactions of the state ν_e produce an electron

$$\nu_e, \nu_\mu, \nu_\tau \quad = \text{weak eigenstates}$$

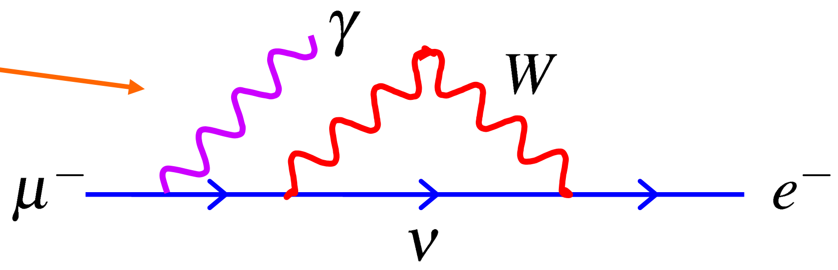
★ For many years, assumed that ν_e, ν_μ, ν_τ were massless fundamental particles

- **Experimental evidence:** neutrinos produced along with an electron always produced an electron in CC Weak interactions, etc.



- **Experimental evidence:** absence $\mu^- \rightarrow e^- \gamma$ $\quad \mathrm{BR}(\mu^- \rightarrow e^- \gamma) < 10^{-11}$

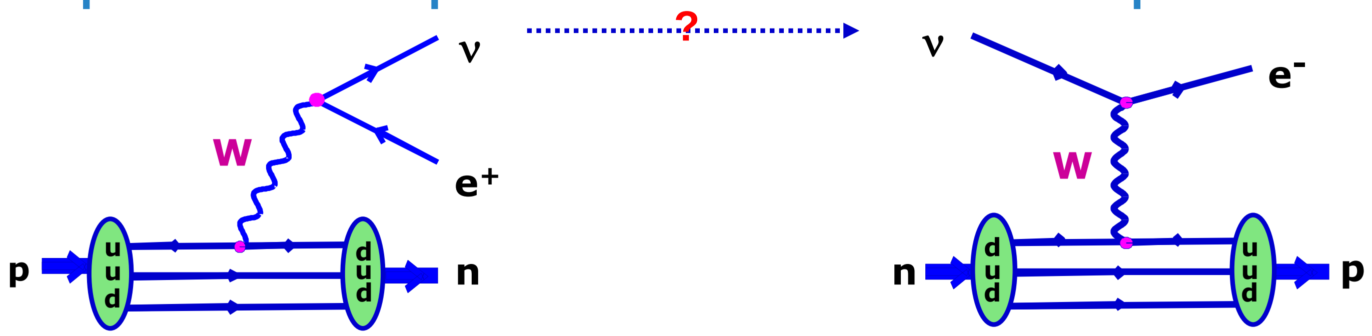
Suggests that ν_e and ν_μ are distinct particles otherwise decay could go via:



MASS EIGENSTATES AND WEAK EIGENSTATES

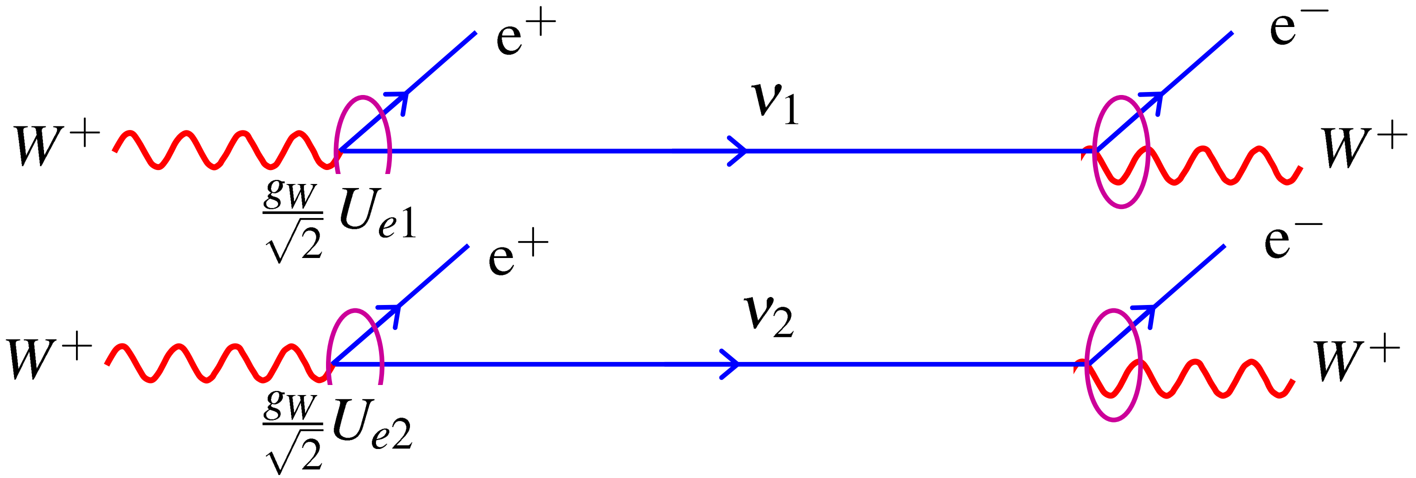
★ The essential feature in understanding the physics of neutrino oscillations is to understand what is meant by weak eigenstates and mass eigenstates ν_1, ν_2

★ Suppose the process below proceeds via two fundamental particle states



i.e.

and



★ Can' t know which **mass eigenstate** (fundamental particle ν_1, ν_2) was involved

★ In Quantum mechanics treat as a coherent state $\psi = \nu_e = U_{e1}\nu_1 + U_{e2}\nu_2$

★ ν_e represents the wave-function of the coherent state produced along with an electron in the weak interaction, i.e. the **weak eigenstate**

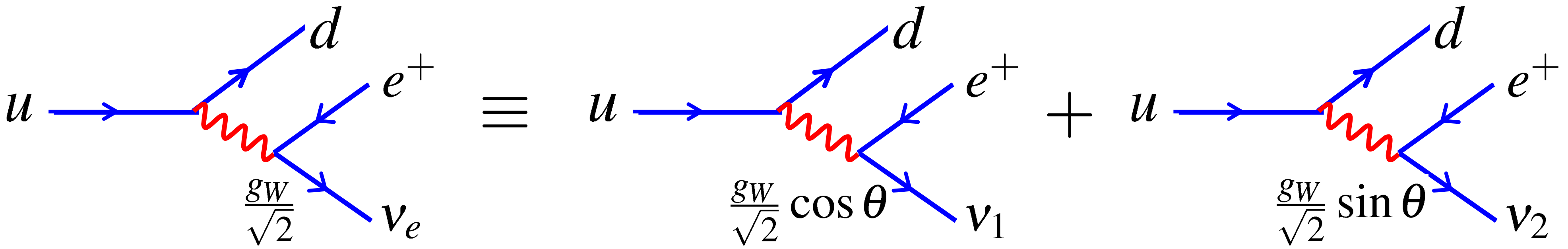
NEUTRINO OSCILLATIONS FOR TWO FLAVOURS

- ★ Neutrinos are produced and interact as weak eigenstates, ν_e, ν_μ
- ★ The weak eigenstates as coherent linear combinations of the fundamental "mass eigenstates" ν_1, ν_2
- ★ The mass eigenstates are the free particle solutions to the wave-equation and will be taken to propagate as plane waves

$$|\nu_1(t)\rangle = |\nu_1\rangle e^{i\vec{p}_1.\vec{x}-iE_1t} \qquad |\nu_2(t)\rangle = |\nu_2\rangle e^{i\vec{p}_2.\vec{x}-iE_2t}$$

- ★ The weak and mass eigenstates are related by the unitary 2x2 matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \qquad (1)$$



- ★ Equation (1) can be inverted to give

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \qquad (2)$$

- Suppose at time $t=0$ a neutrino is produced in a pure ν_e state, e.g. in a decay $u \to d e^+ \nu_e$

$$|\psi(0)\rangle = |\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

- Take the z-axis to be along the neutrino direction
- The wave-function evolves according to the time-evolution of the **mass eigenstates** (free particle solutions to the wave equation)

$$|\psi(t)\rangle = \cos\theta|\nu_1\rangle e^{-ip_1.x} + \sin\theta|\nu_2\rangle e^{-ip_2.x}$$

where $p_i.x = E_i t - \vec{p}_i.\vec{x} = E_i t - |\vec{p}_i| z$

- Suppose the neutrino interacts in a detector at a distance **L** and at a time **T**

$$\phi_i = p_i.x = E_i T - |\vec{p}_i| L$$

gives

$$|\psi(L,T)\rangle = \cos\theta|\nu_1\rangle e^{-i\phi_1} + \sin\theta|\nu_2\rangle e^{-i\phi_2}$$

★ Expressing the mass eigenstates, $|\nu_1\rangle, |\nu_2\rangle$, in terms of weak eigenstates (eq 2)

$$|\psi(L,T)\rangle = \cos\theta(\cos\theta|\nu_e\rangle - \sin\theta|\nu_\mu\rangle)e^{-i\phi_1} + \sin\theta(\sin\theta|\nu_e\rangle + \cos\theta|\nu_\mu\rangle)e^{-i\phi_2}$$

$$|\psi(L,T)\rangle = |\nu_e\rangle(\cos^2\theta e^{-i\phi_1} + \sin^2\theta e^{-i\phi_2}) + |\nu_\mu\rangle \sin\theta\cos\theta(-e^{-i\phi_1} + e^{-i\phi_2})$$

★ If the masses of $|\nu_1\rangle, |\nu_2\rangle$ **are the same**, the mass eigenstates remain in phase, $\phi_1 = \phi_2$, and the state remains the linear combination corresponding to $|\nu_e\rangle$ and in a weak interaction will produce an electron

★ If the masses are different, the wave-function no longer remains a pure $|\nu_e\rangle$

$$
\begin{aligned}
P(\nu_e \to \nu_\mu) &= |\langle \nu_\mu | \psi(L,T)\rangle|^2 \\
&= \cos^2\theta \sin^2\theta(-e^{-i\phi_1} + e^{-i\phi_2})(-e^{+i\phi_1} + e^{+i\phi_2}) \\
&= \frac{1}{4}\sin^2 2\theta(2 - 2\cos(\phi_1 - \phi_2)) \\
&= \sin^2 2\theta \sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)
\end{aligned}
$$

$$\frac{{}^2_2L}{E_2} - \frac{m_1^2 L}{4E_1} \approx \frac{(m_2^2 - m_1^2)L}{4E}$$

★ The treatment of the phase difference

$$\Delta\phi_{12} = \phi_1 - \phi_2 = (E_1 - E_2)T - (|p_1| - |p_2|)L$$

in most text books is dubious. Here we will be more careful…

★ One could assume $|p_1| = |p_2| = \mathrm{p}$ in which case

$$\Delta\phi_{12} = (E_1 - E_2)T = \left[(\mathrm{p}^2 + m_1^2)^{1/2} - (\mathrm{p}^2 + m_2^2)^{1/2}\right] L \qquad L \approx (c)T$$

$$\Delta\phi_{12} = \mathrm{p}\left[\left(1+\frac{m_1^2}{\mathrm{p}^2}\right)^{1/2} - \left(1+\frac{m_2^2}{\mathrm{p}^2}\right)^{1/2}\right]L \approx \frac{m_1^2 - m_2^2}{2\mathrm{p}}L$$

★ However we have neglected that fact that for the same momentum, different mass eigenstates will propagate at different velocities and be observed at different times

★ The full derivation requires a wave-packet treatment and gives the same result

★ Nevertheless it is worth noting that the phase difference can be written

$$\Delta\phi_{12} = (E_1 - E_2)T - \left(\frac{|p_1|^2 - |p_2|^2}{|p_1| + |p_2|}\right)L$$

$$\Delta\phi_{12} = (E_1 - E_2)\left[T - \left(\frac{E_1 + E_2}{|p_1| + |p_2|}\right)L\right] + \left(\frac{m_1^2 - m_2^2}{|p_1| + |p_2|}\right)L$$

★ The first term on the RHS vanishes if we assume $E_1 = E_2$ or $\beta_1 = \beta_2$

in all cases

$$\Delta\phi_{12} = \frac{m_1^2 - m_2^2}{2\mathrm{p}}L = \frac{\Delta m^2}{2E}\,\mathbf{L}$$

★ **Hence the two-flavour oscillation probability is:**

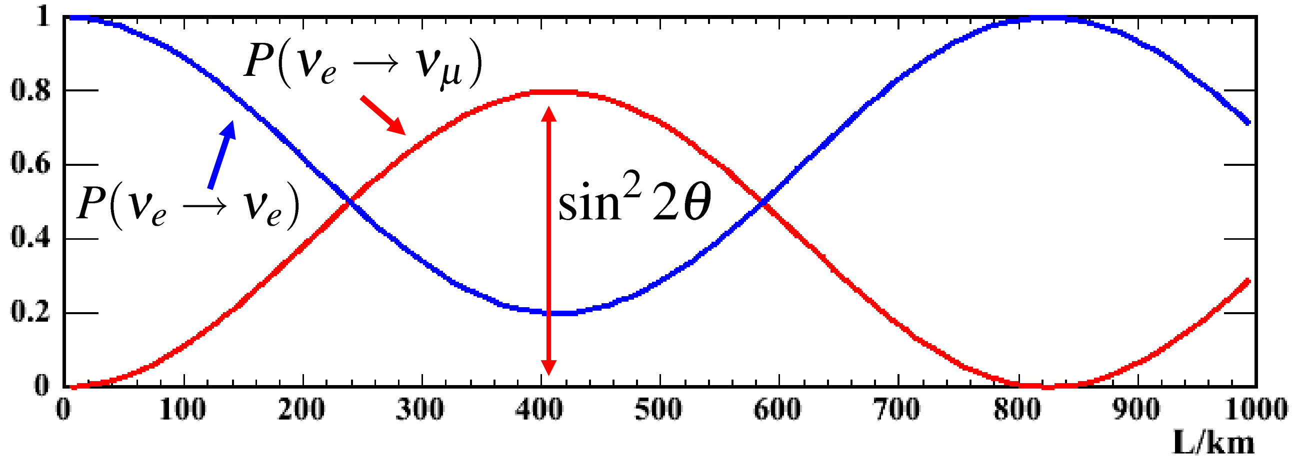
$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

with $\Delta m^2_{21} = m_2^2 - m_1^2$

★ **The corresponding two-flavour survival probability is:**

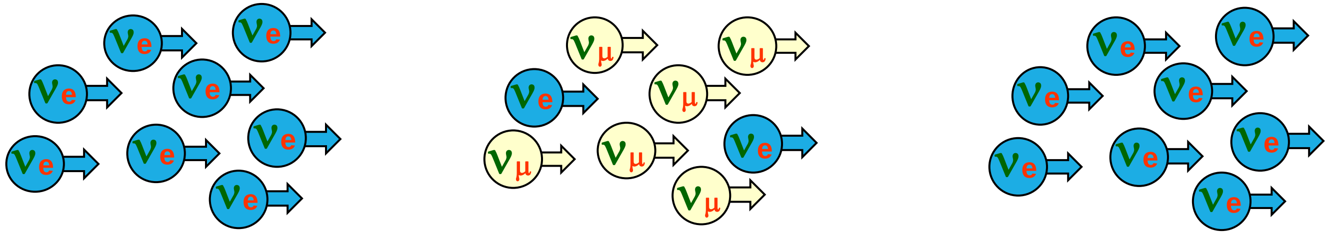
$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

- **e.g.** $\Delta m^2 = 0.003\,\mathrm{eV}^2, \quad \sin^2 2\theta = 0.8, \quad E_\nu = 1\,\mathrm{GeV}$



- **wavelength**

$$\lambda_{\mathrm{osc}} = \frac{4\pi E}{\Delta m^2}$$

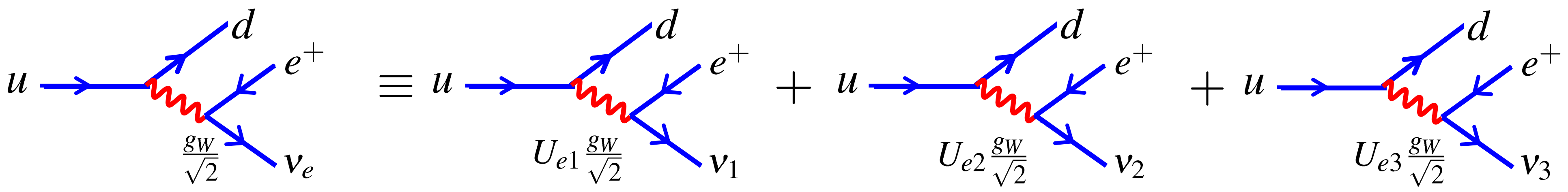


NEUTRINO OSCILLATIONS FOR THREE FLAVOURS

★ It is simple to extend this treatment to three generations of neutrinos.

★ In this case we have:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



★ The 3x3 Unitary matrix U is known as the Pontecorvo-Maki-Nakagawa-Sakata matrix, usually abbreviated **PMNS**

★ Note : has to be unitary to conserve probability

•Using $U^\dagger U = I \quad \Rightarrow \quad U^{-1} = U^\dagger = (U^*)^T$

gives

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U^*_{e1} & U^*_{\mu 1} & U^*_{\tau 1} \\ U^*_{e2} & U^*_{\mu 2} & U^*_{\tau 2} \\ U^*_{e3} & U^*_{\mu 3} & U^*_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

UNITARITY RELATIONS

★The Unitarity of the PMNS matrix gives several useful relations: $UU^\dagger = I \quad \Rightarrow$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} U^*_{e1} & U^*_{\mu 1} & U^*_{\tau 1} \\ U^*_{e2} & U^*_{\mu 2} & U^*_{\tau 2} \\ U^*_{e3} & U^*_{\mu 3} & U^*_{\tau 3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

gives:

$$U_{e1}U^*_{e1} + U_{e2}U^*_{e2} + U_{e3}U^*_{e3} = 1 \qquad \text{(U1)}$$

$$U_{\mu 1}U^*_{\mu 1} + U_{\mu 2}U^*_{\mu 2} + U_{\mu 3}U^*_{\mu 3} = 1 \qquad \text{(U2)}$$

$$U_{\tau 1}U^*_{\tau 1} + U_{\tau 2}U^*_{\tau 2} + U_{\tau 3}U^*_{\tau 3} = 1 \qquad \text{(U3)}$$

$$U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2} + U_{e3}U^*_{\mu 3} = 0 \qquad \text{(U4)}$$

$$U_{e1}U^*_{\tau 1} + U_{e2}U^*_{\tau 2} + U_{e3}U^*_{\tau 3} = 0 \qquad \text{(U5)}$$

$$U_{\mu 1}U^*_{\tau 1} + U_{\mu 2}U^*_{\tau 2} + U_{\mu 3}U^*_{\tau 3} = 0 \qquad \text{(U6)}$$

★To calculate the oscillation probability proceed as before…

- Consider a state which is produced at $t = 0$ as a $|\nu_e\rangle$ (i.e. with an electron)

$$|\psi(t=0)\rangle = |\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle$$

- The wave-function evolves as:

$$|\psi(t)\rangle = U_{e1}|\nu_1\rangle e^{-ip_1.x} + U_{e2}|\nu_2\rangle e^{-ip_2.x} + U_{e3}|\nu_3\rangle e^{-ip_3.x}$$

where $p_i.x = E_i t - \vec{p}_i.\vec{x} = E_i t - |\vec{p}|z$

z axis in direction of propagation

- After a travelling a distance L

$$|\psi(L)\rangle = U_{e1}|\nu_1\rangle e^{-i\phi_1} + U_{e2}|\nu_2\rangle e^{-i\phi_2} + U_{e3}|\nu_3\rangle e^{-i\phi_3}$$

where $\phi_i = p_i.x = E_i t - |\vec{p}|L = (E_i - |\vec{p}_i|)L$

- As before we can approximate

$$\phi_i \approx \frac{m_i^2}{2E_i}L$$

- Expressing the mass eigenstates in terms of the weak eigenstates

$$\begin{aligned}
|\psi(L)\rangle \quad = \quad & U_{e1}(U_{e1}^*|\nu_e\rangle + U_{\mu 1}^*|\nu_\mu\rangle + U_{\tau 1}^*|\nu_\tau\rangle)e^{-i\phi_1} \\
+ \quad & U_{e2}(U_{e2}^*|\nu_e\rangle + U_{\mu 2}^*|\nu_\mu\rangle + U_{\tau 2}^*|\nu_\tau\rangle)e^{-i\phi_2} \\
+ \quad & U_{e3}(U_{e3}^*|\nu_e\rangle + U_{\mu 3}^*|\nu_\mu\rangle + U_{\tau 3}^*|\nu_\tau\rangle)e^{-i\phi_3}
\end{aligned}$$

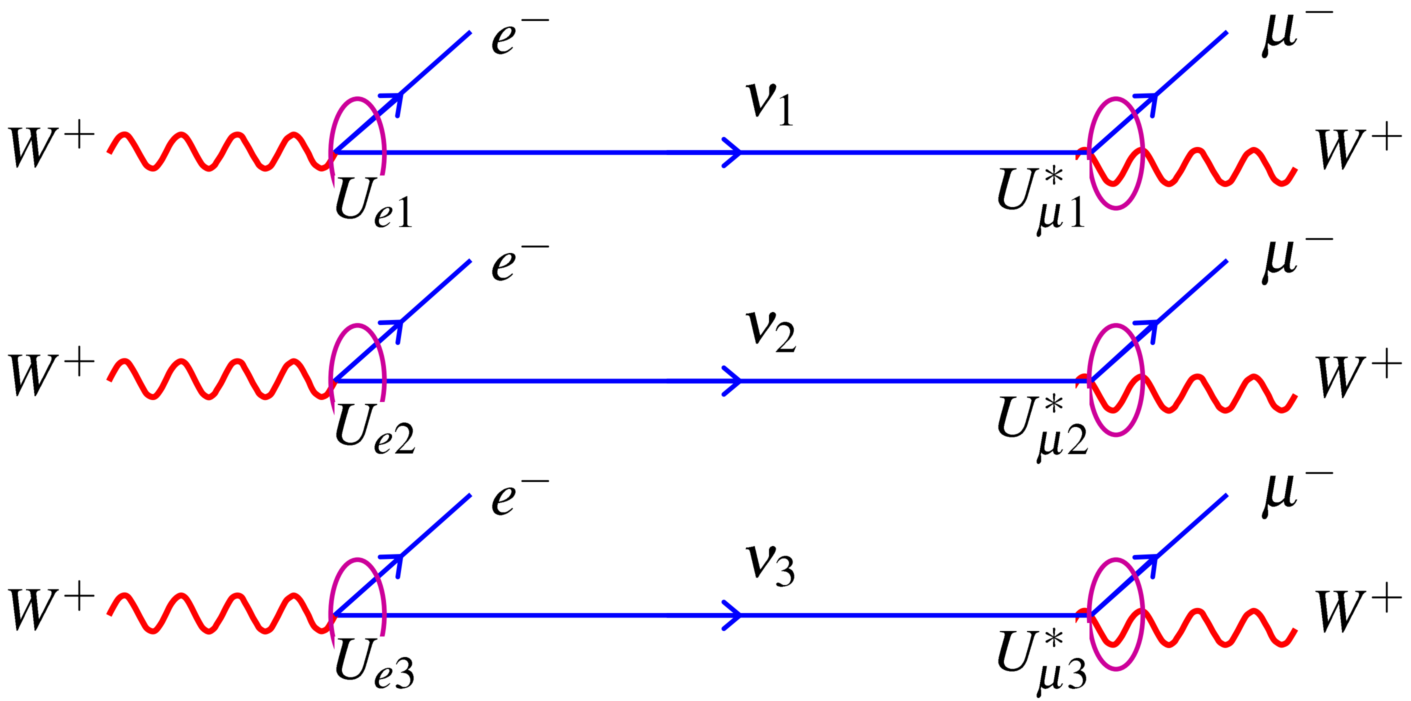
- Which can be rearranged to give

$$\begin{aligned}
|\psi(L)\rangle \quad = \quad & (U_{e1}U_{e1}^* e^{-i\phi_1} + U_{e2}U_{e2}^* e^{-i\phi_2} + U_{e3}U_{e3}^* e^{-i\phi_3})|\nu_e\rangle \\
+ \quad & (U_{e1}U_{\mu 1}^* e^{-i\phi_1} + U_{e2}U_{\mu 2}^* e^{-i\phi_2} + U_{e3}U_{\mu 3}^* e^{-i\phi_3})|\nu_\mu\rangle \qquad (3) \\
+ \quad & (U_{e1}U_{\tau 1}^* e^{-i\phi_1} + U_{e2}U_{\tau 2}^* e^{-i\phi_2} + U_{e3}U_{\tau 3}^* e^{-i\phi_3})|\nu_\tau\rangle
\end{aligned}$$

- From which

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \psi(L) \rangle|^2$$
$$= |U_{e1}U_{\mu1}^* e^{-i\phi_1} + U_{e2}U_{\mu2}^* e^{-i\phi_2} + U_{e3}U_{\mu3}^* e^{-i\phi_3}|^2$$

- The terms in this expression can be represented as:



- Because of the unitarity of the PMNS matrix we have (U4):

$$U_{e1}U_{\mu1}^* + U_{e2}U_{\mu2}^* + U_{e3}U_{\mu3}^* = 0$$

and, consequently, unless the phases of the different components are different, the sum of these three diagrams is zero, i.e., require different neutrino masses for osc.

• Evaluate

$$P(\nu_e \to \nu_\mu) = |U_{e1}U^*_{\mu 1}e^{-i\phi_1} + U_{e2}U^*_{\mu 2}e^{-i\phi_2} + U_{e3}U^*_{\mu 3}e^{-i\phi_3}|^2$$

using $|z_1 + z_2 + z_3|^2 \equiv |z_1|^2 + |z_2|^2 + |z_3|^2 + 2\Re(z_1 z_2^* + z_1 z_3^* + z_2 z_3^*)$ **(4)**

which gives:

$$P(\nu_e \to \nu_\mu) = |U_{e1}U^*_{\mu 1}|^2 + |U_{e2}U^*_{\mu 2}|^2 + |U_{e3}U^*_{\mu 3}|^2 + \quad (5)$$

$$2\Re(U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}e^{-i(\phi_1-\phi_2)} + U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3}e^{-i(\phi_1-\phi_3)} + U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3}e^{-i(\phi_2-\phi_3)})$$

• This can be simplified by applying identity (4) to $|(U4)|^2$

$$|U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2} + U_{e3}U^*_{\mu 3}|^2 = 0$$

$$\Rightarrow \quad |U_{e1}U^*_{\mu 1}|^2 + |U_{e2}U^*_{\mu 2}|^2 + |U_{e3}U^*_{\mu 3}|^2 =$$

$$-2\Re(U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2} + U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3} + U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3})$$

• Substituting into equation (5) gives

$$\begin{aligned} P(\nu_e \to \nu_\mu) &= 2\Re\{U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}[e^{-i(\phi_1-\phi_2)} - 1]\} \\ &+ 2\Re\{U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3}[e^{-i(\phi_1-\phi_3)} - 1]\} \\ &+ 2\Re\{U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3}[e^{-i(\phi_2-\phi_3)} - 1]\} \end{aligned} \quad (6)$$

★ This expression for the electron survival probability is obtained from the coefficient for $|\nu_e\rangle$ in eqn. (3):

$$P(\nu_e \to \nu_e) = |\langle \nu_e|\psi(L)\rangle|^2 = |U_{e1}U_{e1}^* e^{-i\phi_1} + U_{e2}U_{e2}^* e^{-i\phi_2} + U_{e3}U_{e3}^* e^{-i\phi_3}|^2$$

which using the unitarity relation (U1)

$$|U_{e1}U_{e1}^* + U_{e2}U_{e2}^* + U_{e3}U_{e3}^*|^2 = 1$$

can be written

$$\begin{aligned} P(\nu_e \to \nu_e) = 1 &+ 2|U_{e1}|^2|U_{e2}|^2 \Re\{[e^{-i(\phi_1-\phi_2)} - 1]\} \\ &+ 2|U_{e1}|^2|U_{e3}|^2 \Re\{[e^{-i(\phi_1-\phi_3)} - 1]\} \\ &+ 2|U_{e2}|^2|U_{e3}|^2 \Re\{[e^{-i(\phi_2-\phi_3)} - 1]\} \end{aligned} \quad (7)$$

★ This expression can simplified using

$$\Re\{e^{-i(\phi_1-\phi_2)} - 1\} = \cos(\phi_2-\phi_1) - 1 = -2\sin^2\left(\frac{\phi_2-\phi_1}{2}\right) = -2\sin^2\left(\frac{(m_2^2-m_1^2)L}{4E}\right)$$

with $\phi_i \approx \frac{m_i^2}{2E}L$

Phase of mass eigenstate i at $z = L$

•Define:

$$\Delta_{21} = \frac{(m_2^2 - m_1^2)L}{4E} = \frac{\Delta m_{21}^2 L}{4E}$$

with

$$\Delta m_{21}^2 = m_2^2 - m_1^2$$

NOTE: $\Delta_{21} = (\phi_2 - \phi_1)/2$ **is a phase difference (i.e. dimensionless)**

•Which gives the electron neutrino survival probability

$$P(\nu_e \to \nu_e) = 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2 \Delta_{21} - 4|U_{e1}|^2|U_{e3}|^2 \sin^2 \Delta_{31} - 4|U_{e2}|^2|U_{e3}|^2 \sin^2 \Delta_{32}$$

•Similar expressions can be obtained for the muon and tau neutrino survival probabilities for muon and tau neutrinos.

★ Note that since we only have three neutrino generations there are only two independent mass-squared differences, i.e.

$$m_3^2 - m_1^2 = (m_3^2 - m_2^2) + (m_2^2 - m_1^2)$$

and in the above equation only two of the Δ_{ij} are independent

★All expressions are in Natural Units, conversion to more useful units here gives:

$$\Delta_{21} = 1.27 \frac{\Delta m_{21}^2(\text{eV}^2) L(\text{km})}{E(\text{GeV})}$$

and

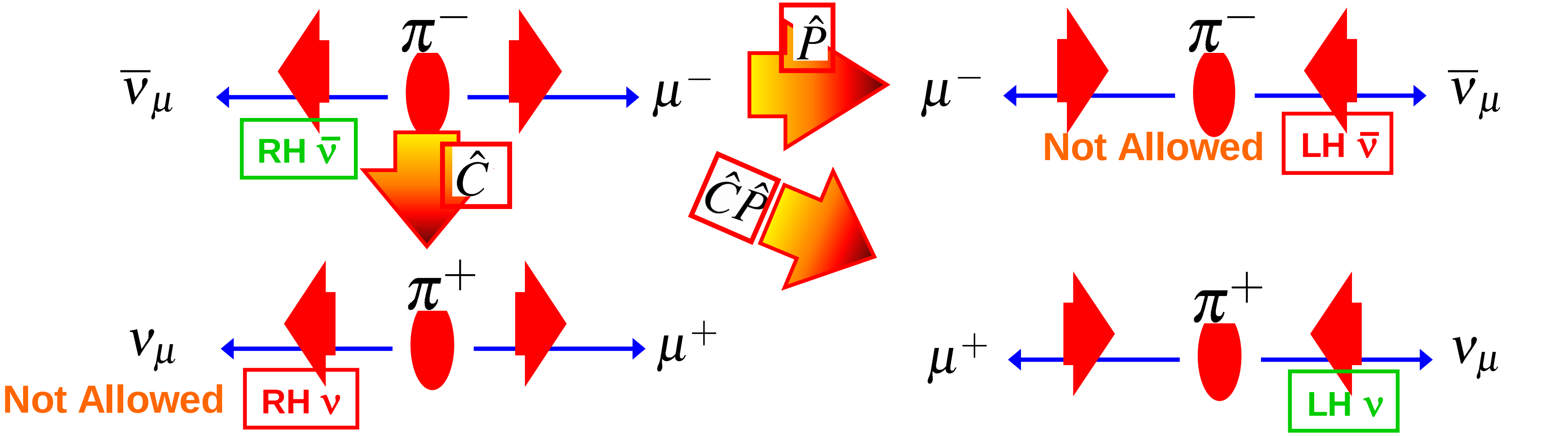
$$\lambda_{\text{osc}}(\text{km}) = 2.47 \frac{E(\text{GeV})}{\Delta m^2(\text{eV}^2)}$$

CP AND CPT IN THE WEAK INTERACTION

★ In addition to parity there are two other important discrete symmetries:

Parity	$\hat{P}:$	$\vec{r} \rightarrow -\vec{r}$
Time Reversal	$\hat{T}:$	$t \rightarrow -t$
Charge Conjugation	$\hat{C}:$	Particle ⟷ Anti-particle

★ The weak interaction violates parity conservation, but what about **C** ? Consider pion decay remembering that the neutrino is ultra-relativistic and only **left-handed neutrinos** and **right-handed anti-neutrinos** participate in WI



★ Hence weak interaction also **violates charge conjugation** symmetry but appears to be invariant under combined effect of **C** and **P**

CP transforms:

RH Particles ⟺ LH Anti-particles

LH Particles ⟺ RH Anti-particles

★ If the weak interaction were invariant under CP expect

$$\Gamma(\pi^+ \to \mu^+ \nu_\mu) = \Gamma(\pi^- \to \mu^- \overline{\nu}_\mu)$$

★ All Lorentz invariant Quantum Field Theories can be shown to be invariant under CPT (charge conjugation + parity + time reversal)

⇒ Particles/anti-particles have identical mass, lifetime, magnetic moments,…

Best current experimental test: $m_{K^0} - m_{\overline{K}^0} < 6 \times 10^{-19} m_{K^0}$

★ Believe CPT has to hold:

- if CP invariance holds ⇒ time reversal symmetry
- if CP is violated ⇒ time reversal symmetry violated

★ To account for the small excess of matter over anti-matter that must have existed early in the universe require CP violation in particle physics !

★ CP violation can arise in the weak interaction (see also handout 12).

CP AND T VIOLATION IN NEUTRINO OSCILLATIONS

- Previously derived the oscillation probability for $\nu_e \rightarrow \nu_\mu$

$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu) &= 2\Re\{U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}[e^{-i(\varphi_1-\varphi_2)} - 1]\} \\ &+ 2\Re\{U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3}[e^{-i(\phi_1-\phi_3)} - 1]\} \\ &+ 2\Re\{U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3}[e^{-i(\phi_2-\phi_3)} - 1]\} \end{aligned}$$

- The oscillation probability for $\nu_\mu \rightarrow \nu_e$ can be obtained in the same manner or by simply exchanging the labels $(e) \leftrightarrow (\mu)$

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) &= 2\Re\{U_{\mu 1}U^*_{e1}U^*_{\mu 2}U_{e2}[e^{-i(\phi_1-\phi_2)} - 1]\} \\ &+ 2\Re\{U_{\mu 1}U^*_{e1}U^*_{\mu 3}U_{e3}[e^{-i(\phi_1-\phi_3)} - 1]\} \\ &+ 2\Re\{U_{\mu 2}U^*_{e2}U^*_{\mu 3}U_{e3}[e^{-i(\phi_2-\phi_3)} - 1]\} \end{aligned} \quad \text{(8)}$$

★ Unless the elements of the PMNS matrix are real (see note below)

$$P(\nu_e \rightarrow \nu_\mu) \neq P(\nu_\mu \rightarrow \nu_e) \quad \text{(9)}$$

- If any of the elements of the PMNS matrix are complex, neutrino oscillations are not invariant under time reversal $t \rightarrow -t$

NOTE: can multiply entire PMNS matrix by a complex phase without changing the oscillation prob. T is violated if one of the elements has a different complex phase than the others

- Consider the effects of T, CP and CPT on neutrino oscillations

T	$\nu_e \to \nu_\mu$	$\xrightarrow{\hat{T}}$	$\nu_\mu \to \nu_e$
CP	$\nu_e \to \nu_\mu$	$\xrightarrow{\hat{C}\hat{P}}$	$\overline{\nu}_e \to \overline{\nu}_\mu$
CPT	$\nu_e \to \nu_\mu$	$\xrightarrow{\hat{C}\hat{P}\hat{T}}$	$\overline{\nu}_\mu \to \overline{\nu}_e$

Note C alone is not sufficient as it transforms LH neutrinos into LH anti-neutrinos (not involved in Weak Interaction)

- If the weak interactions is invariant under CPT

$$P(\nu_e \to \nu_\mu) = P(\overline{\nu}_\mu \to \overline{\nu}_e)$$

and similarly

$$P(\nu_\mu \to \nu_e) = P(\overline{\nu}_e \to \overline{\nu}_\mu) \qquad (10)$$

- If the PMNS matrix is not purely real, then (9)

$$P(\nu_e \to \nu_\mu) \neq P(\nu_\mu \to \nu_e)$$

and from (10)

$$\boxed{P(\nu_e \to \nu_\mu) \neq P(\overline{\nu}_e \to \overline{\nu}_\mu)}$$

★Hence unless the PMNS matrix is real, **CP is violated** in neutrino oscillations!

Future experiments, e.g. "a neutrino factory", are being considered as a way to investigate CP violation in neutrino oscillations. However, CP violating effects are well below the current experimental sensitivity. In the following discussion we will take the PMNS matrix to be real.

NEUTRINO MASS HIERARCHY

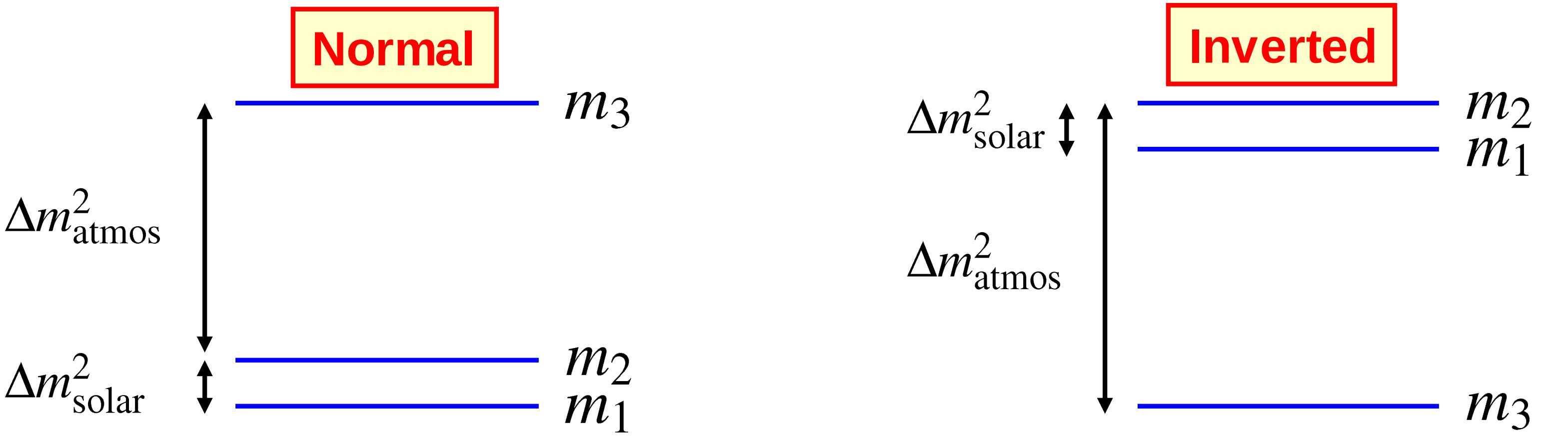
★ **To date, results on neutrino oscillations only determine**

$$|\Delta m^2_{ji}| = |m_j^2 - m_i^2|$$

★ **Two distinct and very different mass scales:**

- Atmospheric neutrino oscillations : $|\Delta m^2|_{\rm atmos} \sim 2.5 \times 10^{-3}\,{\rm eV}^2$
- Solar neutrino oscillations: $|\Delta m^2|_{\rm solar} \sim 8 \times 10^{-5}\,{\rm eV}^2$

•**Two possible assignments of mass hierarchy:**



•**In both cases:** $\Delta m^2_{21} \sim 8 \times 10^{-5}\,{\rm eV}^2$ **(solar)**

$|\Delta m^2_{31}| \approx |\Delta m^2_{32}| \sim 2.5 \times 10^{-3}\,{\rm eV}^2$ **(atmospheric)**

•**Hence we can approximate** $\Delta m^2_{31} \approx \Delta m^2_{32}$

THREE FLAVOUR OSCILLATIONS NEGLECTING CP VIOLATION

- Neglecting CP violation considerably simplifies the algebra of three flavour neutrino oscillations. Taking the PMNS matrix to be real, equation (6) becomes:

$$P(\nu_e \to \nu_\mu) = -4U_{e1}U_{\mu 1}U_{e2}U_{\mu 2}\sin^2\Delta_{21} - 4U_{e1}U_{\mu 1}U_{e3}U_{\mu 3}\sin^2\Delta_{31} - 4U_{e2}U_{\mu 2}U_{e3}U_{\mu 3}\sin^2\Delta_{32}$$

with $\Delta_{ji} = \frac{(m_j^2 - m_i^2)L}{4E} = \frac{\Delta m_{ji}^2 L}{4E}$

- Using: $\Delta_{31} \approx \Delta_{32}$ (see p. 365)

$$P(\nu_e \to \nu_\mu) \approx -4U_{e1}U_{\mu 1}U_{e2}U_{\mu 2}\sin^2\Delta_{21} - 4(U_{e1}U_{\mu 1} + U_{e2}U_{\mu 2})U_{e3}U_{\mu 3}\sin^2\Delta_{32}$$

- Which can be simplified using (U4) $U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0$

$$\Rightarrow \boxed{P(\nu_e \to \nu_\mu) \approx -4U_{e1}U_{\mu 1}U_{e2}U_{\mu 2}\sin^2\Delta_{21} + 4U_{e3}^2U_{\mu 3}^2\sin^2\Delta_{32}}$$

- Can apply $\Delta_{31} \approx \Delta_{32}$ to the expression for electron neutrino survival probability

$$P(\nu_e \to \nu_e) = 1 - 4U_{e1}^2U_{e2}^2\sin^2\Delta_{21} - 4U_{e1}^2U_{e3}^2\sin^2\Delta_{31} - 4U_{e2}^2U_{e3}^2\sin^2\Delta_{32}$$

$$\approx 1 - 4U_{e1}^2U_{e2}^2\sin^2\Delta_{21} - 4(U_{e1}^2 + U_{e2}^2)U_{e3}^2\sin^2\Delta_{32}$$

- Which can be simplified using (U1) $U_{e1}^2 + U_{e2}^2 + U_{e3}^2 = 1$

$$\Rightarrow \boxed{P(\nu_e \to \nu_e) \approx 1 - 4U_{e1}^2U_{e2}^2\sin^2\Delta_{21} - 4(1 - U_{e3}^2)U_{e3}^2\sin^2\Delta_{32}}$$

★ Neglecting CP violation (i.e. taking the PMNS matrix to be real) and making the approximation that $|\Delta m^2_{31}| \approx |\Delta m^2_{32}|$ obtain the following expressions for neutrino oscillation probabilities:

$$P(\nu_e \to \nu_e) \approx 1 - 4U^2_{e1}U^2_{e2}\sin^2\Delta_{21} - 4(1-U^2_{e3})U^2_{e3}\sin^2\Delta_{32} \quad (11)$$

$$P(\nu_\mu \to \nu_\mu) \approx 1 - 4U^2_{\mu1}U^2_{\mu2}\sin^2\Delta_{21} - 4(1-U^2_{\mu3})U^2_{\mu3}\sin^2\Delta_{32} \quad (12)$$

$$P(\nu_\tau \to \nu_\tau) \approx 1 - 4U^2_{\tau1}U^2_{\tau2}\sin^2\Delta_{21} - 4(1-U^2_{\tau3})U^2_{\tau3}\sin^2\Delta_{32} \quad (13)$$

$$P(\nu_e \to \nu_\mu) = P(\nu_\mu \to \nu_e) \approx -4U_{e1}U_{\mu1}U_{e2}U_{\mu2}\sin^2\Delta_{21} + 4U^2_{e3}U^2_{\mu3}\sin^2\Delta_{32} \quad (14)$$

$$P(\nu_e \to \nu_\tau) = P(\nu_\tau \to \nu_e) \approx -4U_{e1}U_{\tau1}U_{e2}U_{\tau2}\sin^2\Delta_{21} + 4U^2_{e3}U^2_{\tau3}\sin^2\Delta_{32} \quad (15)$$

$$P(\nu_\mu \to \nu_\tau) = P(\nu_\tau \to \nu_\mu) \approx -4U_{\mu1}U_{\tau1}U_{\mu2}U_{\tau2}\sin^2\Delta_{21} + 4U^2_{\mu3}U^2_{\tau3}\sin^2\Delta_{32} \quad (16)$$

★ The wavelengths associated with $\sin^2\Delta_{21}$ and $\sin^2\Delta_{32}$ are:

"SOLAR" $\lambda_{21} = \frac{4\pi E}{\Delta m^2_{21}}$ and $\lambda_{32} = \frac{4\pi E}{\Delta m^2_{32}}$ "ATMOSPHERIC"

"Long"-Wavelength

"Short"-Wavelength

PMNS MATRIX

★ The PMNS matrix is usually expressed in terms of 3 rotation angles $\theta_{12}, \theta_{23}, \theta_{13}$ and a complex phase δ, using the notation $s_{ij} = \sin\theta_{ij}, \quad c_{ij} = \cos\theta_{ij}$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Dominates: "Atmospheric" "Solar"

• Writing this out in full:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

★There are six SM parameters that can be measured in ν oscillation experiments

$\lvert\Delta m_{21}\rvert^2 = \lvert m_2^2 - m_1^2\rvert$	θ_{12}	Solar and reactor neutrino experiments
$\lvert\Delta m_{32}\rvert^2 = \lvert m_3^2 - m_2^2\rvert$	θ_{23}	Atmospheric and beam neutrino experiments
	θ_{13}	Reactor neutrino experiments + future beam
	δ	Future beam experiments