Elementary Particle Physics: theory and experiments

Interaction by particle exchange and QED

Electron-positron annihilation

Detectors for HEP: ATLAS at LHC

Some slides taken from M. A. Thomson lectures at Cambridge University in 2011

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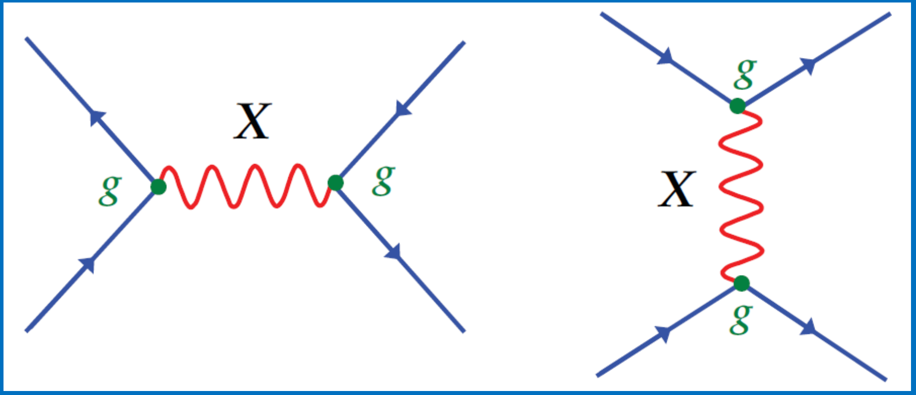
Interaction by particle exchange and QED

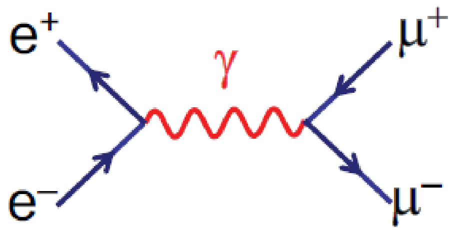


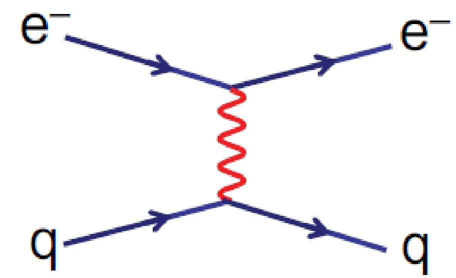
Recap

★ Working towards a proper calculation of decay and scattering processes

Initially concentrate on:

- $e^+e^- \rightarrow \mu^+\mu^-$
- $e^-q \rightarrow e^-q$





⅄ In **Lecture 2** covered the relativistic calculation of particle decay rates and crc s sections

$$\sigma \propto \frac{|M|^2}{\text{flux}} \text{ x (phase space)}$$

⅄ **Skipped** relativistic treatment of spin-half particles

Dirac Equation

⅄ **In this Lecture will** concentrate on the Lorentz Invariant Matrix Element

- Interaction by particle exchange
- Introduction to Feynman diagrams
- The Feynman rules for QED

Interaction by particel exchange

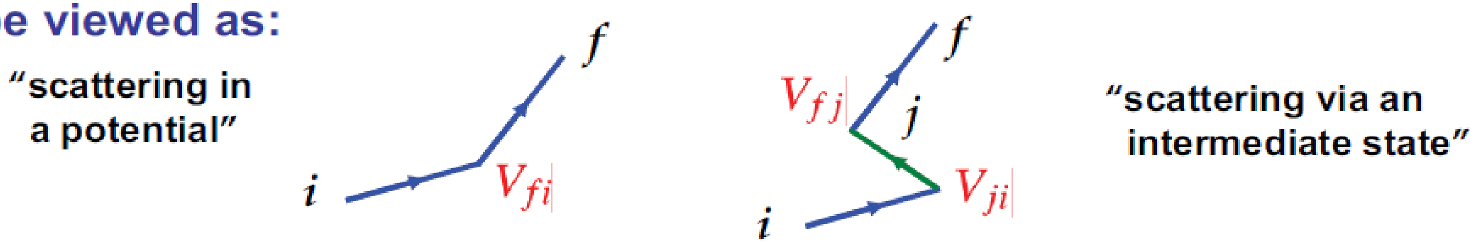
- Calculate transition rates from **Fermi's Golden Rule**

$$\Gamma_{fi} = 2\pi |T_{fi}|^2 \rho(E_f)$$

where T_{fi} is perturbation expansion for the Transition Matrix Element

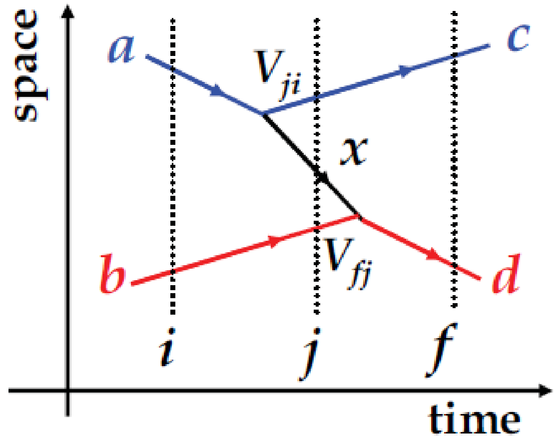
$$T_{fi} = \langle f|V|i\rangle + \sum_{j\neq i} \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j} + \dots$$

- For particle scattering, the first two terms in the perturbation series can be viewed as:



- **"Classical picture"** – particles act as sources for fields which give rise a potential in which other particles scatter – **"action at a distance"**
- **"Quantum Field Theory picture"** – forces arise due to the exchange of virtual particles. No action at a distance + forces between particles now due to particles

- Consider the particle interaction $a+b\rightarrow c+d$ which occurs via an intermediate state corresponding to the exchange of particle x
- One possible space-time picture of this process is:



Initial state i : $a+b$
Final state f : $c+d$
Intermediate state j : $c+b+x$

- This **time-ordered** diagram corresponds to a "emitting" x and then b absorbing x

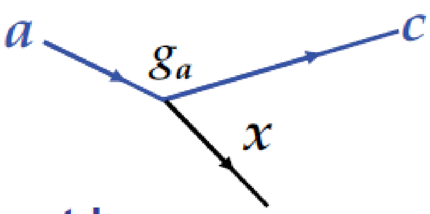
- The corresponding term in the perturbation expansion is:

$$T_{fi} = \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j}$$

$$T_{fi}^{ab} = \frac{\langle d|V|x+b\rangle\langle c+x|V|a\rangle}{(E_a+E_b)-(E_c+E_x+E_b)}$$

- T_{fi}^{ab} refers to the time-ordering where a emits x before b absorbs it

- Need an expression for $\langle c+x|V|a\rangle$ in non-invariant matrix element T_{fi}



- Ultimately aiming to obtain Lorentz Invariant ME
- Recall T_{fi} is related to the invariant matrix element by

$$T_{fi} = \prod_k (2E_k)^{-1/2} M_{fi}$$

where k runs over all particles in the matrix element

- Here we have

$$\langle c+x|V|a\rangle = \frac{M_{(a\to c+x)}}{(2E_a 2E_c 2E_x)^{1/2}}$$

$M_{(a\to c+x)}$ is the "Lorentz Invariant" matrix element for $a \to c + x$

★ The simplest Lorentz Invariant quantity is a scalar, in this case

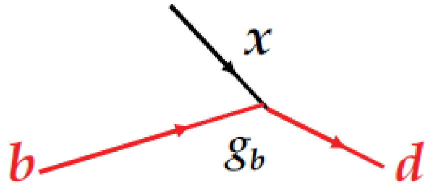
$$\langle c+x|V|a\rangle = \frac{g_a}{(2E_a 2E_c 2E_x)^{1/2}}$$

g_a is a measure of the strength of the interaction $a \to c + x$

Note : the matrix element is only LI in the sense that it is defined in terms of LI wave-function normalisations and that the form of the coupling is LI

Note : in this "illustrative" example g is not dimensionless.

Similarly $\langle d|V|x+b\rangle = \frac{g_b}{(2E_b 2E_d 2E_x)^{1/2}}$



Giving
$$T_{fi}^{ab} = \frac{\langle d|V|x+b\rangle\langle c+x|V|a\rangle}{(E_a+E_b)-(E_c+E_x+E_b)}$$
$$= \frac{1}{2E_x}\cdot\frac{1}{(2E_a 2E_b 2E_c 2E_d)^{1/2}}\cdot\frac{g_a g_b}{(E_a-E_c-E_x)}$$

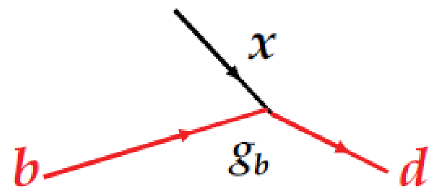
★The "Lorentz Invariant" matrix element for the entire process is

$$M_{fi}^{ab} = (2E_a 2E_b 2E_c 2E_d)^{1/2} T_{fi}^{ab}$$
$$= \frac{1}{2E_x}\cdot\frac{g_a g_b}{(E_a-E_c-E_x)}$$

Note:

- M_{fi}^{ab} refers to the time-ordering where a emits x before b absorbs it
  It is not Lorentz invariant, order of events in time depends on frame
- Momentum is conserved at each interaction vertex but not energy
  $E_j \neq E_i$
- Particle x is "on-mass shell" i.e. $E_x^2 = \vec{p}_x^{\,2} + m^2$

Similarly $\langle d|V|x+b\rangle = \frac{g_b}{(2E_b 2E_d 2E_x)^{1/2}}$



Giving
$$T_{fi}^{ab} = \frac{\langle d|V|x+b\rangle\langle c+x|V|a\rangle}{(E_a+E_b)-(E_c+E_x+E_b)}$$
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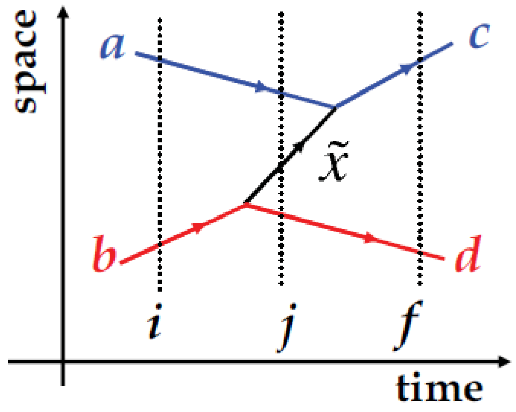
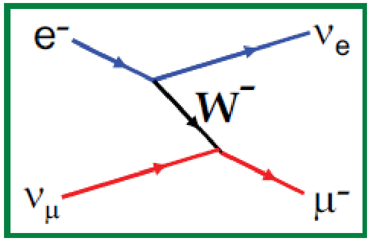
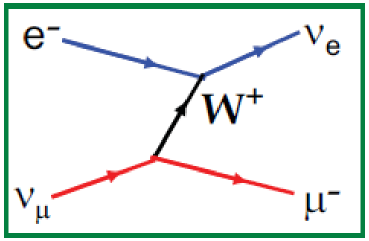
★ But need to consider also the other time ordering for the process



- This **time-ordered** diagram corresponds to b "emitting" $\tilde{x}$ and then a absorbing $\tilde{x}$
- $\tilde{x}$ is the anti-particle of x e.g.





- The **Lorentz invariant matrix element** for this time ordering is:

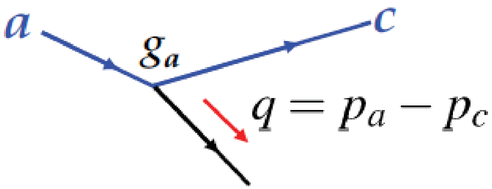
$$M_{fi}^{ba} = \frac{1}{2E_x} \cdot \frac{g_a g_b}{(E_b - E_d - E_x)}$$

★ In QM need to sum over matrix elements corresponding to same final state:

$$\begin{aligned} M_{fi} &= M_{fi}^{ab} + M_{fi}^{ba} \\ &= \frac{g_a g_b}{2E_x} \cdot \left(\frac{1}{E_a - E_c - E_x} + \frac{1}{E_b - E_d - E_x} \right) \\ &= \frac{g_a g_b}{2E_x} \cdot \left(\frac{1}{E_a - E_c - E_x} - \frac{1}{E_a - E_c + E_x} \right) \end{aligned}$$

Energy conservation:
$(E_a + E_b = E_c + E_d)$

- Which gives $$M_{fi} = \frac{g_a g_b}{2E_x} \cdot \frac{2E_x}{(E_a - E_c)^2 - E_x^2}$$ $$= \frac{g_a g_b}{(E_a - E_c)^2 - E_x^2}$$
- From 1st time ordering $E_x^2 = \vec{p}_x^2 + m_x^2 = (\vec{p}_a - \vec{p}_c)^2 + m_x^2$



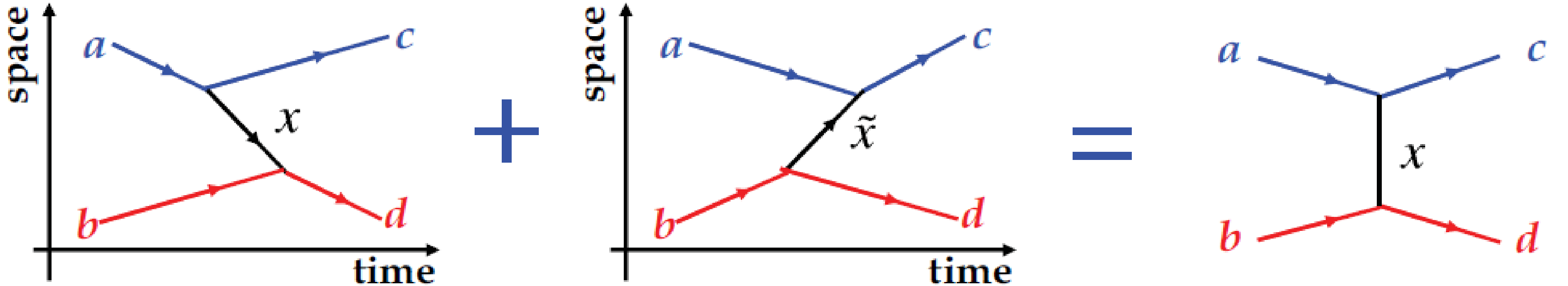
giving $$M_{fi} = \frac{g_a g_b}{(E_a - E_c)^2 - (\vec{p}_a - \vec{p}_c)^2 - m_x^2}$$ $$= \frac{g_a g_b}{(p_a - p_c)^2 - m_x^2}$$

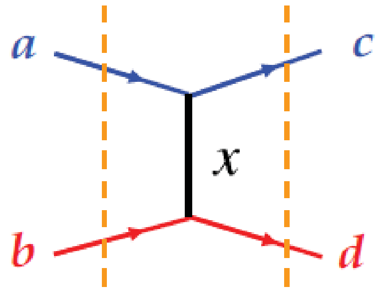
$$\Rightarrow \quad \boxed{M_{fi} = \frac{g_a g_b}{q^2 - m_x^2}}$$

- After summing over all possible time orderings, M_{fi} is (as anticipated) Lorentz invariant. This is a remarkable result – the sum over all time orderings gives a frame independent matrix element.
- Exactly the same result would have been obtained by considering the annihilation process

Feynman diagrams

- The sum over all possible time-orderings is represented by a **FEYNMAN diagram**





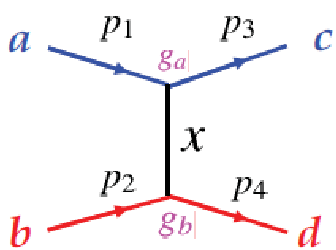
In a Feynman diagram:

- the LHS represents the initial state
- the RHS is the final state
- everything in between is "how the interaction happened"

- It is important to remember that energy and momentum are conserved at each interaction vertex in the diagram.
- The factor $1/(q^2 - m_x^2)$ is the propagator; it arises naturally from the above discussion of interaction by particle exchange

★ The matrix element: $M_{fi} = \frac{g_a g_b}{q^2 - m_x^2}$ depends on:

- The fundamental strength of the interaction at the two vertices g_a, g_b
- The four-momentum, q, carried by the (virtual) particle which is determined from energy/momentum conservation at the vertices. Note q^2 can be either positive or negative.

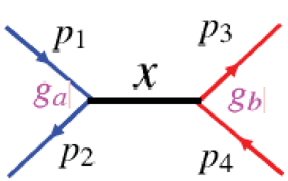


Here $q = p_1 - p_3 = p_4 - p_2 = t$ "t-channel"

For elastic scattering: $p_1 = (E, \vec{p}_1); \quad p_3 = (E, \vec{p}_3)$

$$q^2 = (E-E)^2 - (\vec{p}_1 - \vec{p}_3)^2$$

$q^2 < 0$ termed "space-like"



Here $q = p_1 + p_2 = p_3 + p_4 = s$ "s-channel"

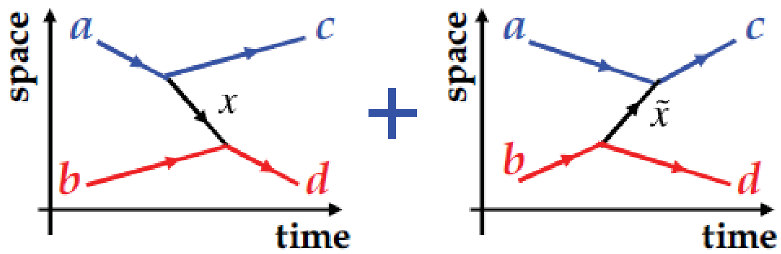
In CoM: $p_1 = (E, \vec{p}); \quad p_2 = (E, -\vec{p})$

$$q^2 = (E+E)^2 - (\vec{p} - \vec{p})^2 = 4E^2$$

$q^2 > 0$ termed "time-like"

Virtual particles

"Time-ordered QM"



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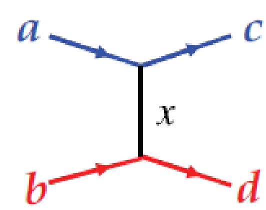
Feynman diagram



$$M_{fi} = \frac{g_a g_b}{q^2 - m_x^2}$$

- Momentum conserved at vertices
- Energy **not** conserved at vertices
- Exchanged particle **"on mass shell"**

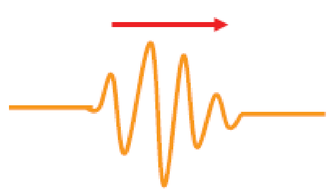
$$E_x^2 - |\vec{p}_x|^2 = m_x^2$$

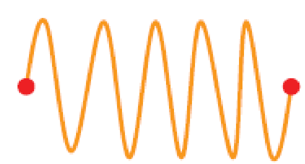
- Momentum **AND** energy conserved at interaction vertices
- Exchanged particle **"off mass shell"**

$$E_x^2 - |\vec{p}_x|^2 = q^2 \neq m_x^2$$

VIRTUAL PARTICLE

- Can think of observable "on mass shell" particles as propagating waves and unobservable virtual particles as normal modes between the source particles:

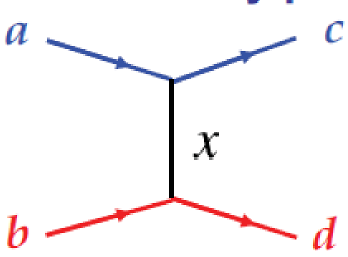




Aside: V(r) from particle exchange

★ Can view the scattering of an electron by a proton at rest in two ways:

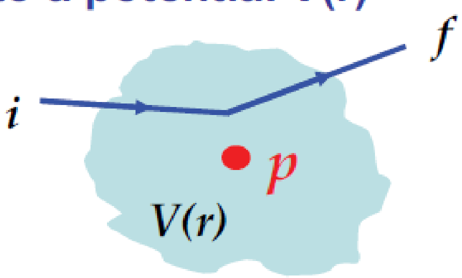
- Interaction by particle exchange in 2nd order perturbation theory.



$$M_{fi} = \frac{g_a g_b}{q^2 - m_x^2}$$

- Could also evaluate the same process in first order perturbation theory treating proton as a fixed source of a field which gives rise to a potential V(r)

$$M = \langle \psi_f | V(r) | \psi_i \rangle$$



Obtain same expression for M_{fi} using

$$V(r) = g_a g_b \frac{e^{-mr}}{r}$$

YUKAWA potential

★ In this way can relate potential and forces to the particle exchange picture

★ However, scattering from a fixed potential $V(r)$ is not a relativistic invariant view

Quantum Electrodynamics (QED)

★ Now consider the interaction of an electron and tau lepton by the exchange of a photon. Although the general ideas we applied previously still hold, we now have to account for the spin of the electron/tau-lepton and also the spin (polarization) of the virtual photon.

- The basic interaction between a photon and a charged particle can be introduced by making the minimal substitution

$$\vec{p} \rightarrow \vec{p} - q\vec{A}; \quad E \rightarrow E - q\phi$$

(here q = charge)

In QM:

$$\vec{p} = -i\vec{\nabla}; \qquad E = i\partial/\partial t$$

Therefore make substitution: $i\partial_\mu \rightarrow i\partial_\mu - qA_\mu$

where $A_\mu = (\phi, -\vec{A}); \qquad \partial_\mu = (\partial/\partial t, +\vec{\nabla})$

- The Dirac equation:

$$\gamma^\mu \partial_\mu \psi + im\psi = 0 \quad \Rightarrow \quad \gamma^\mu \partial_\mu \psi + iq\gamma^\mu A_\mu \psi + im\psi = 0$$

$$(\times i) \quad \Rightarrow \quad i\gamma^0 \frac{\partial \psi}{\partial t} + i\vec{\gamma}.\vec{\nabla}\psi - q\gamma^\mu A_\mu \psi - m\psi = 0$$

$$i\gamma^0 \frac{\partial \psi}{\partial t} = \gamma^0 \hat{H}\psi \quad = \quad m\psi - i\vec{\gamma}.\vec{\nabla}\psi + q\gamma^\mu A_\mu \psi$$

$$\times\gamma^0 : \qquad \hat{H}\psi \quad = \quad \underbrace{(\gamma^0 m - i\gamma^0\vec{\gamma}.\vec{\nabla})}_{\text{Combined rest mass + K.E.}}\psi + \underbrace{q\gamma^0\gamma^\mu A_\mu}_{\text{Potential energy}}\psi$$

Combined rest mass + K.E.

Potential energy

- We can identify the potential energy of a charged spin-half particle in an electromagnetic field as:

$$\boxed{\hat{V}_D = q\gamma^0\gamma^\mu A_\mu}$$

(note the A_0 term is just: $q\gamma^0\gamma^0 A_0 = q\phi$)

- The final complication is that we have to account for the photon polarization states.

$$A_\mu = \varepsilon_\mu^{(\lambda)} e^{i(\vec{p}.\vec{r}-Et)}$$

e.g. for a real photon propagating in the z direction we have two orthogonal transverse polarization states

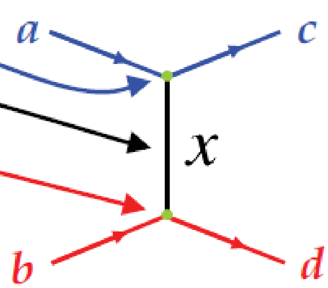
$$\varepsilon^{(1)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \varepsilon^{(2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Could equally have chosen circularly polarized states

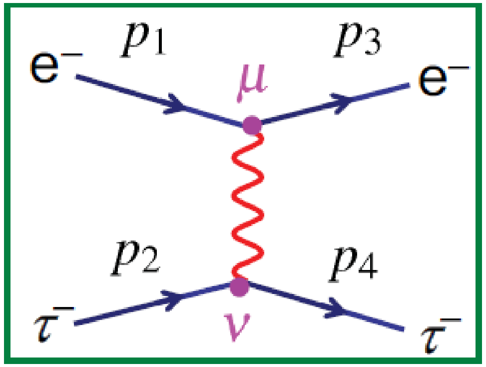
- Previously with the example of a simple spin-less interaction we had:

$$M = \langle \psi_c | V | \psi_a \rangle \frac{1}{q^2 - m_x^2} \langle \psi_d | V | \psi_b \rangle$$

$\langle \psi_c | V | \psi_a \rangle = g_a$, $\langle \psi_d | V | \psi_b \rangle = g_b$



★ In QED we could again go through the procedure of summing the time-orderings using Dirac spinors and the expression for $\hat{V}_D$. If we were to do this, remembering to sum over all photon polarizations, we would obtain:



$$M = [u_e^\dagger(p_3) q_e \gamma^0 \gamma^\mu u_e(p_1)] \sum_\lambda \frac{\varepsilon_\mu^\lambda (\varepsilon_\nu^\lambda)^*}{q^2} [u_\tau^\dagger(p_4) q_\tau \gamma^0 \gamma^\nu u_\tau(p_2)]$$

Interaction of e^- with photon

Massless photon propagator summing over polarizations

Interaction of τ^- with photon

- All the physics of **QED** is in the above expression !

Virtuality in t-channel, not a charge!

- The sum over the polarizations of the VIRTUAL photon has to include longitudinal and scalar contributions, i.e. 4 polarisation states

$$\varepsilon^{(0)} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad \varepsilon^{(1)} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad \varepsilon^{(2)} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad \varepsilon^{(3)} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

and gives: $$\sum_{\lambda} \varepsilon^{\lambda}_{\mu} (\varepsilon^{\lambda}_{\nu})^{*} = -g_{\mu\nu}$$

This is not obvious – for the moment just take it on trust

and the invariant matrix element becomes:

$$M = \left[u_e^{\dagger}(p_3) q_e \gamma^0 \gamma^{\mu} u_e(p_1)\right] \frac{-g_{\mu\nu}}{q^2} \left[u_{\tau}^{\dagger}(p_4) q_{\tau} \gamma^0 \gamma^{\nu} u_{\tau}(p_2)\right]$$

- Using the definition of the adjoint spinor $\overline{\psi} = \psi^{\dagger}\gamma^0$

$$M = \left[\overline{u}_e(p_3) q_e \gamma^{\mu} u_e(p_1)\right] \frac{-g_{\mu\nu}}{q^2} \left[\overline{u}_{\tau}(p_4) q_{\tau} \gamma^{\nu} u_{\tau}(p_2)\right]$$

★ This is a remarkably simple expression !

$\overline{u}_1 \gamma^{\mu} u_2$ transforms as a four vector. Writing

$$j_e^{\mu} = \overline{u}_e(p_3)\gamma^{\mu} u_e(p_1) \qquad j_{\tau}^{\nu} = \overline{u}_{\tau}(p_4)\gamma^{\nu} u_{\tau}(p_2)$$

$$M = -q_e q_{\tau} \frac{j_e \cdot j_{\tau}}{q^2}$$ showing that M is Lorentz Invariant

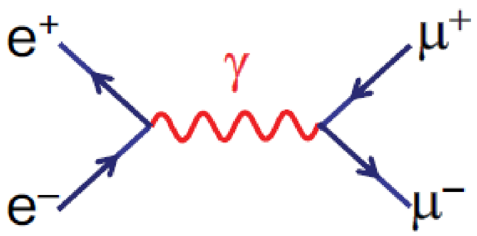
Feynman rules for QED

- It should be remembered that the expression

$$M = [\overline{u}_e(p_3)q_e\gamma^\mu u_e(p_1)]\frac{-g_{\mu\nu}}{q^2}[\overline{u}_\tau(p_4)q_\tau\gamma^\nu u_\tau(p_2)]$$

hides a lot of complexity. We have summed over all possible time-orderings and summed over all polarization states of the virtual photon. If we are then presented with a new Feynman diagram we don't want to go through the full calculation again.

Fortunately this isn't necessary – can just write down matrix element using a set of simple rules



Basic Feynman Rules:

- Propagator factor for each internal line (i.e. each internal virtual particle)
- Dirac Spinor for each external line (i.e. each real incoming or outgoing particle)
- Vertex factor for each vertex

Basic rules for QED

- **External Lines**

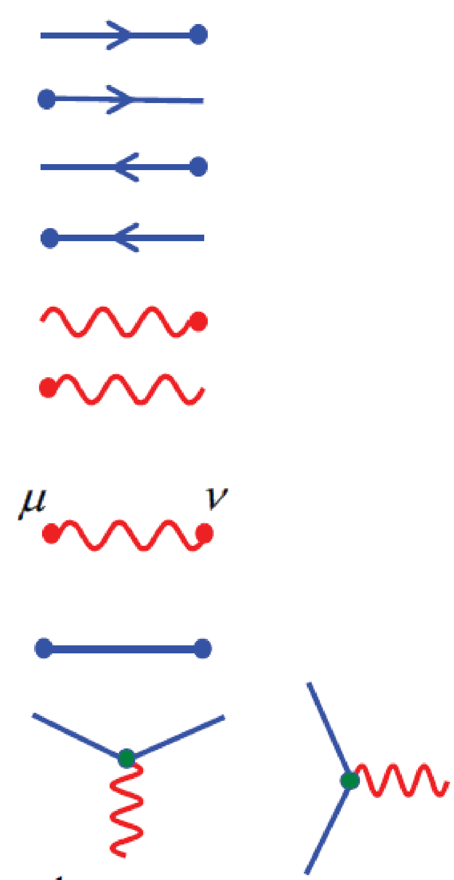
spin 1/2	incoming particle	$u(p)$	
	outgoing particle	$\bar{u}(p)$	
	incoming antiparticle	$\bar{v}(p)$	
	outgoing antiparticle	$v(p)$	
spin 1	incoming photon	$\varepsilon^{\mu}(p)$	
	outgoing photon	$\varepsilon^{\mu}(p)^{*}$	

- **Internal Lines (propagators)**

spin 1	photon	$-\dfrac{ig_{\mu\nu}}{q^2}$
spin 1/2	fermion	$\dfrac{i(\gamma^{\mu}q_{\mu}+m)}{q^2-m^2}$

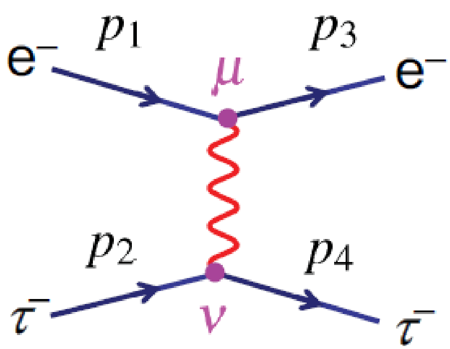
- **Vertex Factors**

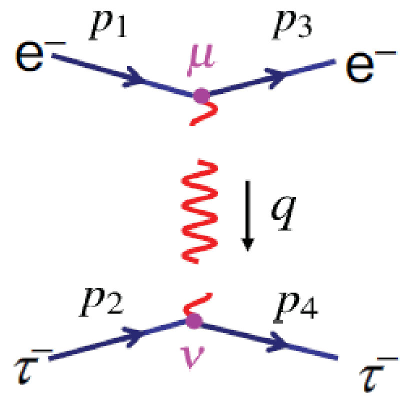
spin 1/2	fermion (charge -|e|)	$ie\gamma^{\mu}$



- **Matrix Element** $-iM$ = product of all factors

e.g.





$$\overline{u}_e(p_3)[ie\gamma^\mu]u_e(p_1)$$

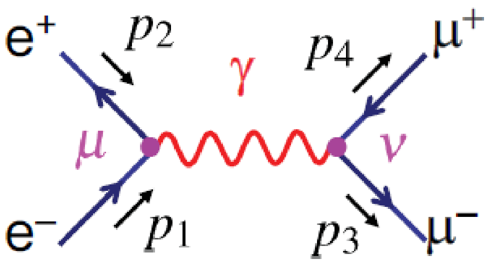
$$\frac{-ig_{\mu\nu}}{q^2}$$

$$\overline{u}_\tau(p_4)[ie\gamma^\nu]u_\tau(p_2)$$

$$-iM = [\overline{u}_e(p_3)ie\gamma^\mu u_e(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\overline{u}_\tau(p_4)ie\gamma^\nu u_\tau(p_2)]$$

- Which is the same expression as we obtained previously

e.g.



$$-iM = [\overline{v}(p_2)ie\gamma^\mu u(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\overline{u}(p_3)ie\gamma^\nu v(p_4)]$$

Note:

- At each vertex the adjoint spinor is written first
- Each vertex has a different index
- The $g_{\mu\nu}$ of the propagator connects the indices at the vertices

Summary

★ Interaction by particle exchange naturally gives rise to **Lorentz Invariant Matrix Element** of the form

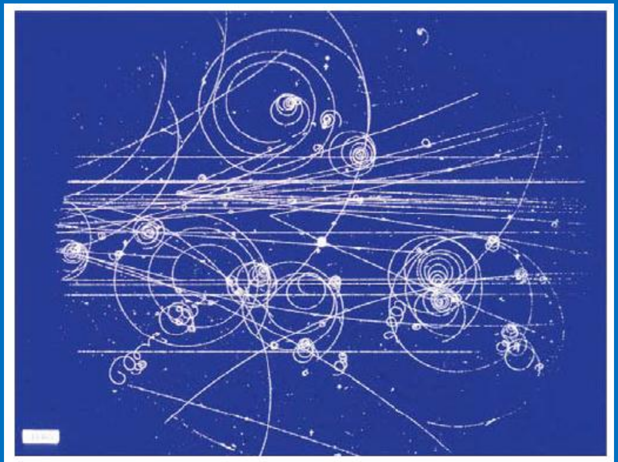
$$M_{fi} = \frac{g_a g_b}{q^2 - m_x^2}$$

★ Derived the basic interaction in **QED** taking into account the spins of the fermions and polarization of the virtual photons:

$$-iM = [\overline{u}(p_3) ie\gamma^{\mu} u(p_1)] \frac{-ig_{\mu\nu}}{q^2} [\overline{u}(p_4) ie\gamma^{\nu} u(p_2)]$$

★ We now have all the elements to perform proper calculations in QED !

Electron-positron annihilation

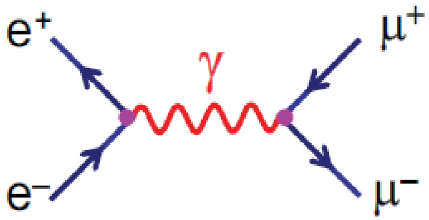


QED calculations

- How to calculate a cross section using QED (e.g. $e^+e^- \to \mu^+\mu^-$):

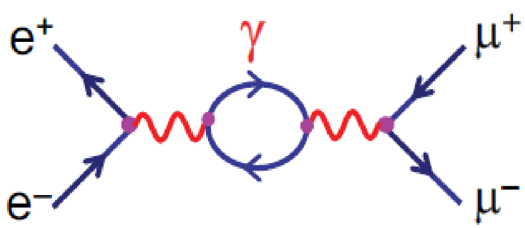
❶ Draw all possible Feynman Diagrams

- For $e^+e^- \to \mu^+\mu^-$ there is just one lowest order diagram

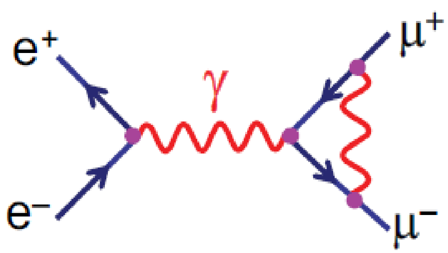


$$M \propto e^2 \propto \alpha_{em}$$

+ many second order diagrams + ...



+



$+\ldots$

$$M \propto e^4 \propto \alpha_{em}^2$$

❷ For each diagram calculate the matrix element using Feynman rules

❸ Sum the individual matrix elements (i.e. sum the amplitudes)

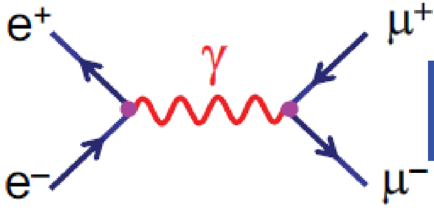
$$M_{fi} = M_1 + M_2 + M_3 + \ldots.$$

- Note: summing amplitudes therefore different diagrams for the same final state can interfere either positively or negatively!

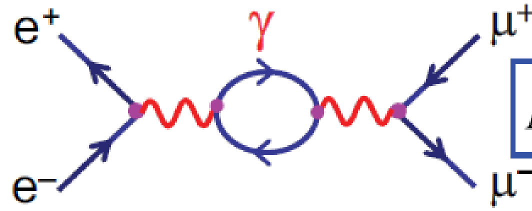
and then square $|M_{fi}|^2 = (M_1 + M_2 + M_3 + \ldots.)(M_1^* + M_2^* + M_3^* + \ldots.)$

➡ this gives the full perturbation expansion in α_{em}

- For QED $\alpha_{em} \sim 1/137$ the lowest order diagram dominates and for most purposes it is sufficient to neglect higher order diagrams.



$M^2 \propto \alpha_{em}^2$



$M^2 \propto \alpha_{em}^4$

❹ Calculate decay rate/cross section

- e.g. for a decay

$$\Gamma = \frac{p^*}{32\pi^2 m_a^2} \int |M_{fi}|^2 d\Omega$$

- For scattering in the centre-of-mass frame

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s} \frac{|\vec{p}_f^*|}{|\vec{p}_i^*|} |M_{fi}|^2 \qquad (1)$$

- For scattering in lab. frame (neglecting mass of scattered particle)

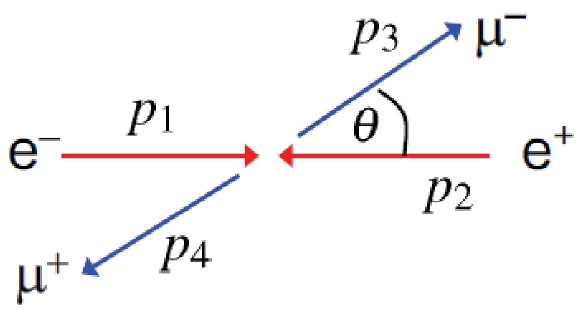
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \left(\frac{E_3}{ME_1}\right)^2 |M_{fi}|^2$$

Electron-positron annihilation

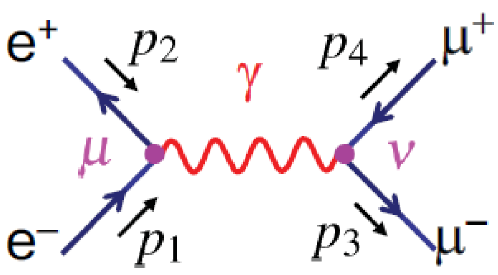
★ **Consider the process:** $e^+e^- \rightarrow \mu^+\mu^-$

- **Work in C.o.M. frame (this is appropriate for most e^+e^- colliders).**

$$p_1 = (E,0,0,p) \qquad p_2 = (E,0,0,-p)$$
$$p_3 = (E,\vec{p}_f) \qquad p_4 = (E,-\vec{p}_f)$$



- **Only consider the lowest order Feynman diagram:**



- **Feynman rules give:**

$$-iM = [\overline{v}(p_2)ie\gamma^\mu u(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\overline{u}(p_3)ie\gamma^\nu v(p_4)]$$

NOTE:
- **Incoming anti-particle** $\overline{v}$
- **Incoming particle** u
- **Adjoint spinor written first**

- **In the C.o.M. frame have**

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{p}_f|}{|\vec{p}_i|}|M_{fi}|^2 \qquad \text{with} \qquad s = (p_1+p_2)^2 = (E+E)^2 = 4E^2$$

Electron and muon currents

- Here $q^2 = (p_1 + p_2)^2 = s$ and matrix element

$$-iM = [\overline{v}(p_2) ie\gamma^{\mu} u(p_1)] \frac{-ig_{\mu\nu}}{q^2} [\overline{u}(p_3) ie\gamma^{\nu} v(p_4)]$$

$$\Rightarrow \quad M = -\frac{e^2}{s} g_{\mu\nu} [\overline{v}(p_2)\gamma^{\mu} u(p_1)][\overline{u}(p_3)\gamma^{\nu} v(p_4)]$$

- Introduced the four-vector current

$$j^{\mu} = \overline{\psi}\gamma^{\mu}\psi$$

which has same form as the two terms in [ ] in the matrix element

- The matrix element can be written in terms of the electron and muon currents

$$(j_e)^{\mu} = \overline{v}(p_2)\gamma^{\mu} u(p_1) \quad \text{and} \quad (j_{\mu})^{\nu} = \overline{u}(p_3)\gamma^{\nu} v(p_4)$$

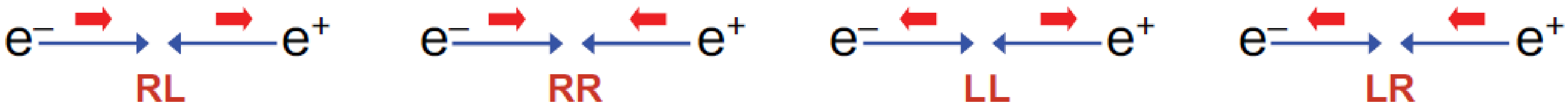
$$\Rightarrow \quad M = -\frac{e^2}{s} g_{\mu\nu} (j_e)^{\mu} (j_{\mu})^{\nu}$$

$$\boxed{M = -\frac{e^2}{s} j_e \cdot j_{\mu}}$$

- Matrix element is a four-vector scalar product – confirming it is Lorentz Invariant

Spin in e^+e^- annihilation

- In general the electron and positron will not be polarized, i.e. there will be equal numbers of positive and negative helicity states
- There are four possible combinations of spins in the initial state !



- Similarly there are four possible helicity combinations in the final state
- In total there are 16 combinations e.g. RL→RR, RL→RL, ...
- To account for these states we need to sum over all 16 possible helicity combinations and then average over the number of initial helicity states:

$$\langle |M|^2 \rangle = \frac{1}{4} \sum_{\text{spins}} |M_i|^2 = \frac{1}{4} \left(|M_{LL\to LL}|^2 + |M_{LL\to LR}|^2 + \dots \right)$$

★ i.e. need to evaluate:

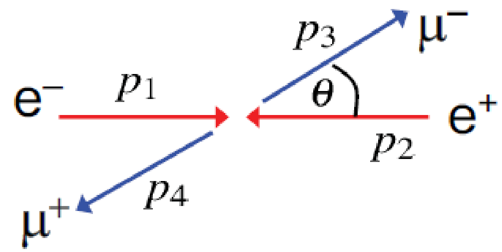
$$M = -\frac{e^2}{s} j_e \cdot j_\mu$$

for all 16 helicity combinations !

★ Fortunately, in the limit $E \gg m_\mu$ only 4 helicity combinations give non-zero matrix elements – we will see that this is an important feature of QED/QCD

- In the C.o.M. frame in the limit $E \gg m$

$$p_1 = (E,0,0,E); \quad p_2 = (E,0,0,-E)$$
$$p_3 = (E, E\sin\theta, 0, E\cos\theta);$$
$$p_4 = (E, -\sin\theta, 0, -E\cos\theta)$$



- Left- and right-handed helicity spinors for particles/anti-particles are:

$$u_\uparrow = N\begin{pmatrix} c \\ e^{i\phi}s \\ \frac{|\vec{p}|}{E+m}c \\ \frac{|\vec{p}|}{E+m}e^{i\phi}s \end{pmatrix} \quad u_\downarrow = N\begin{pmatrix} -s \\ e^{i\phi}c \\ \frac{|\vec{p}|}{E+m}s \\ -\frac{|\vec{p}|}{E+m}e^{i\phi}c \end{pmatrix} \quad v_\uparrow = N\begin{pmatrix} \frac{|\vec{p}|}{E+m}s \\ -\frac{|\vec{p}|}{E+m}e^{i\phi}c \\ -s \\ e^{i\phi}c \end{pmatrix} \quad v_\downarrow = N\begin{pmatrix} \frac{|\vec{p}|}{E+m}c \\ \frac{|\vec{p}|}{E+m}e^{i\phi}s \\ c \\ e^{i\phi}s \end{pmatrix}$$

where $s = \sin\frac{\theta}{2}; \quad c = \cos\frac{\theta}{2}$ and $N = \sqrt{E+m}$

- In the limit $E \gg m$ these become:

$$u_\uparrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}; \; u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix}; \; v_\uparrow = \sqrt{E}\begin{pmatrix} s \\ -ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix}; \; v_\downarrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}$$

- The initial-state electron can either be in a left- or right-handed helicity state

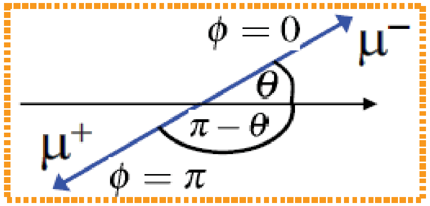
$$u_\uparrow(p_1) = \sqrt{E}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \; u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix};$$

- For the initial state positron $(\theta = \pi)$ can have either:

$$v_\uparrow(p_2) = \sqrt{E}\begin{pmatrix}1\\0\\-1\\0\end{pmatrix};\ v_\downarrow(p_2) = \sqrt{E}\begin{pmatrix}0\\1\\0\\1\end{pmatrix}$$

- Similarly for the final state μ^- which has polar angle θ and choosing $\phi = 0$

$$u_\uparrow(p_3) = \sqrt{E}\begin{pmatrix}c\\s\\c\\s\end{pmatrix};\ u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix}-s\\c\\s\\-c\end{pmatrix};$$



- And for the final state μ^+ replacing $\theta \to \pi - \theta;\quad \phi \to \pi$ obtain

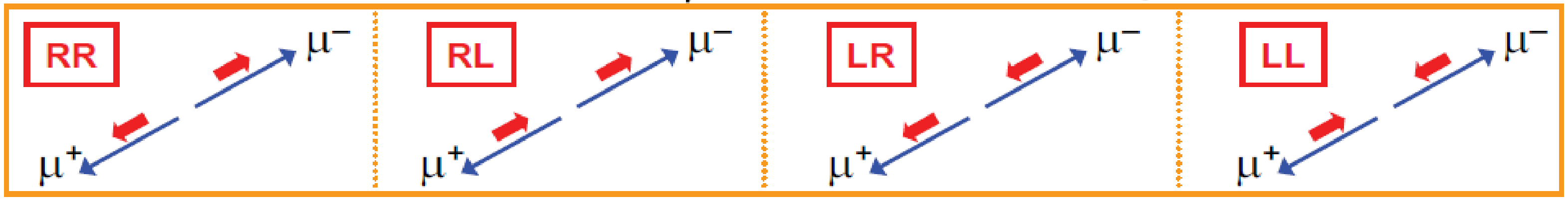
$$v_\uparrow(p_4) = \sqrt{E}\begin{pmatrix}c\\s\\-c\\-s\end{pmatrix};\ v_\downarrow(p_4) = \sqrt{E}\begin{pmatrix}s\\-c\\s\\-c\end{pmatrix};$$

using

$$\sin\left(\frac{\pi-\theta}{2}\right) = \cos\frac{\theta}{2}$$
$$\cos\left(\frac{\pi-\theta}{2}\right) = \sin\frac{\theta}{2}$$
$$e^{i\pi} = -1$$

- Wish to calculate the matrix element $M = -\frac{e^2}{s} j_e . j_\mu$

★ first consider the muon current j_μ for 4 possible helicity combinations



The muon current

- Want to evaluate $(j_\mu)^\nu = \overline{u}(p_3)\gamma^\nu v(p_4)$ for all four helicity combinations
- For arbitrary spinors $\psi,\ \phi$ with it is straightforward to show that the components of $\overline{\psi}\gamma^\mu\phi$ are

$$\begin{aligned}
\overline{\psi}\gamma^0\phi &= \psi^\dagger\gamma^0\gamma^0\phi = \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4 && (3)\\
\overline{\psi}\gamma^1\phi &= \psi^\dagger\gamma^0\gamma^1\phi = \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1 && (4)\\
\overline{\psi}\gamma^2\phi &= \psi^\dagger\gamma^0\gamma^2\phi = -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1) && (5)\\
\overline{\psi}\gamma^3\phi &= \psi^\dagger\gamma^0\gamma^3\phi = \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2 && (6)
\end{aligned}$$

- Consider the $\mu_R^-\mu_L^+$ combination using $\psi = u_\uparrow$ $\phi = v_\downarrow$

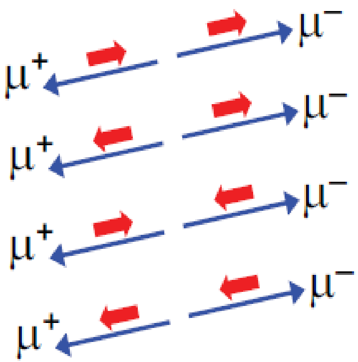
with $v_\downarrow = \sqrt{E}\begin{pmatrix} s \\ -c \\ s \\ -c \end{pmatrix};\ u_\uparrow = \sqrt{E}\begin{pmatrix} c \\ s \\ c \\ s \end{pmatrix};$

$$\begin{aligned}
\overline{u}_\uparrow(p_3)\gamma^0 v_\downarrow(p_4) &= E(cs - sc + cs - sc) = 0\\
\overline{u}_\uparrow(p_3)\gamma^1 v_\downarrow(p_4) &= E(-c^2 + s^2 - c^2 + s^2) = 2E(s^2 - c^2) = -2E\cos\theta\\
\overline{u}_\uparrow(p_3)\gamma^2 v_\downarrow(p_4) &= -iE(-c^2 - s^2 - c^2 - s^2) = 2iE\\
\overline{u}_\uparrow(p_3)\gamma^3 v_\downarrow(p_4) &= E(cs + sc + cs + sc) = 4Esc = 2E\sin\theta
\end{aligned}$$

- Hence the four-vector muon current for the RL combination is

$$\overline{u}_{\uparrow}(p_3)\gamma^{\nu}v_{\downarrow}(p_4) \quad = \quad 2E(0,-\cos\theta,i,\sin\theta)$$

- The results for the 4 helicity combinations (obtained in the same manner) are:



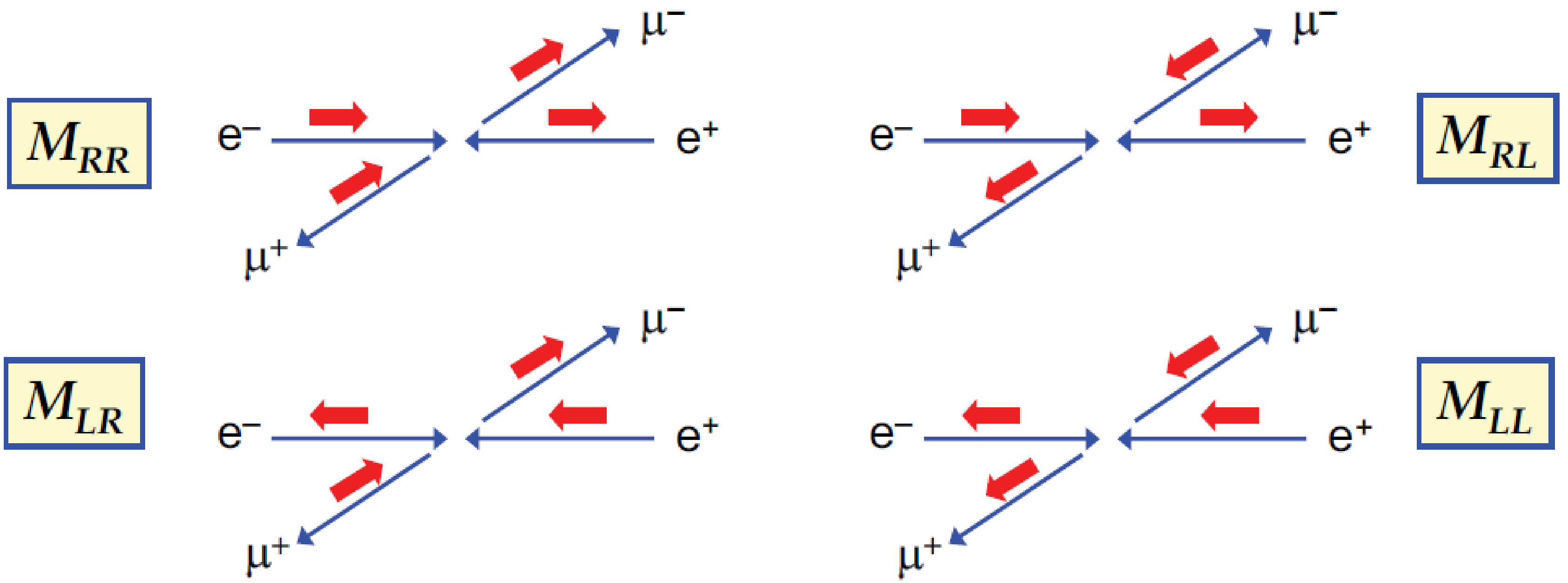
$$\begin{aligned}
\overline{u}_{\uparrow}(p_3)\gamma^{\nu}v_{\downarrow}(p_4) &= 2E(0,-\cos\theta,i,\sin\theta) && \text{RL}\\
\overline{u}_{\uparrow}(p_3)\gamma^{\nu}v_{\uparrow}(p_4) &= (0,0,0,0) && \text{RR}\\
\overline{u}_{\downarrow}(p_3)\gamma^{\nu}v_{\downarrow}(p_4) &= (0,0,0,0) && \text{LL}\\
\overline{u}_{\downarrow}(p_3)\gamma^{\nu}v_{\uparrow}(p_4) &= 2E(0,-\cos\theta,-i,\sin\theta) && \text{LR}
\end{aligned}$$

★ **IN THE LIMIT** $E \gg m$ **only two helicity combinations are non-zero !**

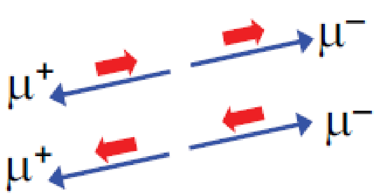
- This is an important feature of QED. It applies equally to QCD.
- In the Weak interaction only one helicity combination contributes.
- The origin of this will be discussed in the last part of this lecture
- But as a consequence of the 16 possible helicity combinations only four given non-zero matrix elements

Electron-positron annihilation cont.

★ For $e^+e^- \rightarrow \mu^+\mu^-$ now only have to consider the 4 matrix elements:



- Previously we derived the muon currents for the allowed helicities:



$$\mu_R^-\mu_L^+ \;:\quad \bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4) = 2E(0,-\cos\theta,i,\sin\theta)$$
$$\mu_L^-\mu_R^+ \;:\quad \bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) = 2E(0,-\cos\theta,-i,\sin\theta)$$

- Now need to consider the electron current

The electron current

- The incoming electron and positron spinors (L and R helicities) are:

$$u_\uparrow = \sqrt{E}\begin{pmatrix}1\\0\\1\\0\end{pmatrix};\ u_\downarrow = \sqrt{E}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix};\quad v_\uparrow = \sqrt{E}\begin{pmatrix}1\\0\\-1\\0\end{pmatrix};\ v_\downarrow = \sqrt{E}\begin{pmatrix}0\\1\\0\\1\end{pmatrix}$$

- The electron current can either be obtained from equations (3)-(6) as before or it can be obtained directly from the expressions for the muon current.

$$(j_e)^\mu = \overline{v}(p_2)\gamma^\mu u(p_1) \qquad\qquad (j_\mu)^\mu = \overline{u}(p_3)\gamma^\mu v(p_4)$$

- Taking the Hermitian conjugate of the muon current gives

$$\begin{aligned}
[\overline{u}(p_3)\gamma^\mu v(p_4)]^\dagger &= \left[u(p_3)^\dagger\gamma^0\gamma^\mu v(p_4)\right]^\dagger \\
&= v(p_4)^\dagger\gamma^{\mu\dagger}\gamma^{0\dagger}u(p_3) && (AB)^\dagger = B^\dagger A^\dagger \\
&= v(p_4)^\dagger\gamma^{\mu\dagger}\gamma^0 u(p_3) && \gamma^{0\dagger} = \gamma^0 \\
&= v(p_4)^\dagger\gamma^0\gamma^\mu u(p_3) && \gamma^{\mu\dagger}\gamma^0 = \gamma^0\gamma^\mu \\
&= \overline{v}(p_4)\gamma^\mu u(p_3)
\end{aligned}$$

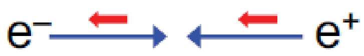
- Taking the complex conjugate of the muon currents for the two non-zero helicity configurations:

$$\overline{v}_{\downarrow}(p_4)\gamma^{\mu}u_{\uparrow}(p_3) = \left[\overline{u}_{\uparrow}(p_3)\gamma^{\nu}v_{\downarrow}(p_4)\right]^* = 2E(0,-\cos\theta,-i,\sin\theta)$$

$$\overline{v}_{\uparrow}(p_4)\gamma^{\mu}u_{\downarrow}(p_3) = \left[\overline{u}_{\downarrow}(p_3)\gamma^{\nu}v_{\uparrow}(p_4)\right]^* = 2E(0,-\cos\theta,i,\sin\theta)$$

To obtain the electron currents we simply need to set $\theta = 0$

$$e_R^- e_L^+ \; : \quad \overline{v}_{\downarrow}(p_2)\gamma^{\nu}u_{\uparrow}(p_1) = 2E(0,-1,-i,0)$$

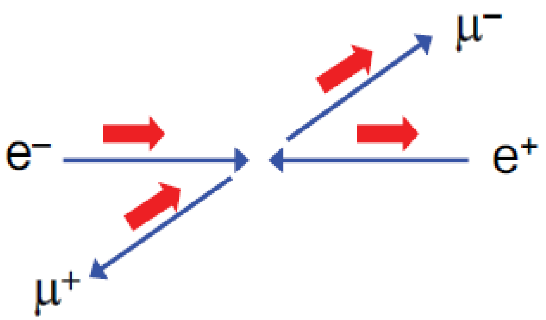


$$e_L^- e_R^+ \; : \quad \overline{v}_{\uparrow}(p_2)\gamma^{\nu}u_{\downarrow}(p_1) = 2E(0,-1,i,0)$$

Matrix element calculation

- We can now calculate $M = -\frac{e^2}{s} j_e . j_\mu$ for the four possible helicity combinations.

e.g. the matrix element for $e_R^- e_L^+ \rightarrow \mu_R^- \mu_L^+$ which will denote M_{RR}



Here the first subscript refers to the helicity of the e⁻ and the second to the helicity of the μ⁻. Don't need to specify other helicities due to "helicity conservation", only certain chiral combinations are non-zero.

★Using:

$$e_R^- e_L^+ : \quad (j_e)^\mu = \bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1) = 2E(0,-1,-i,0)$$

$$\mu_R^- \mu_L^+ : \quad (j_\mu)^\nu = \bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4) = 2E(0,-\cos\theta, i, \sin\theta)$$

gives

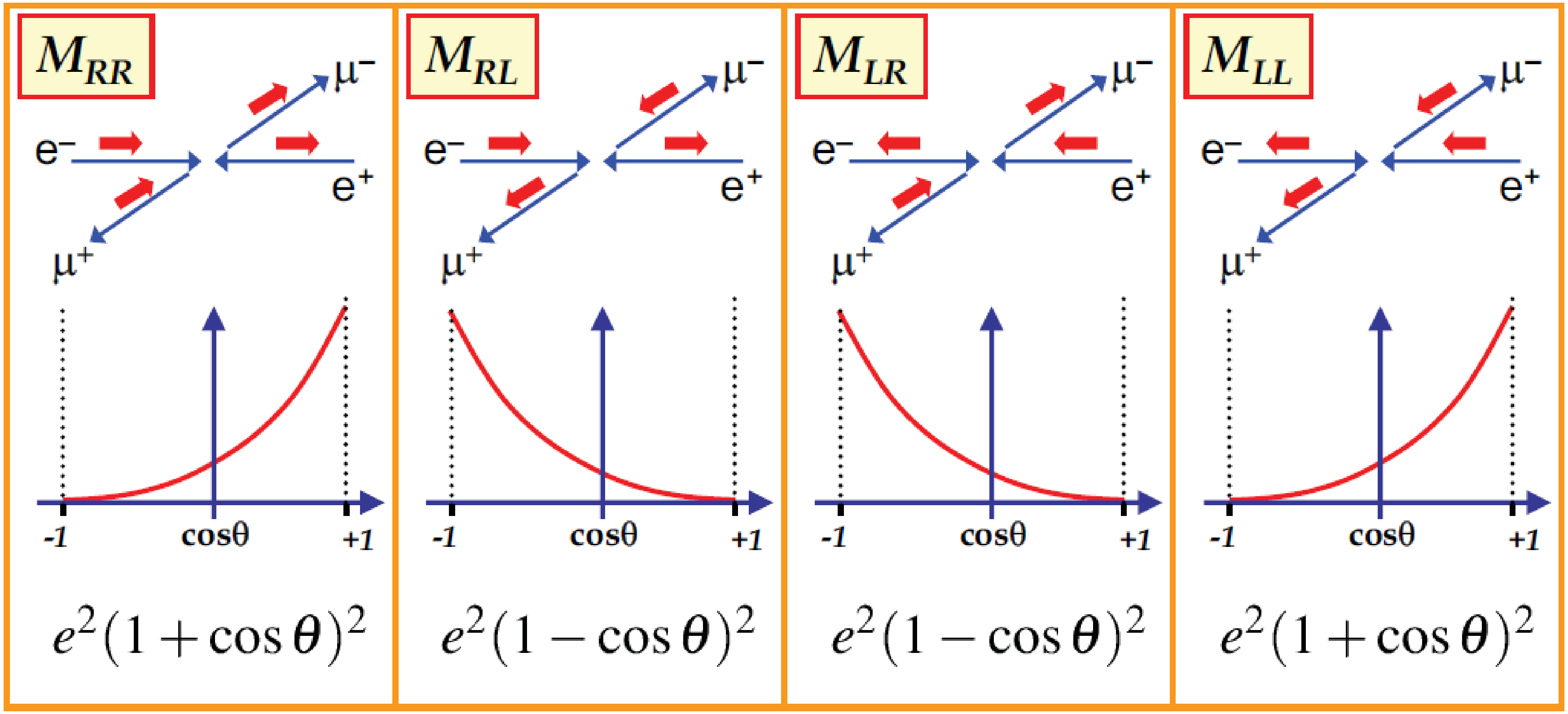
$$M_{RR} = -\frac{e^2}{s}[2E(0,-1,-i,0)].[2E(0,-\cos\theta,i,\sin\theta)]$$

$$= -e^2(1+\cos\theta)$$

$$= -4\pi\alpha(1+\cos\theta)$$

where $\alpha = e^2/4\pi \approx 1/137$

Similarly $$|M_{RR}|^2 = |M_{LL}|^2 = (4\pi\alpha)^2(1+\cos\theta)^2$$
$$|M_{RL}|^2 = |M_{LR}|^2 = (4\pi\alpha)^2(1-\cos\theta)^2$$



- Assuming that the incoming electrons and positrons are unpolarized, all 4 possible initial helicity states are equally likely.

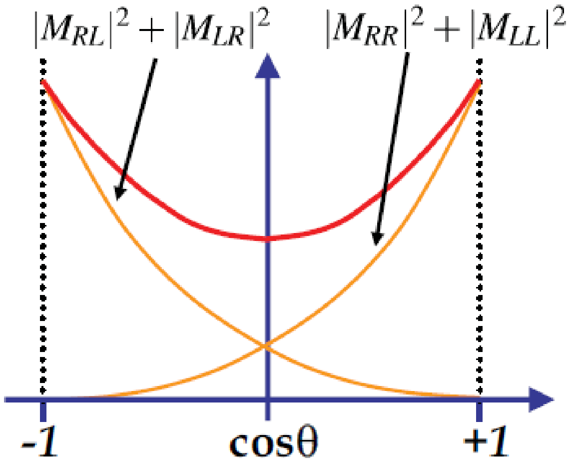
Differential cross-section

- The cross section is obtained by averaging over the initial spin states and summing over the final spin states:

$$\frac{d\sigma}{d\Omega} = \frac{1}{4} \times \frac{1}{64\pi^2 s}(|M_{RR}|^2 + |M_{RL}|^2 + |M_{LR}|^2 + |M_{LL}^2|)$$

$$= \frac{(4\pi\alpha)^2}{256\pi^2 s}(2(1+\cos\theta)^2 + 2(1-\cos\theta)^2)$$

$$\Rightarrow \boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta)}$$

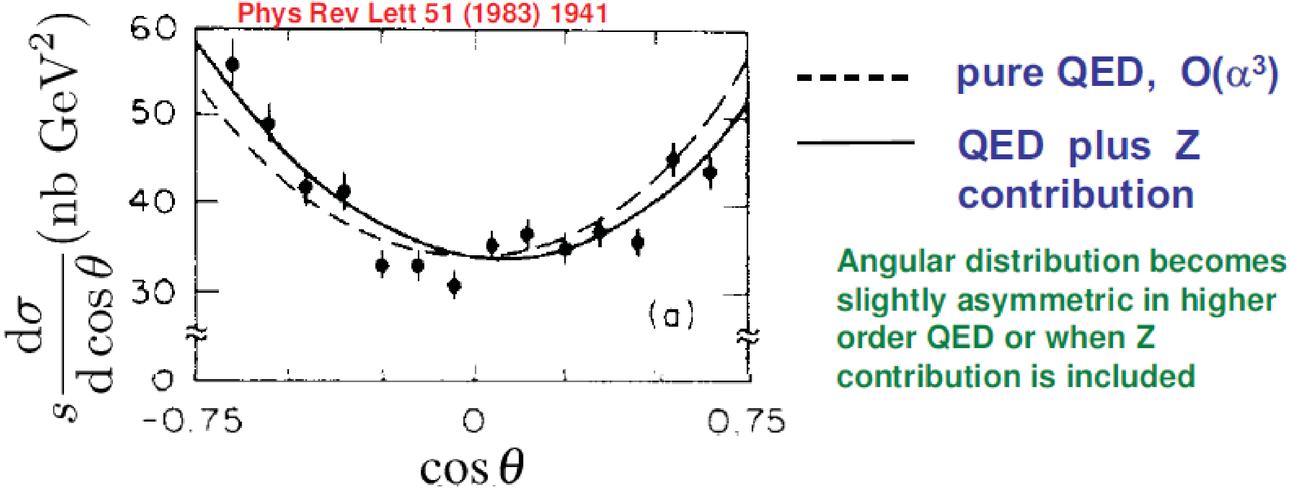


Example:

$e^+e^- \rightarrow \mu^+\mu^-$

$\sqrt{s} = 29$ GeV

Mark II Expt., M.E.Levi et al., Phys Rev Lett 51 (1983) 1941



Angular distribution becomes slightly asymmetric in higher order QED or when Z contribution is included

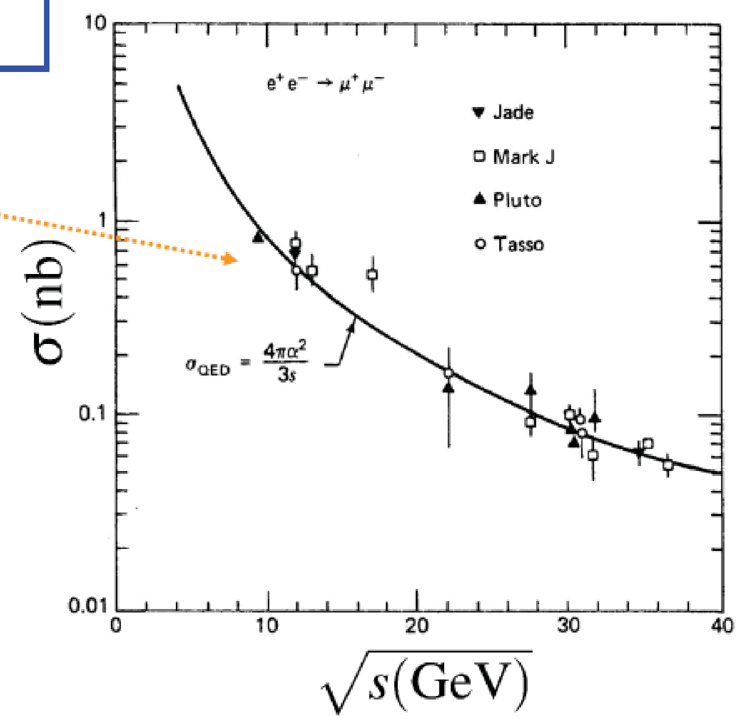
- The total cross section is obtained by integrating over $\theta, \ \phi$ using

$$\int (1+\cos^2\theta)\mathrm{d}\Omega = 2\pi \int_{-1}^{+1} (1+\cos^2\theta)\mathrm{d}\cos\theta = \frac{16\pi}{3}$$

giving the **QED** total cross-section for the process $e^+e^- \rightarrow \mu^+\mu^-$

$$\sigma = \frac{4\pi\alpha^2}{3s}$$

★ Lowest order cross section calculation provides a good description of the data !



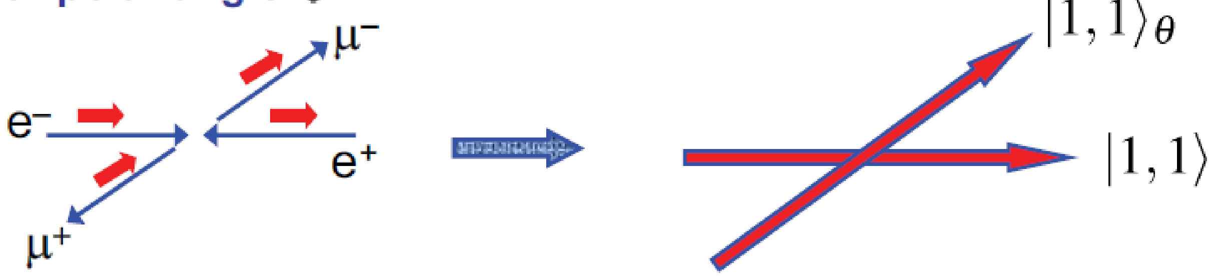
This is an impressive result. From first principles we have arrived at an expression for the electron-positron annihilation cross section which is good to 1%

Spin considerations (E>>m)

★ The angular dependence of the QED electron-positron matrix elements can be understood in terms of angular momentum

- Because of the allowed helicity states, the electron and positron interact in a spin state with $S_z = \pm 1$, i.e. in a total spin 1 state aligned along the z axis: $|1,+1\rangle$ or $|1,-1\rangle$
- Similarly the muon and anti-muon are produced in a total spin 1 state aligned along an axis with polar angle θ

e.g. M_{RR}



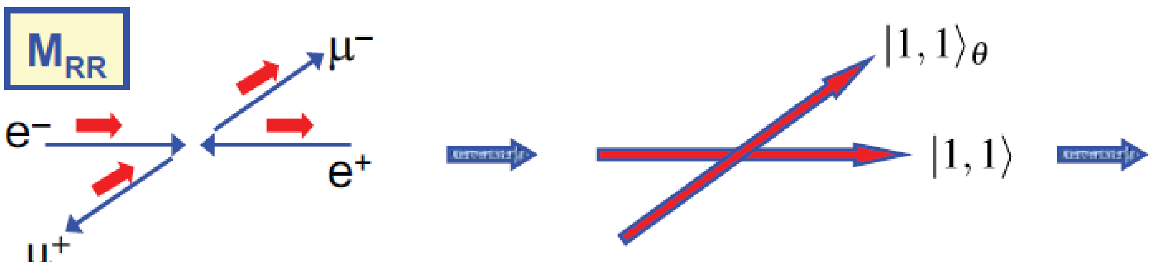
- Hence $M_{RR} \propto \langle \psi | 1, 1 \rangle$ where ψ corresponds to the spin state, $|1,1\rangle_\theta$, of the muon pair.
- To evaluate this need to express $|1,1\rangle_\theta$ in terms of eigenstates of S_z
- In the appendix it is shown that

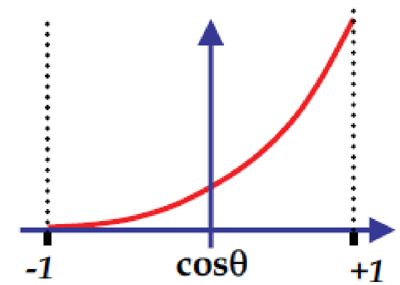
$$|1,1\rangle_\theta = \tfrac{1}{2}(1-\cos\theta)|1,-1\rangle + \tfrac{1}{\sqrt{2}}\sin\theta|1,0\rangle + \tfrac{1}{2}(1+\cos\theta)|1,+1\rangle$$

- Using the wave-function for a spin 1 state along an axis at angle θ

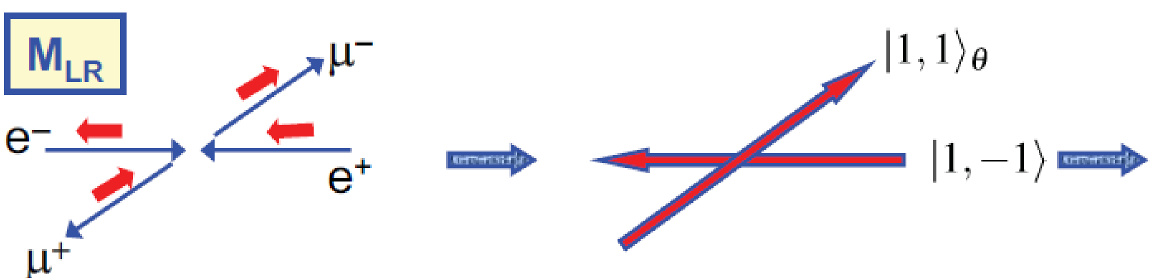
$$\psi = |1,1\rangle_\theta = \tfrac{1}{2}(1-\cos\theta)|1,-1\rangle + \tfrac{1}{\sqrt{2}}\sin\theta|1,0\rangle + \tfrac{1}{2}(1+\cos\theta)|1,+1\rangle$$

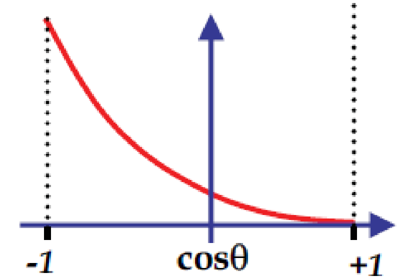
can immediately understand the angular dependence





$$|M_{RR}|^2 \propto |\langle\psi|1,+1\rangle|^2 = \tfrac{1}{4}(1+\cos\theta)^2$$



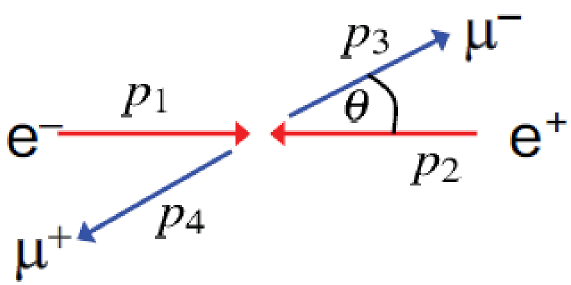


$$|M_{LR}|^2 \propto |\langle\psi|1,-1\rangle|^2 = \tfrac{1}{4}(1-\cos\theta)^2$$

Lorentz invariant form of ME

- Before concluding this discussion, note that the spin-averaged Matrix Element derived above is written in terms of the muon angle in the C.o.M. frame.

$$\begin{aligned}\langle|M_{fi}|^2\rangle &= \frac{1}{4}\times(|M_{RR}|^2+|M_{RL}|^2+|M_{LR}|^2+|M_{LL}^2|)\\ &= \frac{1}{4}e^4(2(1+\cos\theta)^2+2(1-\cos\theta)^2)\\ &= e^4(1+\cos^2\theta)\end{aligned}$$



- The matrix element is Lorentz Invariant (scalar product of 4-vector currents) and it is desirable to write it in a frame-independent form, i.e. express in terms of Lorentz Invariant 4-vector scalar products
- In the C.o.M. $\quad p_1=(E,0,0,E) \quad p_2=(E,0,0,-E)$

$$p_3=(E,E\sin\theta,0,E\cos\theta) \quad p_4=(E,-E\sin\theta,0,-E\cos\theta)$$

giving: $\quad p_1.p_2=2E^2; \quad p_1.p_3=E^2(1-\cos\theta); \quad p_1.p_4=E^2(1+\cos\theta)$

- Hence we can write

$$\boxed{\langle|M_{fi}|^2\rangle = 2e^4\frac{(p_1.p_3)^2+(p_1.p_4)^2}{(p_1.p_2)^2}} \qquad \equiv 2e^4\left(\frac{t^2+u^2}{s^2}\right)$$

★Valid in any frame !

Chirality

- The helicity eigenstates for a particle/anti-particle for $E \gg m$ are:

$$u_\uparrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix};\ u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix};\ v_\uparrow = \sqrt{E}\begin{pmatrix} s \\ -ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix};\ v_\downarrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}$$

where $s = \sin\frac{\theta}{2};\quad c = \cos\frac{\theta}{2}$

- Define the matrix

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

- In the limit $E \gg m$ the helicity states are also eigenstates of γ^5

$$\gamma^5 u_\uparrow = +u_\uparrow;\ \ \gamma^5 u_\downarrow = -u_\downarrow;\ \ \gamma^5 v_\uparrow = -v_\uparrow;\ \ \gamma^5 v_\downarrow = +v_\downarrow$$

★ In general, define the eigenstates of γ^5 as LEFT and RIGHT HANDED CHIRAL states $u_R;\quad u_L;\quad v_R;\quad v_L$

i.e. $\gamma^5 u_R = +u_R;\ \ \gamma^5 u_L = -u_L;\ \ \gamma^5 v_R = -v_R;\ \ \gamma^5 v_L = +v_L$

- In the LIMIT $E \gg m$ (and ONLY IN THIS LIMIT):

$$u_R \equiv u_\uparrow;\quad u_L \equiv u_\downarrow;\quad v_R \equiv v_\uparrow;\quad v_L \equiv v_\downarrow$$

Chirality

★ This is a subtle but important point: in general the **HELICITY** and **CHIRAL** eigenstates are not the same. It is only in the ultra-relativistic limit that the chiral eigenstates correspond to the helicity eigenstates.

★ Chirality is an import concept in the structure of QED, and any interaction of the form $\overline{u}\gamma^{\nu}u$

- In general, the eigenstates of the chirality operator are:

$$\gamma^5 u_R = +u_R; \quad \gamma^5 u_L = -u_L; \quad \gamma^5 v_R = -v_R; \quad \gamma^5 v_L = +v_L$$

- Define the projection operators:

$$P_R = \tfrac{1}{2}(1+\gamma^5); \qquad P_L = \tfrac{1}{2}(1-\gamma^5)$$

- The projection operators, project out the chiral eigenstates

$$P_R u_R = u_R; \quad P_R u_L = 0; \quad P_L u_R = 0; \quad P_L u_L = u_L$$
$$P_R v_R = 0; \quad P_R v_L = v_L; \quad P_L v_R = v_R; \quad P_L v_L = 0$$

- Note P_R projects out right-handed particle states and left-handed anti-particle states
- We can then write any spinor in terms of it left and right-handed chiral components:

$$\psi = \psi_R + \psi_L = \tfrac{1}{2}(1+\gamma^5)\psi + \tfrac{1}{2}(1-\gamma^5)\psi$$

Chirality in QED

- In QED the basic interaction between a fermion and photon is:

$$ie\overline{\psi}\gamma^{\mu}\phi$$

- Can decompose the spinors in terms of Left and Right-handed chiral components:

$$\begin{aligned} ie\overline{\psi}\gamma^{\mu}\phi &= ie(\overline{\psi}_L + \overline{\psi}_R)\gamma^{\mu}(\phi_R + \phi_L) \\ &= ie(\overline{\psi}_R\gamma^{\mu}\phi_R + \overline{\psi}_R\gamma^{\mu}\phi_L + \overline{\psi}_L\gamma^{\mu}\phi_R + \overline{\psi}_L\gamma^{\mu}\phi_L) \end{aligned}$$

- Using the properties of γ^5

$$(\gamma^5)^2 = 1; \quad \gamma^{5\dagger} = \gamma^5; \quad \gamma^5\gamma^{\mu} = -\gamma^{\mu}\gamma^5$$

it is straightforward to show

$$\overline{\psi}_R\gamma^{\mu}\phi_L = 0; \quad \overline{\psi}_L\gamma^{\mu}\phi_R = 0$$

★ Hence only certain combinations of chiral eigenstates contribute to the interaction. This statement is **ALWAYS** true.

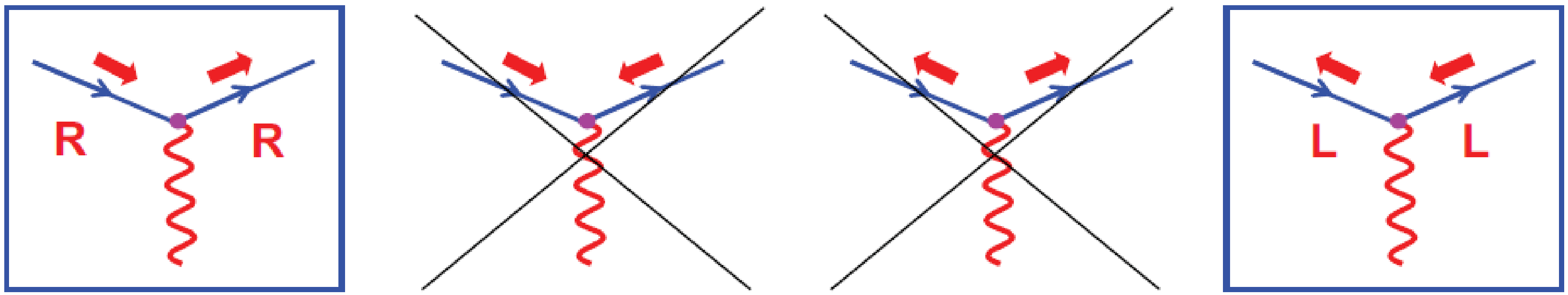
- For $E \gg m$, the chiral and helicity eigenstates are equivalent. This implies that for $E \gg m$ only certain helicity combinations contribute to the QED vertex ! This is why previously we found that for two of the four helicity combinations for the muon current were zero

Allowed QED helicity combinations

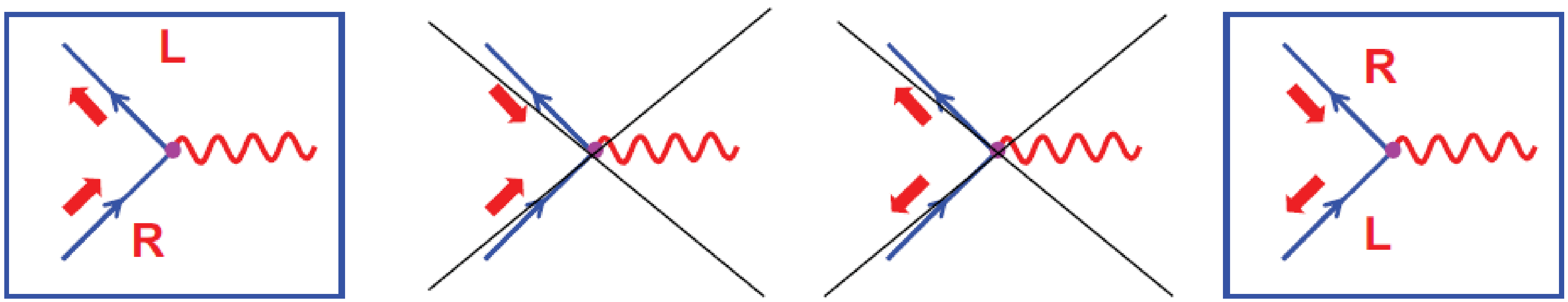
- In the ultra-relativistic limit the helicity eigenstates ≡ chiral eigenstates
- In this limit, the only non-zero helicity combinations in QED are:

Scattering:

"Helicity conservation"



Annihilation:



Summary

★ In the centre-of-mass frame the $e^+e^- \rightarrow \mu^+\mu^-$ differential cross-section is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta)$$

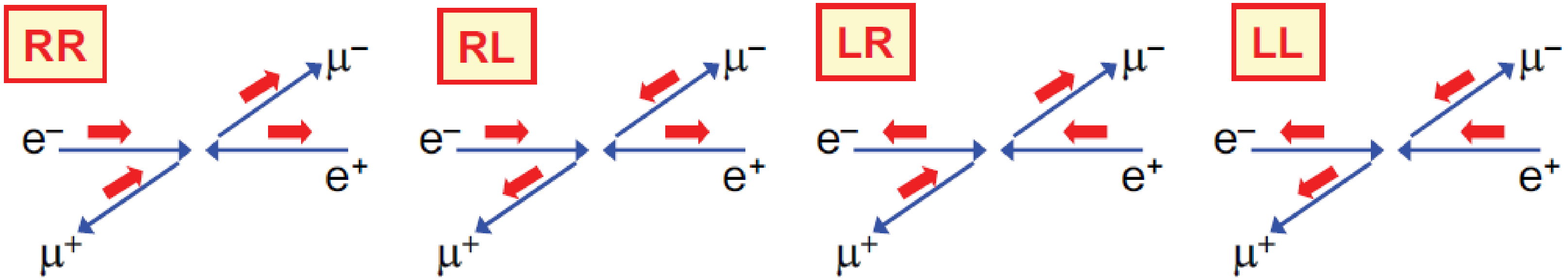
NOTE: neglected masses of the muons, i.e. assumed $E \gg m_\mu$

★ In QED only certain combinations of LEFT- and RIGHT-HANDED CHIRAL states give non-zero matrix elements

★ CHIRAL states defined by chiral projection operators

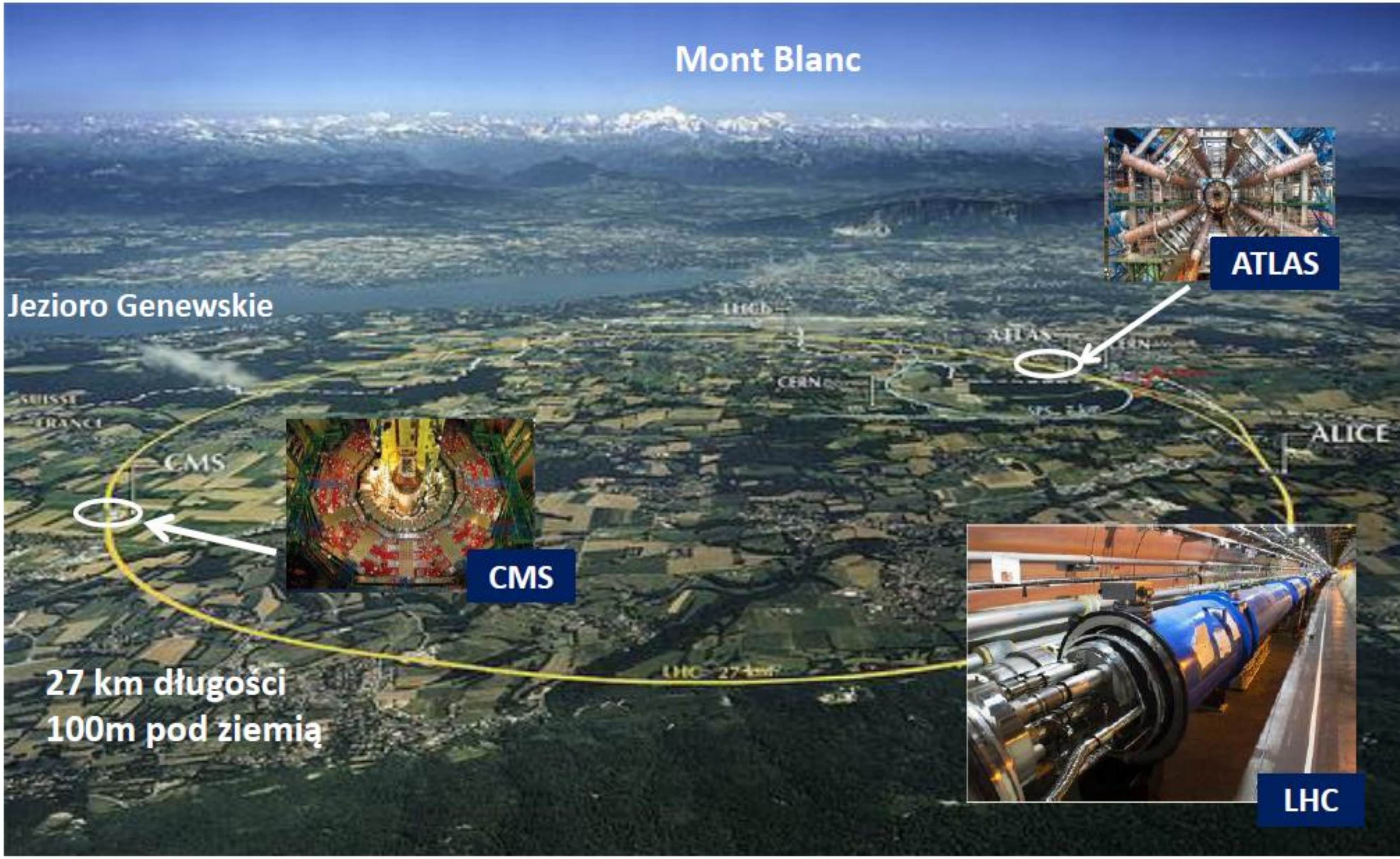
$$P_R = \tfrac{1}{2}(1+\gamma^5); \qquad P_L = \tfrac{1}{2}(1-\gamma^5)$$

★ In limit $E \gg m$ the chiral eigenstates correspond to the HELICITY eigenstates and only certain HELICITY combinations give non-zero matrix elements



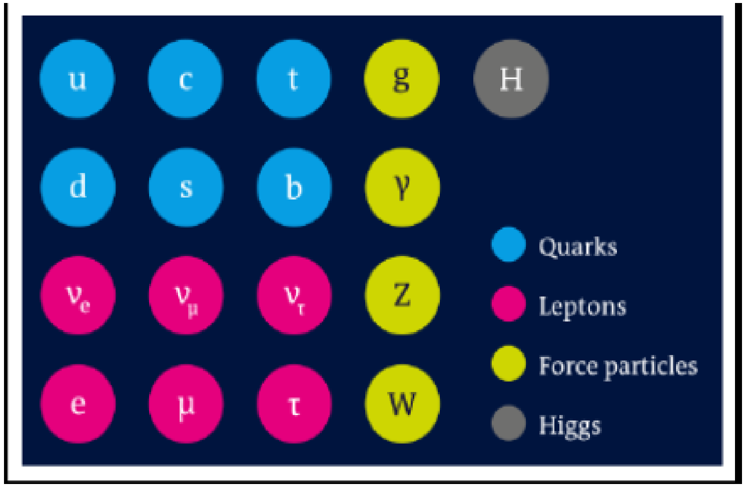
Detectors for high energy physics (ATLAS at LHC)

LHC (Large Hadron Collider)

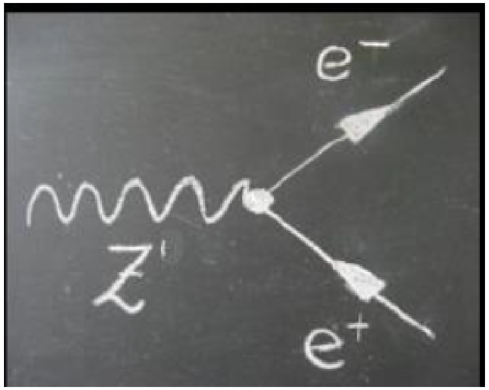


Which particles are detected?

1) **Charged leptons, photons and hadrons: e, μ,γ,π,K,p,n…** (maybe new long-lived particles, i.e. particles which enter detector)
2) B (and D) mesons and τ leptons have cτ ~ 0.09 ..0. x 10^{-3}m large enough for additional vertex reconstruction
3) Neutrinos (maybe also new particles) are reconstructed as mising transverse momentum
4) All other particles which decay or hadronise in primary vertex (top quark decays before hadronises)

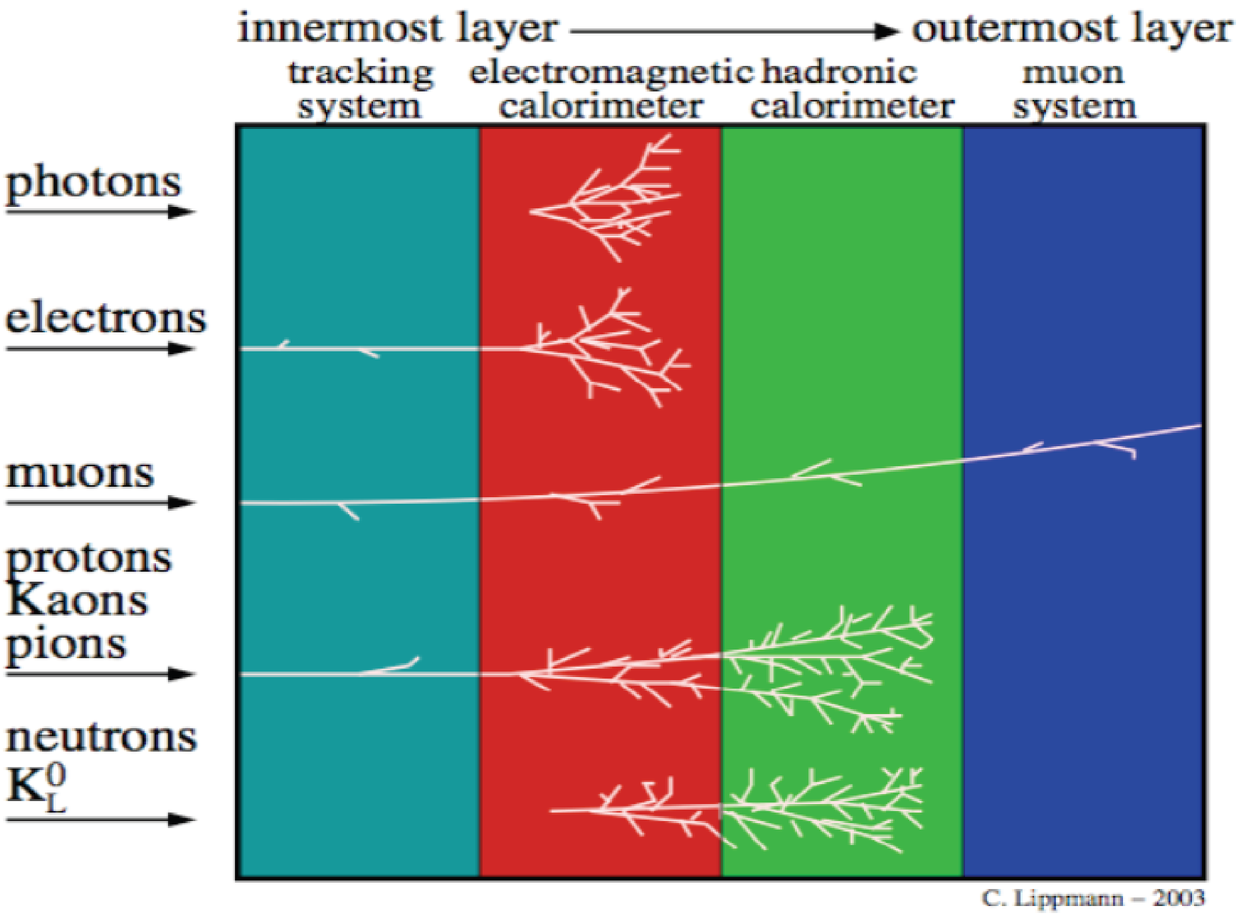


Only e, μ, γ of the fundamental Standard Model Particles are directly detected



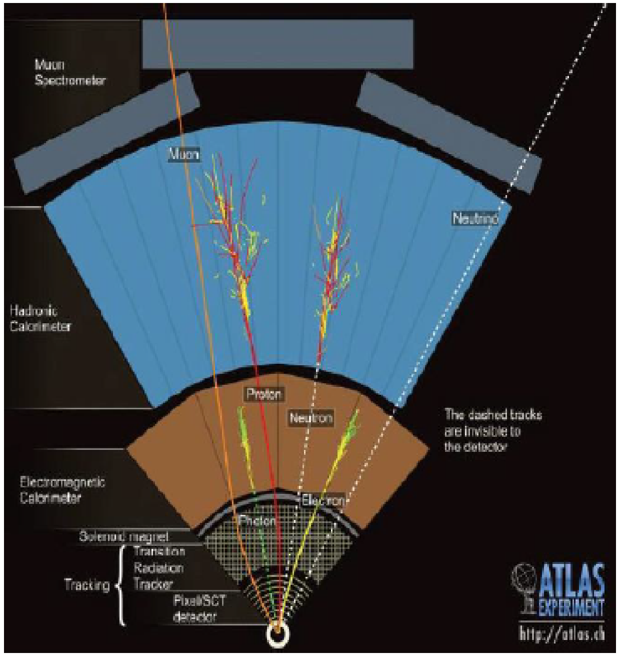
Heavy particles W, Z decay immediatelly

Sketch of particles interaction with detector



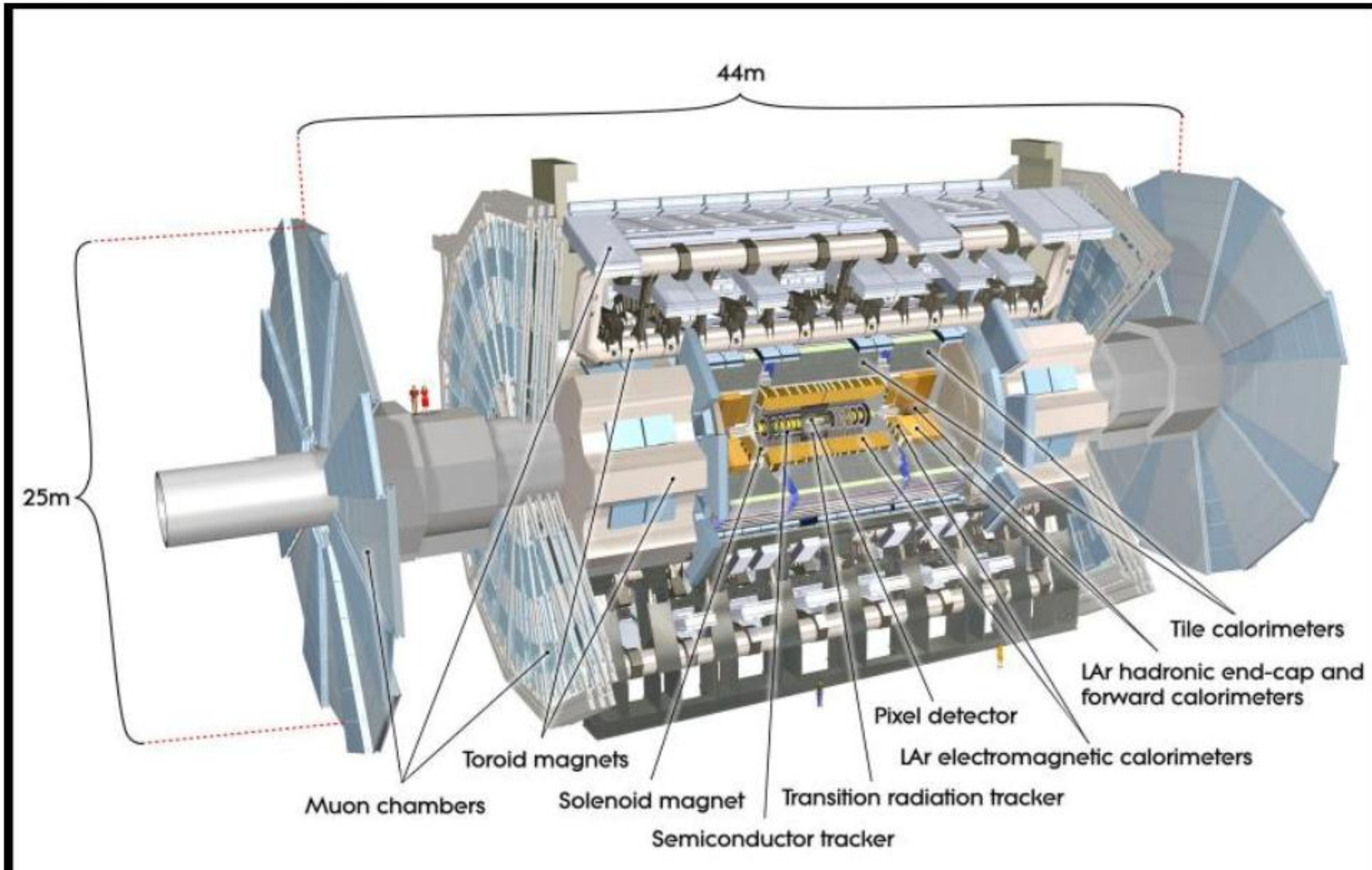
The observables?

1) Photon makes photo-efect, Compton scattering and **pair production**. It has no track but an **electromagnetic cascade** in the calorimeter.
2) Charged particles makes scattering, **ionisation**, excitation and bremsstrahlung, transition and cherenkov radiation. They produce **tracks**.
3) Electrons make **electromagnetic cascades** (clusters) in the calorimeter
4) Hadrons also interact strongly via inelastic interactions, e.g. neutron capture, induced fission, etc. They make **hadronic cascades** (clusters) int he hadronic calorimeter.
5) Only weakly interacting particles (neutrinos) are reconstructed as **missing transverse momentum** („missing energy”).



The ATLAS example

Typical 4π cylindrical onion structure



Reconstructed properties

From the hits, tracks, clusters, missing transverse momentum and vertices we reconstruct the particles properties:

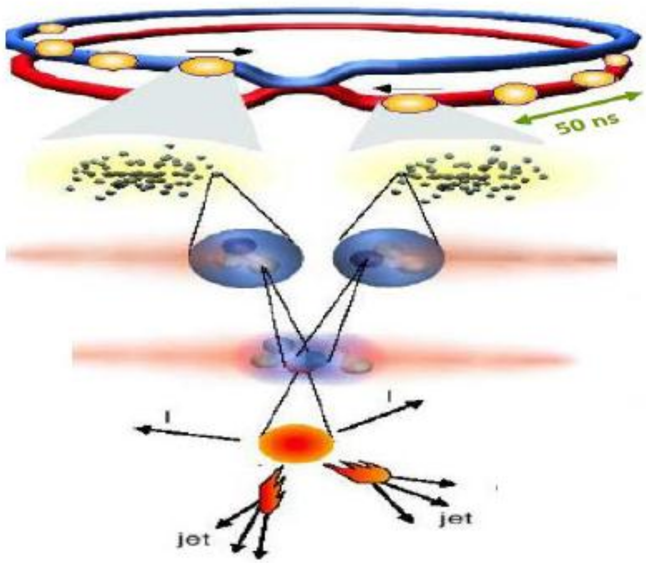
1) Momentum from curved tracks
2) Charge from track curvature
3) Energy from full absorption in calorimeters and curved tracks
4) Spin from angular distributions
5) Mass from invariant mass from decay products
6) Lifetime from time of flight measurement
7) Identity from dE/dx, lifetime or special behaviour (like transition ratiation)

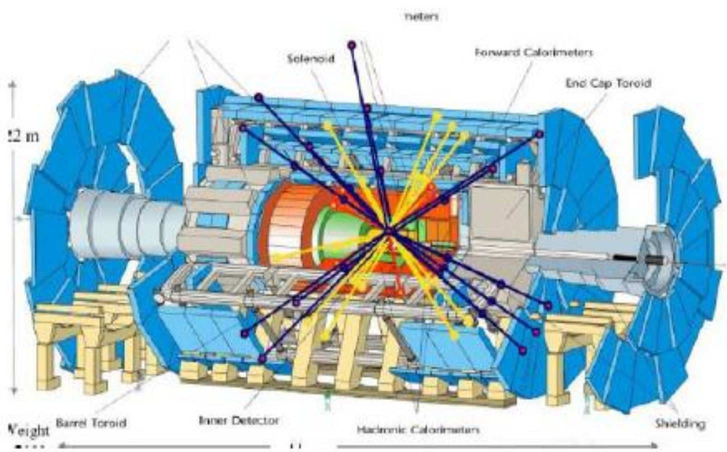
Detector design constraints (I)

- **Constraints from physics:**
  1) High detection efficiency demands minimal cracks and holes, high coverage
  2) High resolution demands little material like support structures, cables, cooling pipes, electronics etc. (avoid multiple scattering)
  3) Irradiation hard active materials to avoid degradation and changes during operation
  4) Low noise
  5) Easy maintenance (materials get radioactive)
  6) …

Detector design constraints (II)

- **Enviromental contraints, i.e. from LHC design parameters:**
  1) Collision events every ~ 25ns
  2) Muons from previous event still in detector when current enters tracker
  3) High occupancy in the inner detector
  4) Pile up (more proton proton collisions in each bunch crosing)
  5) High irradiation
  6) ...





Size and field examples

ATLAS barrel toroid
20.5 kA, 3.9 T

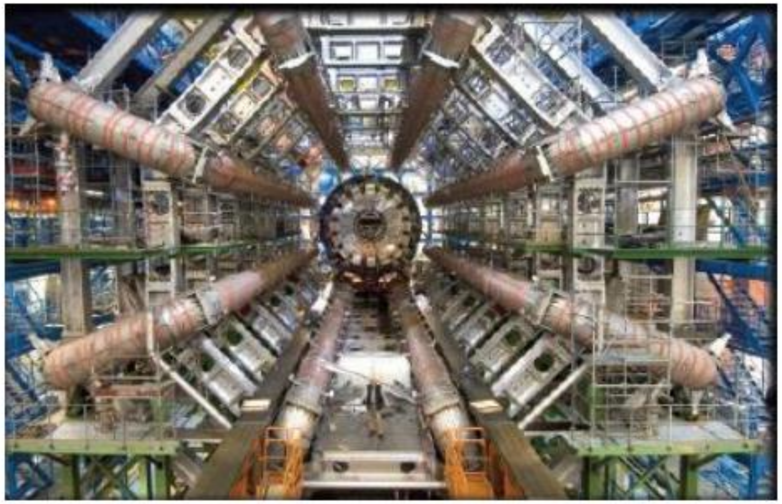
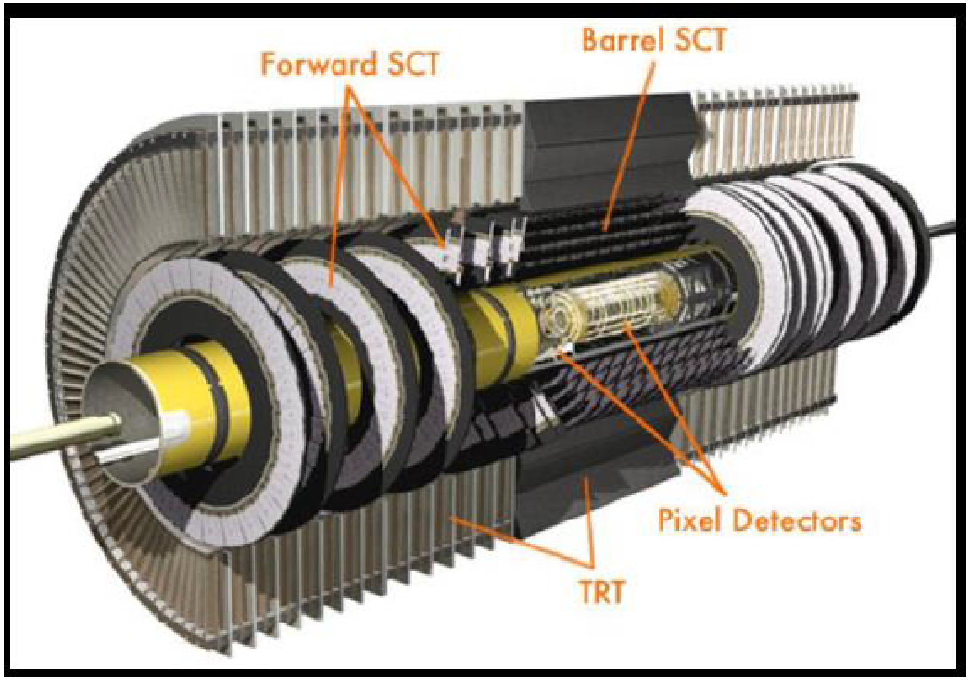


Table 1
Main parameters of some HEP detector magnets (solenoids).

	CDF	CLEO-II	ALEPH	ZEUS	H1	KLOE	BaBar	Atlas	CMS
B (T)	1.5	1.5	1.5	1.8	1.2	0.6	1.5	2.0	4.0
R (m)	1.5	1.55	2.7	1.5	2.8	2.6	1.5	1.25	3.0
L (m)	4.8	3.5	6.3	2.45	5.2	3.9	3.5	3.66	12.5

The magnet layout is a major constraint for the rest of the detector!
See A. Gadi, A magnet system for HEP experiments, NIMA 666 (2012) 10-24

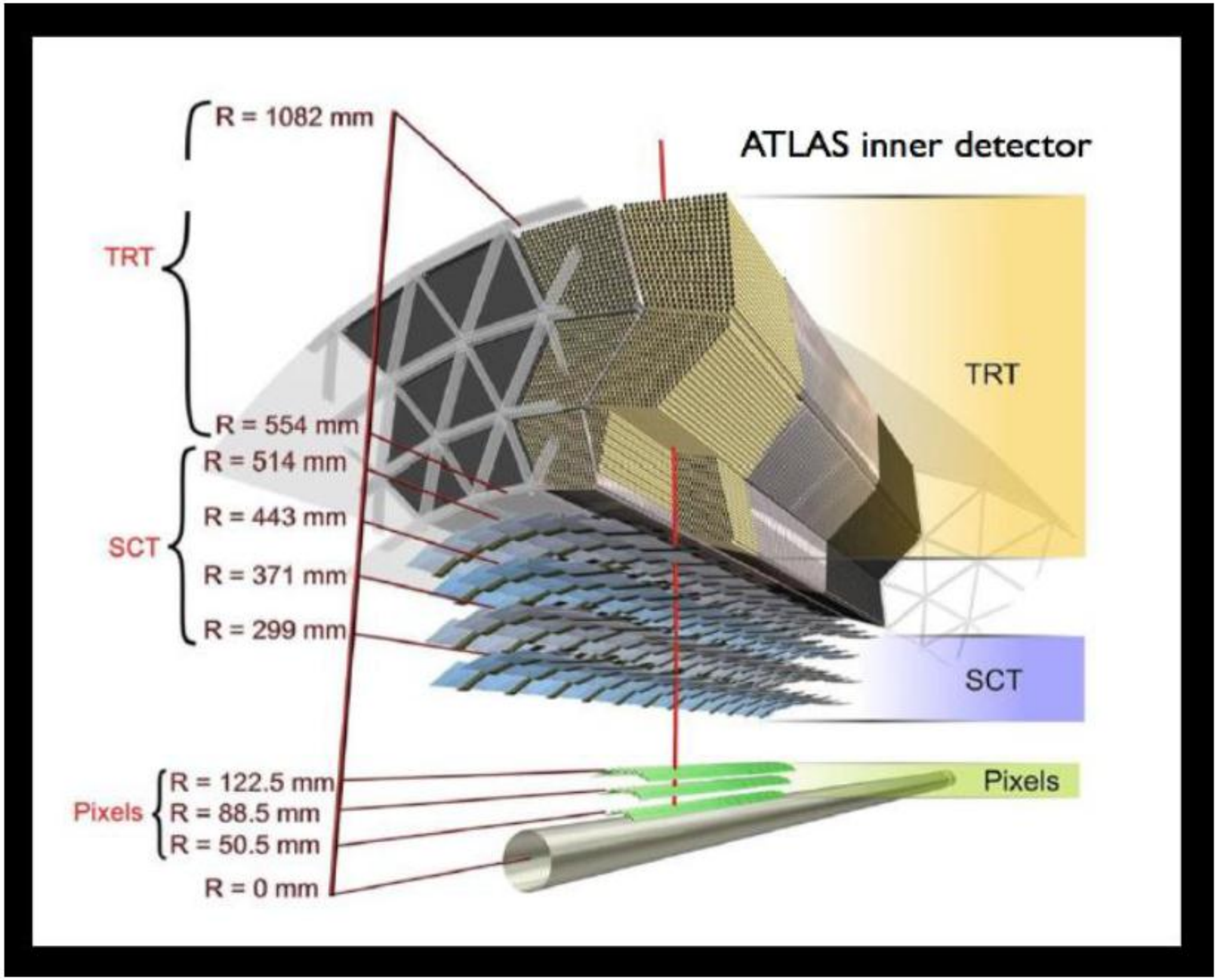
ATLAS Inner Detector



- 3 layers of pixel modules in barrel
- 2x5 disks of forward pixel disks
- 4 layers of strip (SCT) modules in barrel
- 2x9 disks of forward strip modules

Figure : ATLAS Inner detector (ID) in LHC run 1 with pixel and strip (SCT) silicon and transition radiation (TRT) detectors. The length is about 5.5 m.

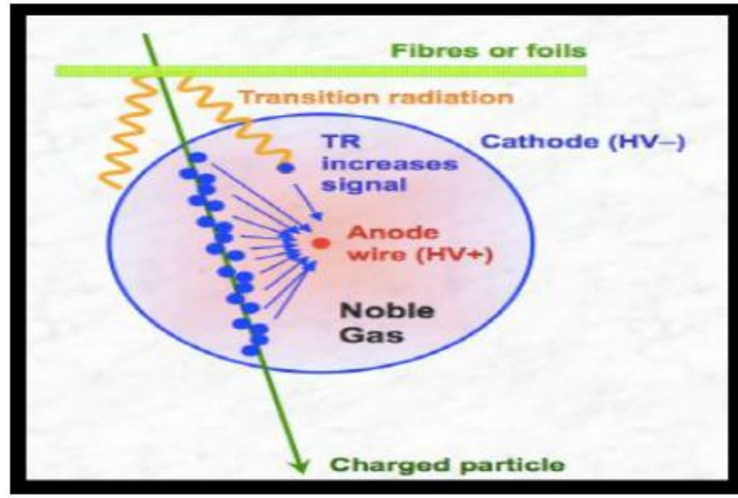
ATLAS Inner Detector

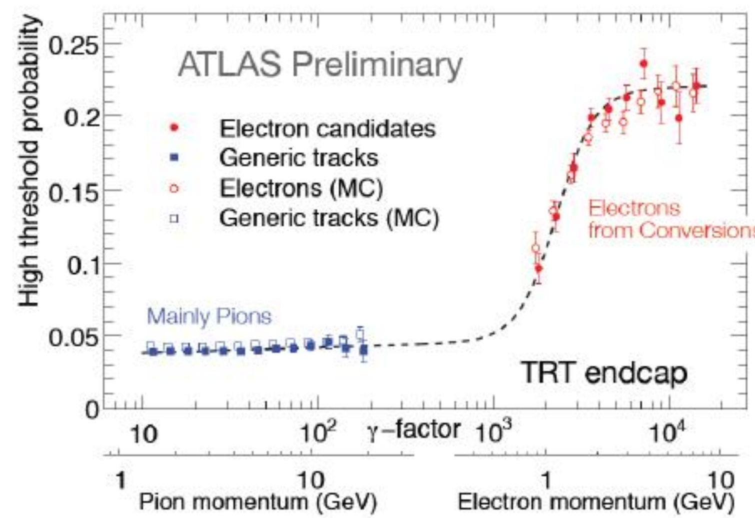


Transition Radiation Tracker

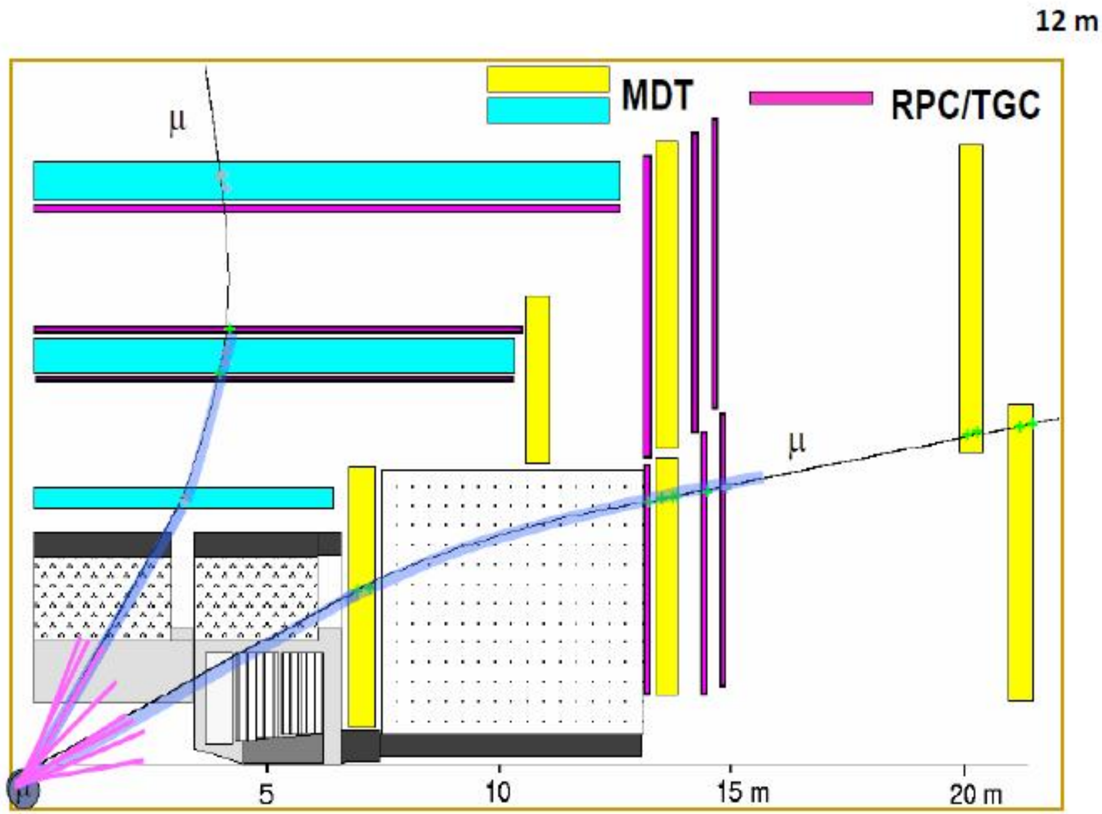
Combine tracking with particle identification (PID)

- Charged particles radiate photons when crossing material borders.
- E^{+-} radiate x-rays more than heavier particles.
- Use this particle PID, i.e. distinguis e^{+-} from hadrons.
- ATLAS has a TR detector in the inner detector. It uses gas for detection.

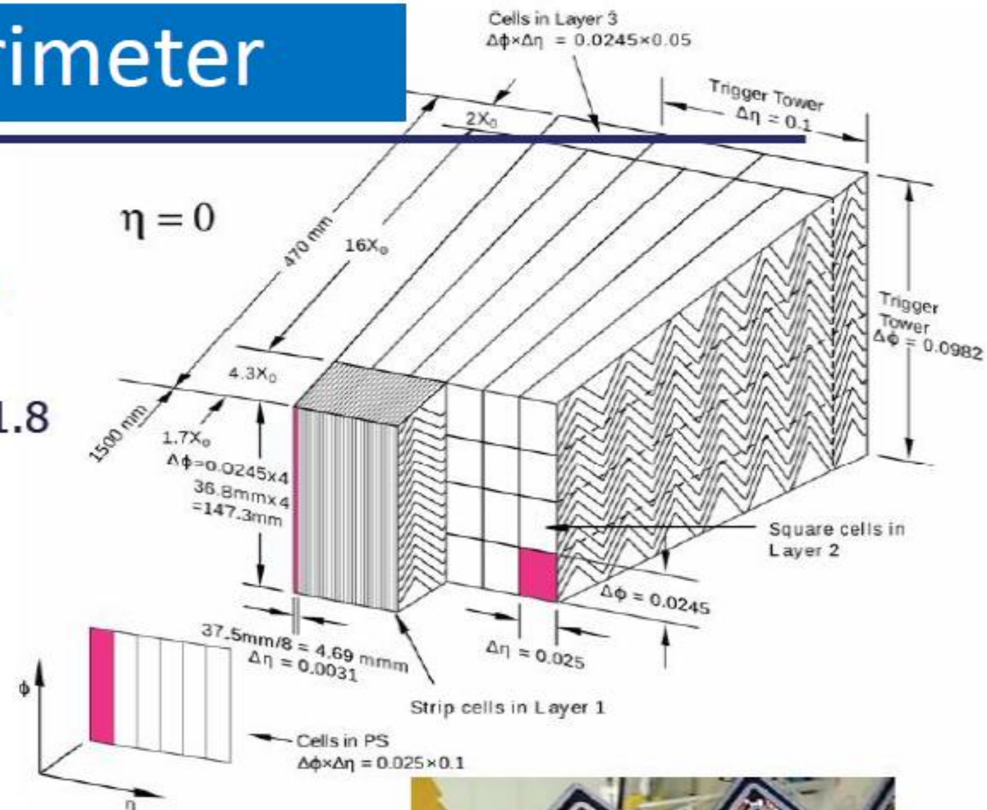




Muon system in ATLAS



ATLAS EM Calorimeter



Accordion Pb/LAr $|\eta|<3.2$ ~170k channels

- Precision measurement $|\eta|<2.5$

3 layers up to $|\eta|=2.5$ + presampler $|\eta|<1.8$

2 layers $2.5<|\eta|<3.2$

- Layer 1 (γ/π^0 rej. + angular meas.)
- $\Delta\eta.\Delta\varphi$ = 0.003 x 0.1
- Layer 2 (shower max)
- $\Delta\eta.\Delta\varphi$ = 0.025 x 0.0.25
- Layer 3 (Hadronic leakage)
- $\Delta\eta.\Delta\varphi$ = 0.05 x 0.0.025

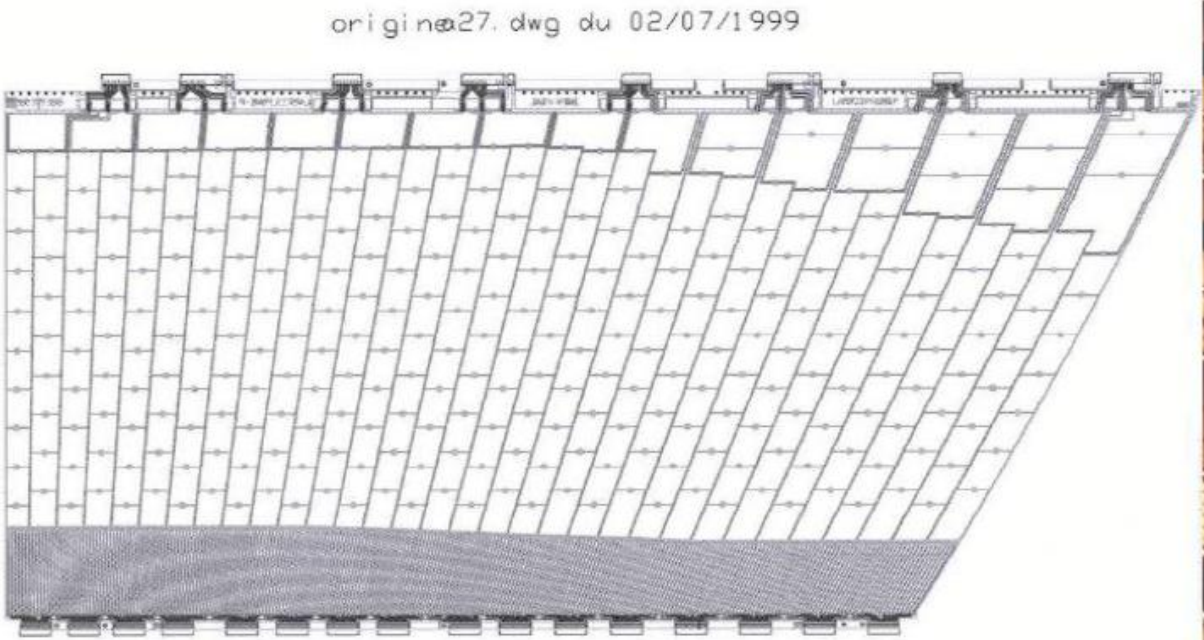
Energy Resolution: design for η~0

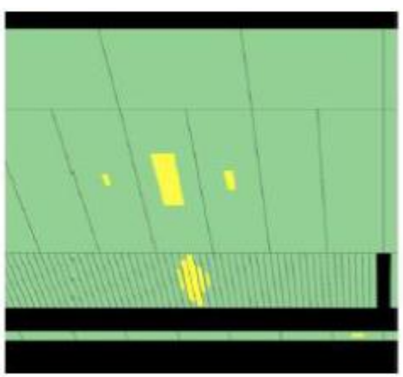
- $\Delta E/E \sim 10\%/\sqrt{E} \oplus 150\ \mathrm{MeV}/E \oplus 0.7\%$

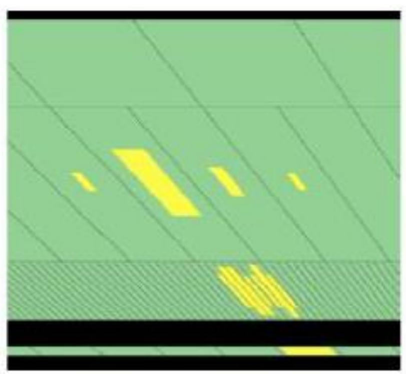
Angular Resolution

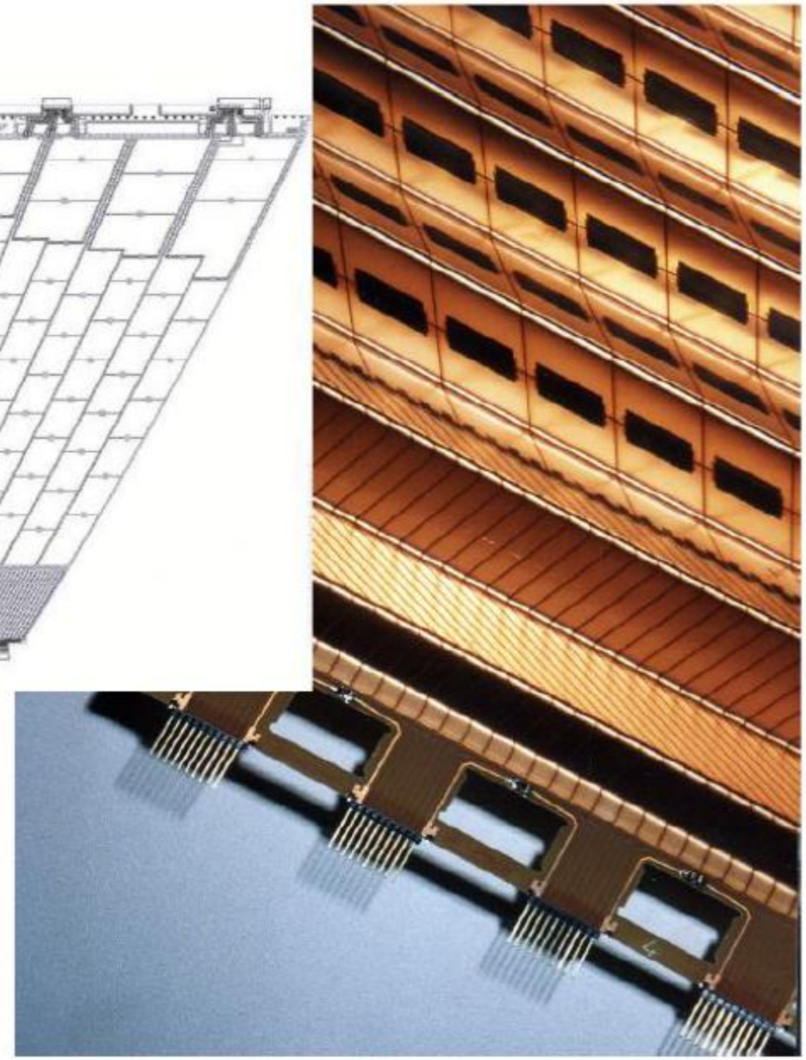
- 50mrad/$\sqrt{E}$(GeV)

The segmentation

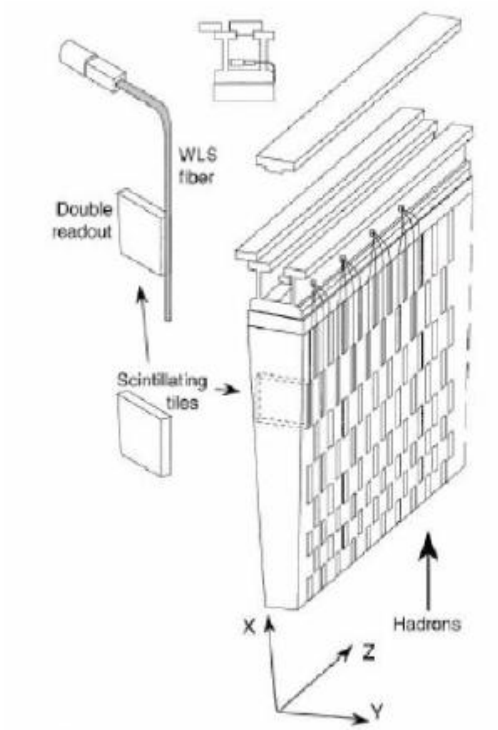








ATLAS Hadronic Calorimeter (Tile)



Fe/Scint with WLS fiber Readout via PMT

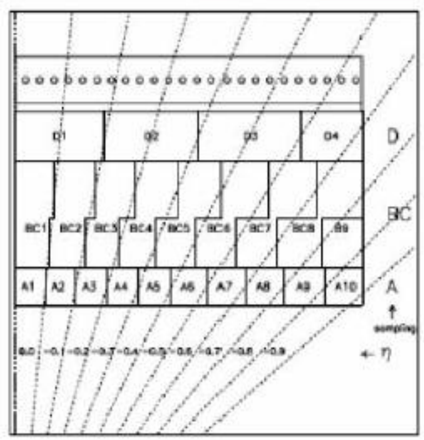


Figure 5-15 Cell geometry of half of a barrel module. The fibres of each cell are routed to one PMT.

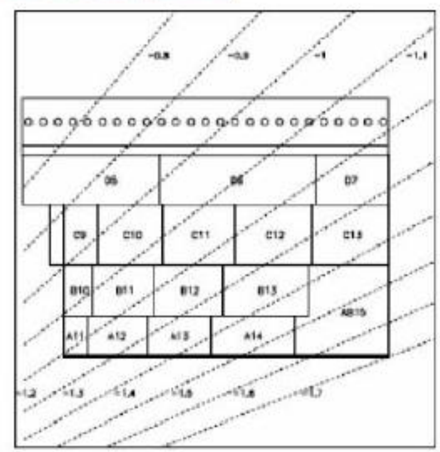


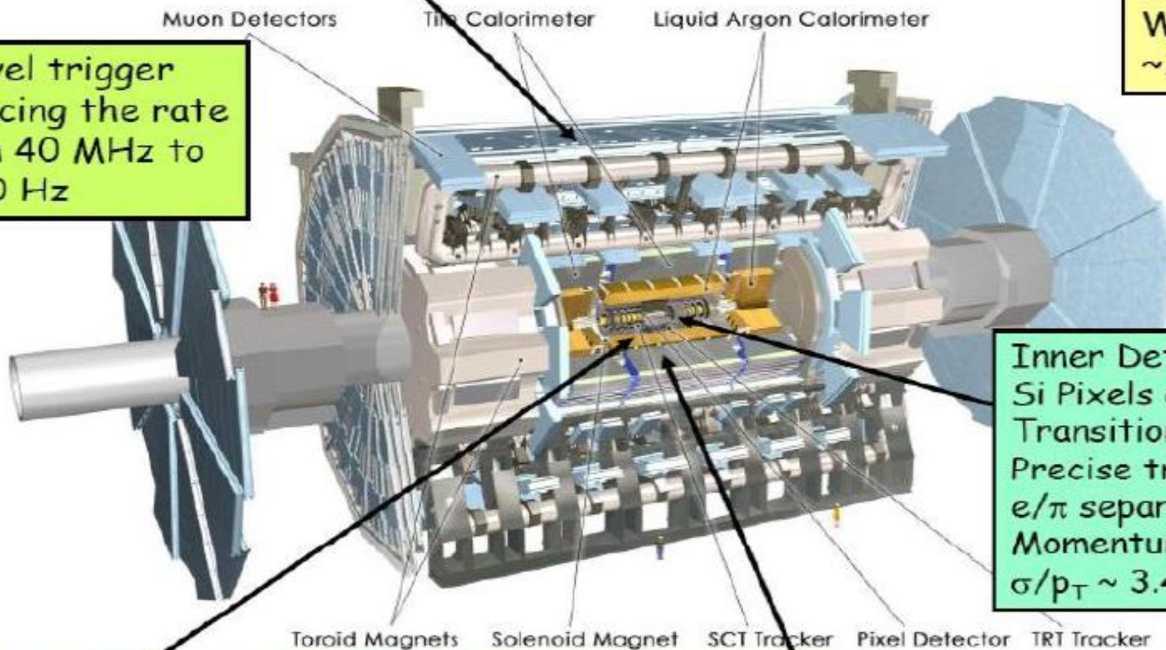
Figure 5-16 Proposed cell geometry for the extended barrel modules (version "a la barrel").

The ATLAS detector

Muon Spectrometer ($|\eta|<2.7$) : air-core toroids with gas-based chambers
Muon trigger and measurement with momentum resolution < 10% up to E_μ ~ TeV

Length : ~ 46 m
Radius : ~ 12 m
Weight : ~ 7000 tons
~10^8 electronic channels

3-level trigger reducing the rate from 40 MHz to ~200 Hz



Inner Detector ($|\eta|<2.5$, B=2T):
Si Pixels and strips (SCT) +
Transition Radiation straws
Precise tracking and vertexing,
e/π separation (TRT).
Momentum resolution:
$\sigma/p_T \sim 3.4\times10^{-4}\, p_T(\text{GeV}) \oplus 0.015$

EM calorimeter: Pb-LAr Accordion
e/γ trigger, identification and measurement
E-resolution: ~ 1% at 100 GeV, 0.5% at 1 TeV

HAD calorimetry ($|\eta|<5$): segmentation, hermeticity
Tilecal Fe/scintillator (central), Cu/W-LAr (fwd)
Trigger and measurement of jets and missing E_T
E-resolution: $\sigma/E \sim 50\%/\sqrt{E} \oplus 0.03$

Nuclear Instruments & Methods in Physics Research

topical issue

Instrumentation and detector technologies for frontier high energy physics

Volume 666, pages 1 - 222 (21 February 2012)

Edited by:
Archana Sharma (CERN)

Technological advances in radiation detection have been pioneered and led by particle physics. The ever increasing complexity of the experiments in high energy physics has driven the need for developments in high performance silicon and gaseous tracking detectors, electromagnetic and hadron calorimetry, transition radiation detectors and novel particle identification techniques. Magnet systems have evolved with superconducting magnets being used in present and, are being designed for use in, future experiments. The alignment system, being critical for the overall detector performance, has become one of the essential design aspects of large experiments. The electronic developments go hand in hand to enable the exploitation of these detectors designed to operate in the hostile conditions of radiation, high rate and luminosity.
This volume provides a panorama of the state-of-the-art in the field of radiation detection and instrumentation for large experiments at the present and future particle accelerators.