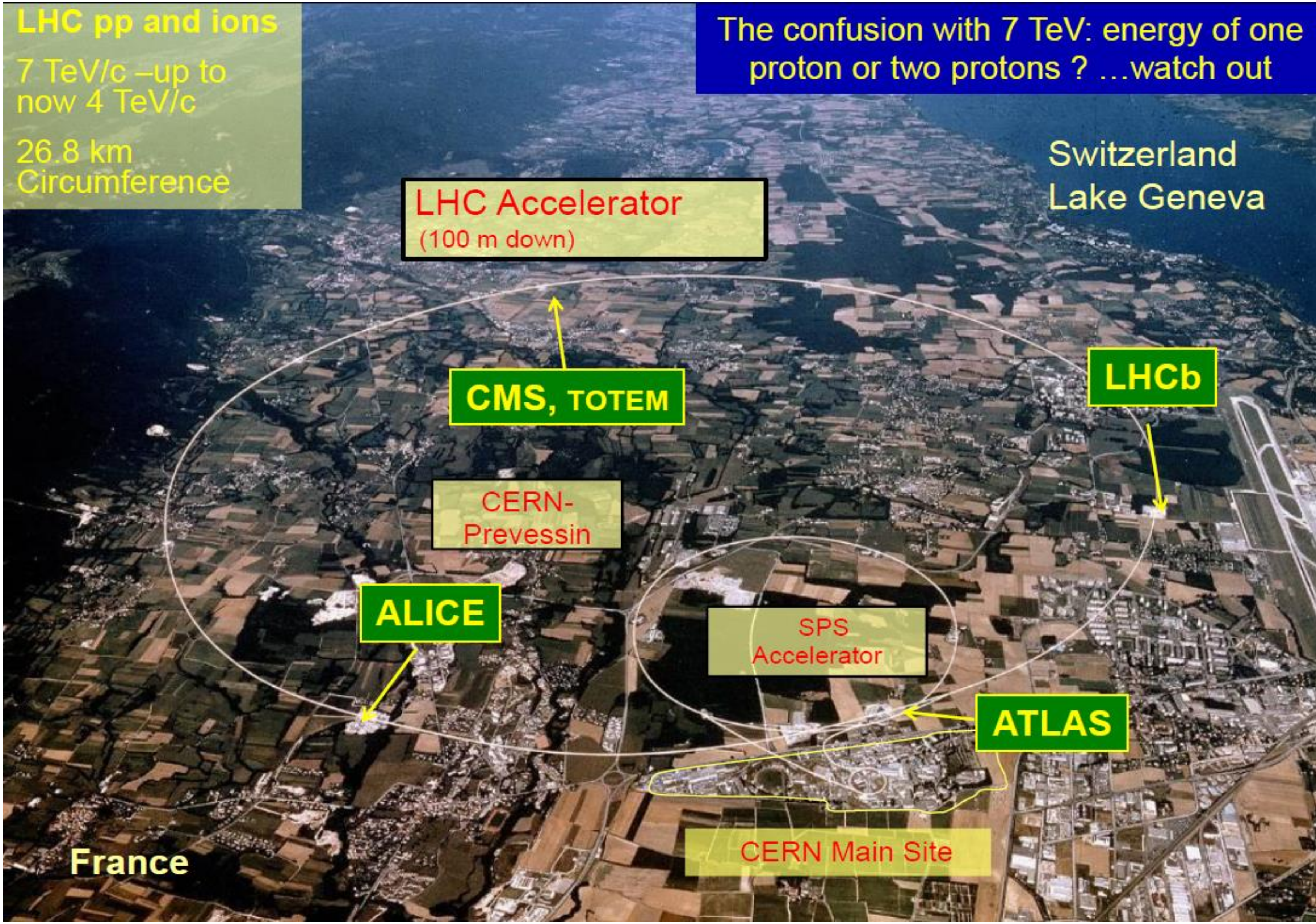
Elementary Particle Physics: theory and experiments

Standard Model measurements at LHC

Matrix elements & Feynman diagrams

Some slides taken from M. A. Thomson lectures at Cambridge University in 2011

Prof. dr hab. Elżbieta Richter-Wąs

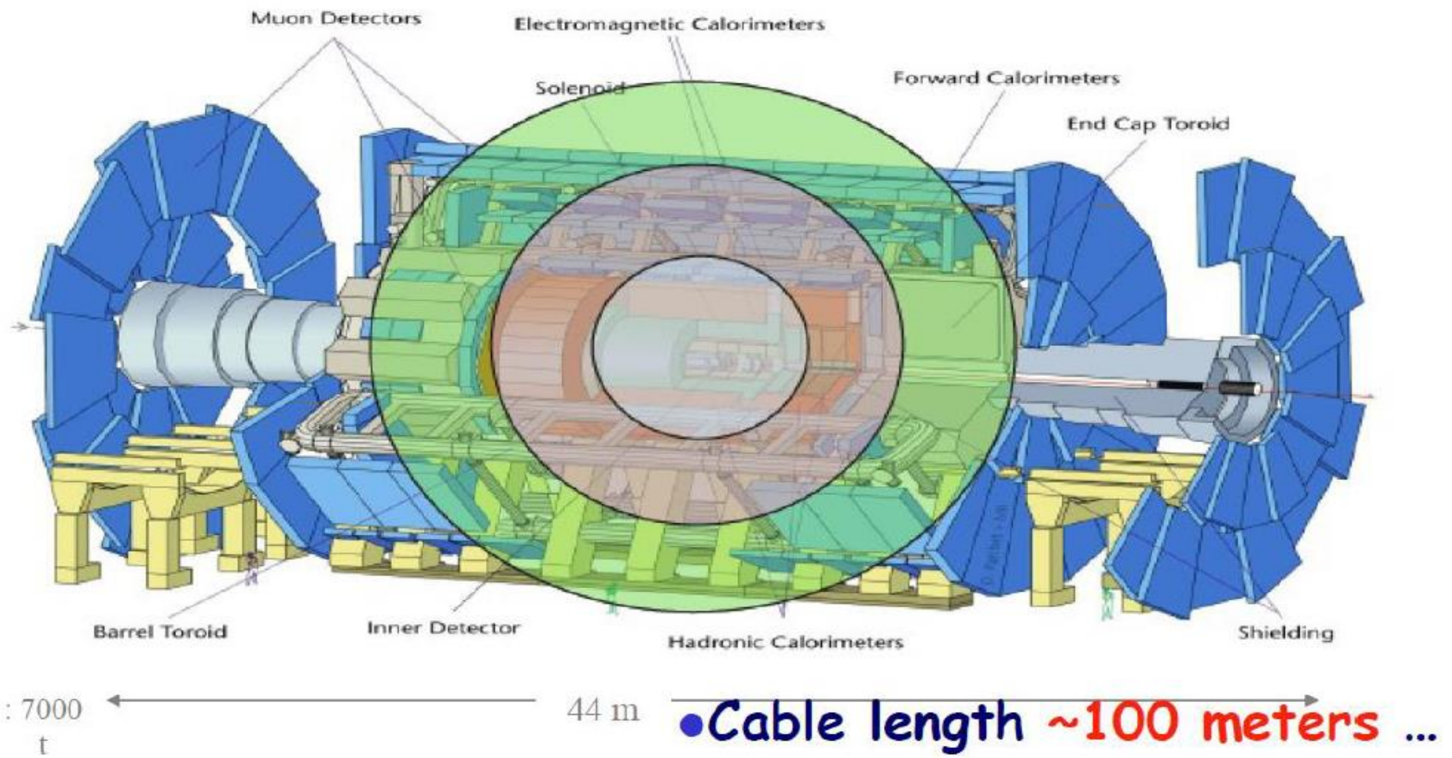


LHC pp and ions
7 TeV/c –up to now 4 TeV/c
26.8 km Circumference
The confusion with 7 TeV: energy of one proton or two protons ? …watch out
Switzerland
Lake Geneva
LHC Accelerator
(100 m down)
CMS, TOTEM
LHCb
CERN-Prevessin
ALICE
SPS Accelerator
ATLAS
CERN Main Site
France

Trigger system

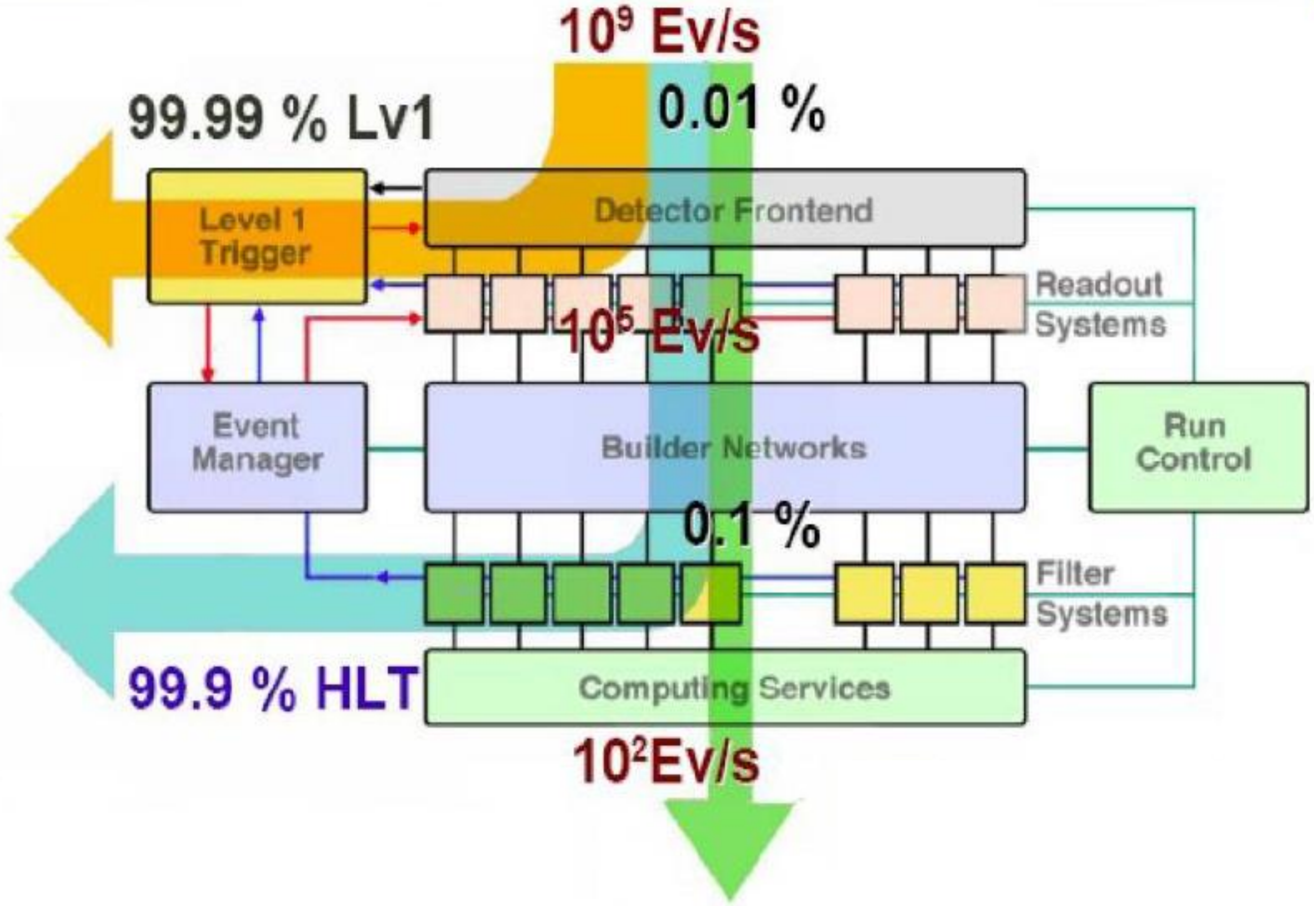
- **Interactions every 25 ns …**
  - In 25 ns particles travel 7.5 m

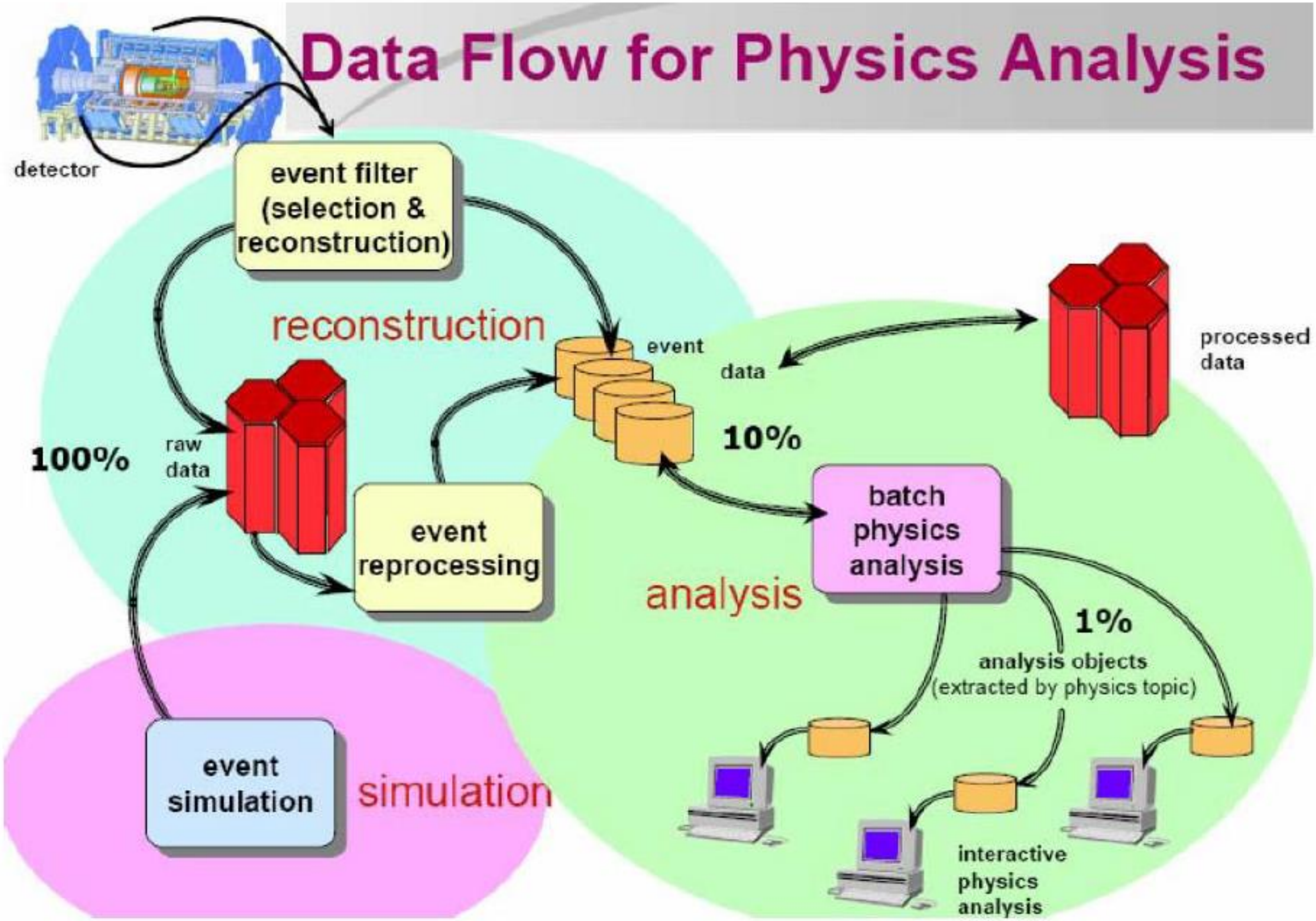
c=30cm/ns; in 25ns, s=7.5m

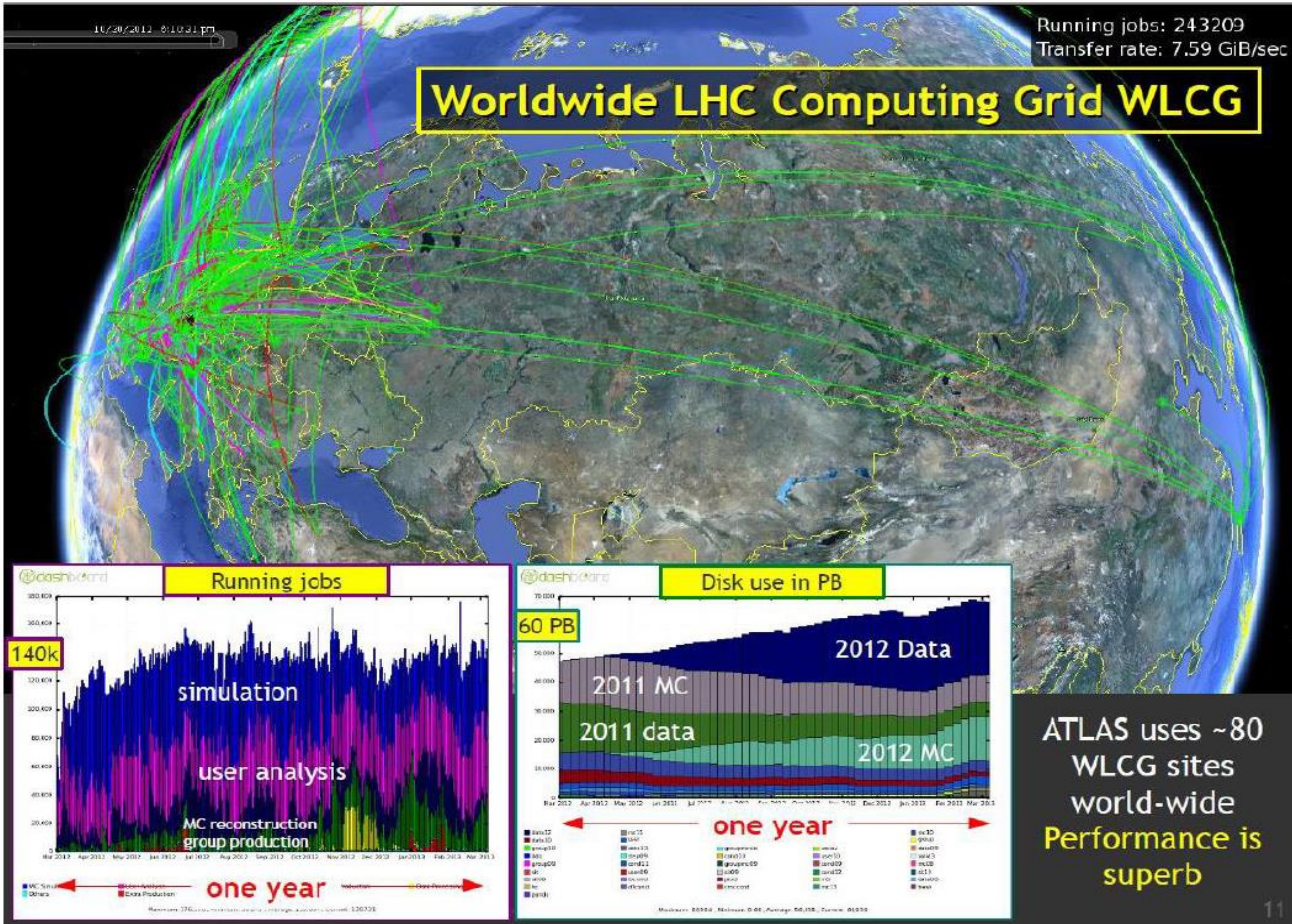


- **Cable length ~100 meters …**
  - In 25 ns signals travel 5 m

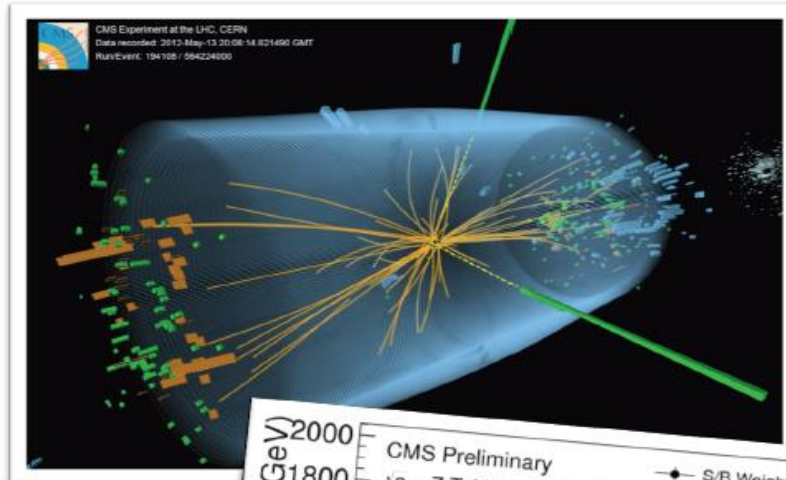
Trigger system

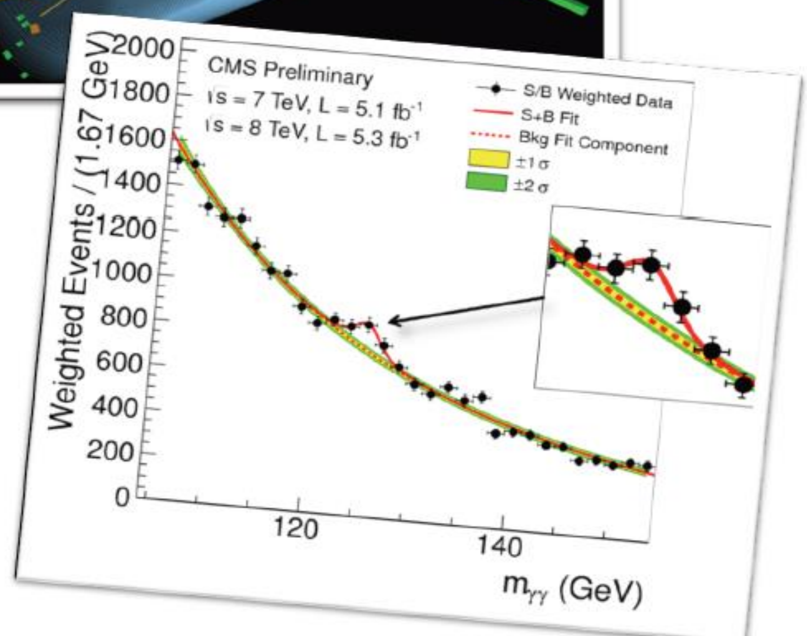



Data Flow for Physics Analysis
detector
event filter (selection & reconstruction)
reconstruction
event data
processed data
100%
raw data
10%
event reprocessing
batch physics analysis
analysis
1%
analysis objects (extracted by physics topic)
event simulation
simulation
interactive physics analysis

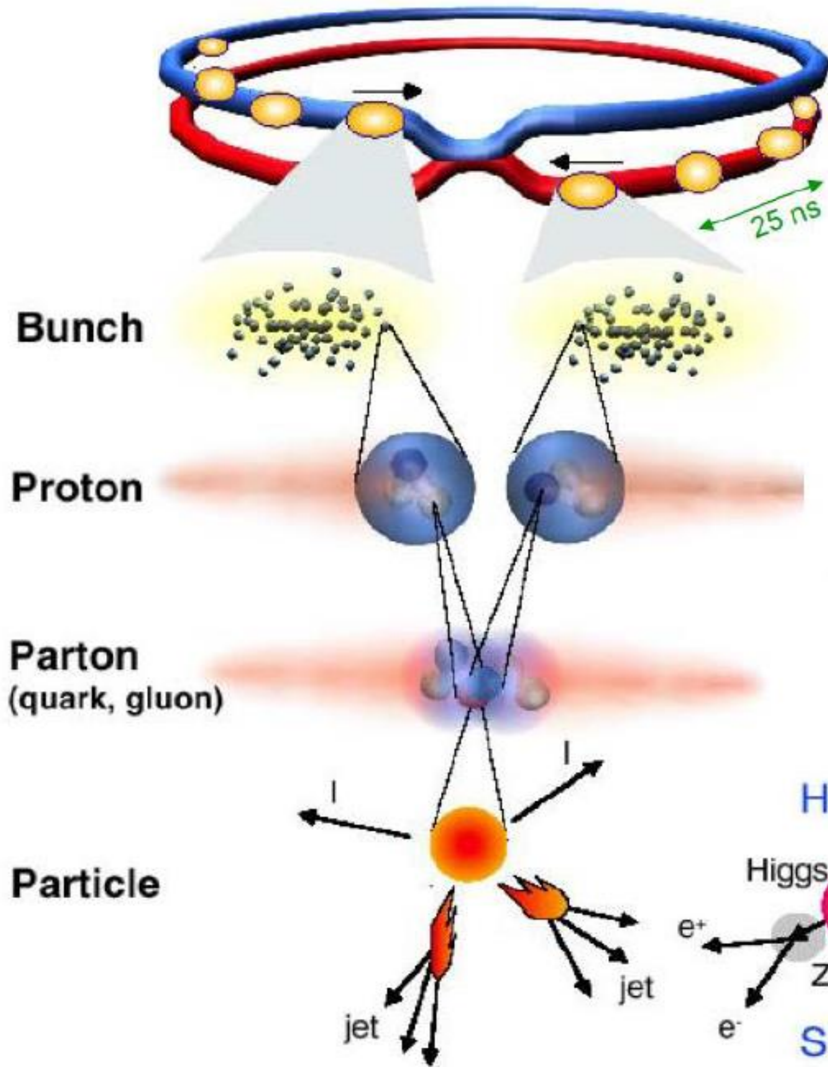

Running jobs: 243209
Transfer rate: 7.59 GiB/sec
Worldwide LHC Computing Grid WLCG
Running jobs
140k
simulation
user analysis
MC reconstruction
group production
one year
Disk use in PB
60 PB
2012 Data
2011 MC
2011 data
2012 MC
one year
ATLAS uses ~80
WLCG sites
world-wide
Performance is
superb
11

Experiment = probing theories with data





Collisions at LHC

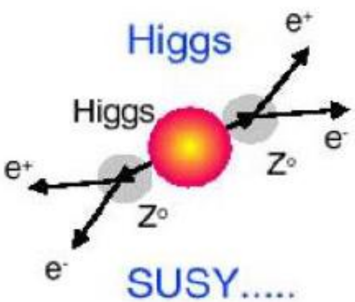


Proton-Proton	2835 bunch/beam
Protons/bunch	10^{11}
Beam energy	7 TeV (7×10^{12} eV)
Luminosity	10^{34} cm^{-2} s^{-1}

In the experiments:

10^9 pp interactions per second

~ 1500 particles (p,n, π) produced in the detectors at each bunch-crossing

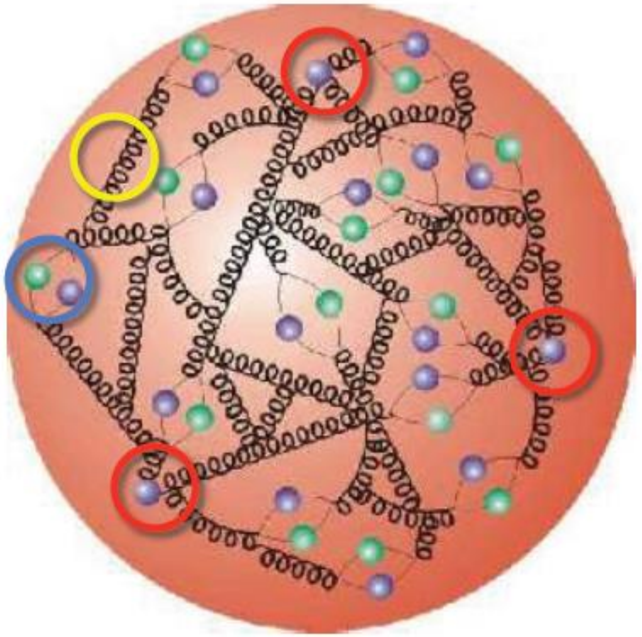


Selection of 1 in 10,000,000,000,000

Inner structure of a proton

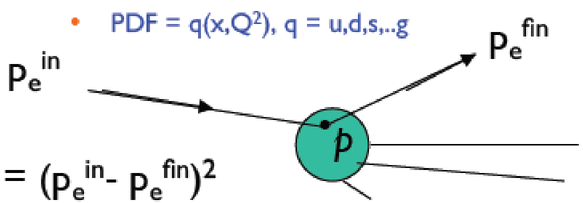
protons have substructures

- ✓ partons = quarks & gluons
- ✓ 3 valence (colored) quarks bound by gluons
- ✓ Gluons (colored) have self-interactions
- ✓ Virtual quark pairs can pop-up (sea-quark)
- ✓ p momentum shared among constituents
  - described by p structure functions



Parton energy not 'monochromatic'

- ✓ Parton Distribution Function
  - PDF = $q(x,Q^2)$, q = u,d,s,..g



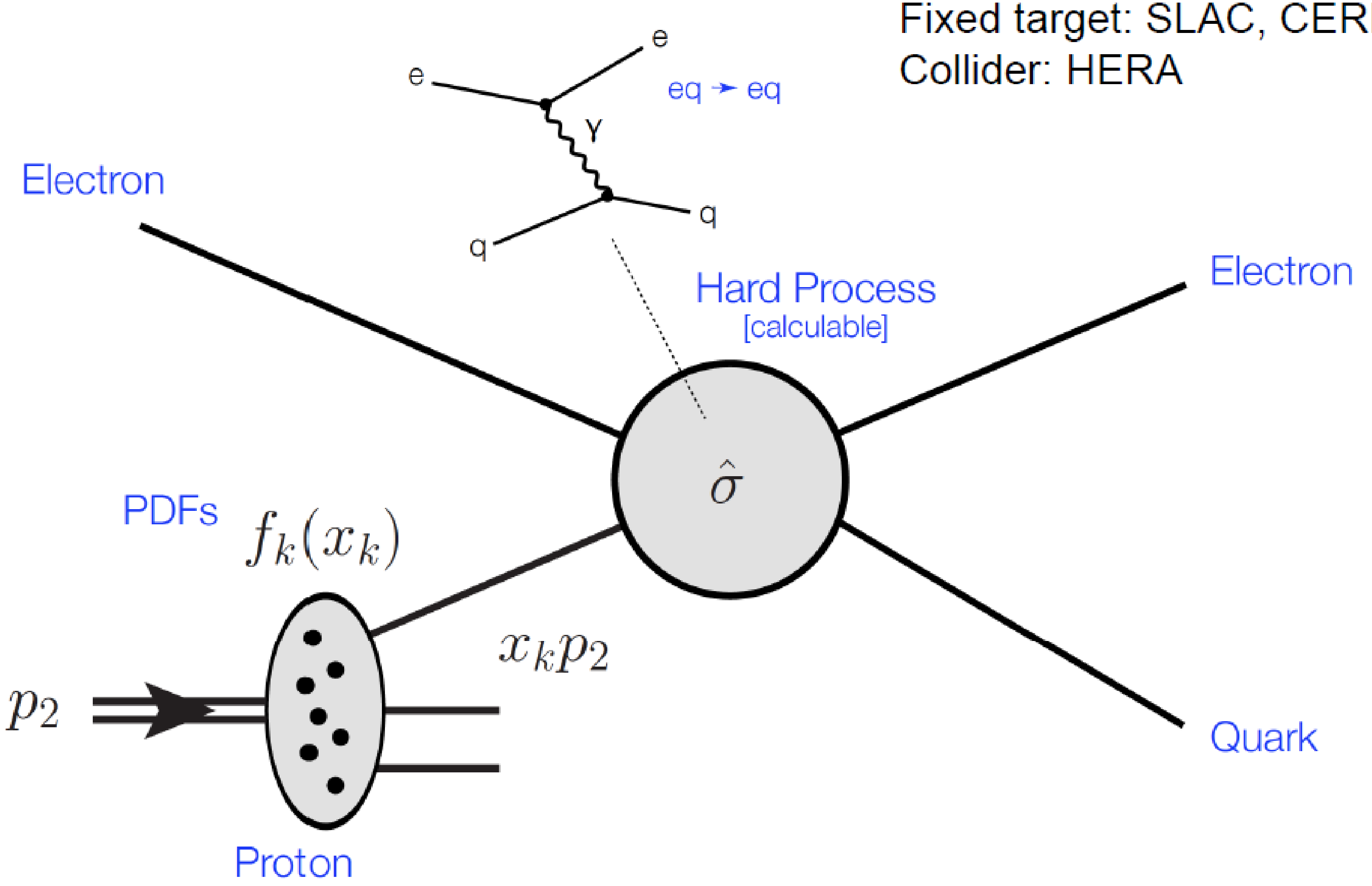
$Q^2 = (p_e^{in} - p_e^{fin})^2$

Kinematic variables

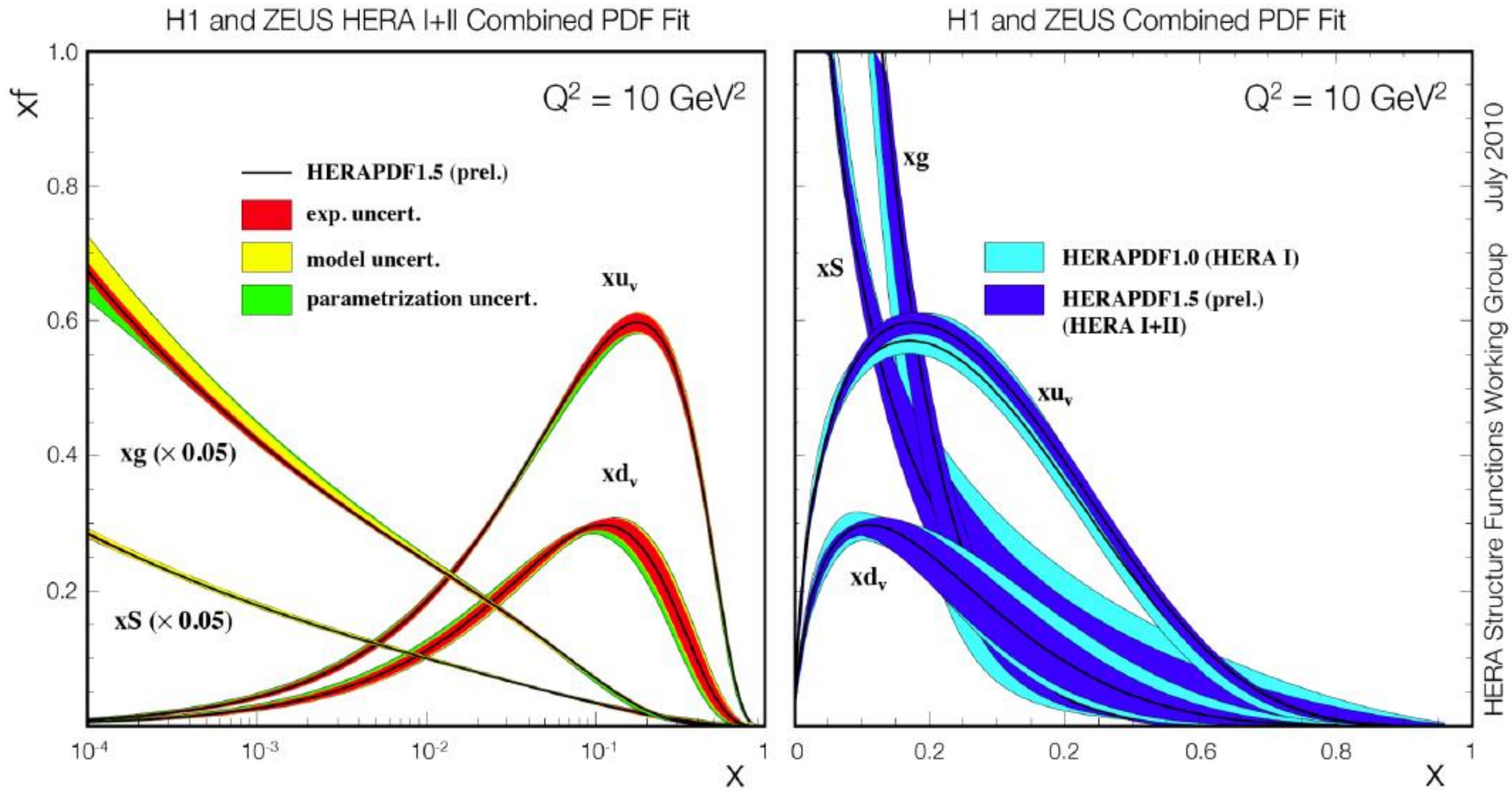
- ✓ Bjorken-x: fraction of the proton momentum carried by struck parton
  - $x = P_{parton}/P_{proton}$
- ✓ Q^2 : 4-momentum^2 transfer

Lepton-proton scattering

Fixed target: SLAC, CERN
Collider: HERA

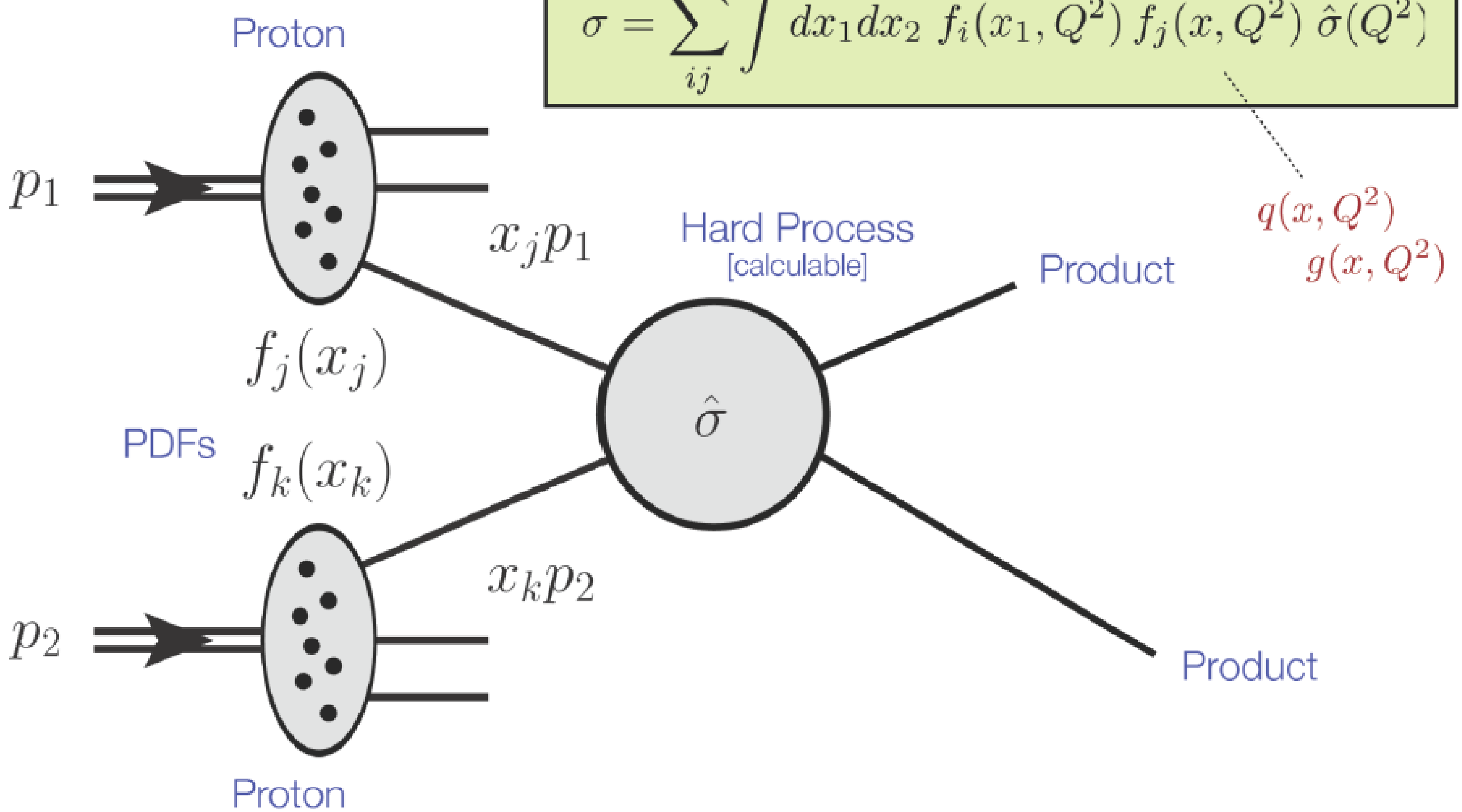


Inner structure of a proton



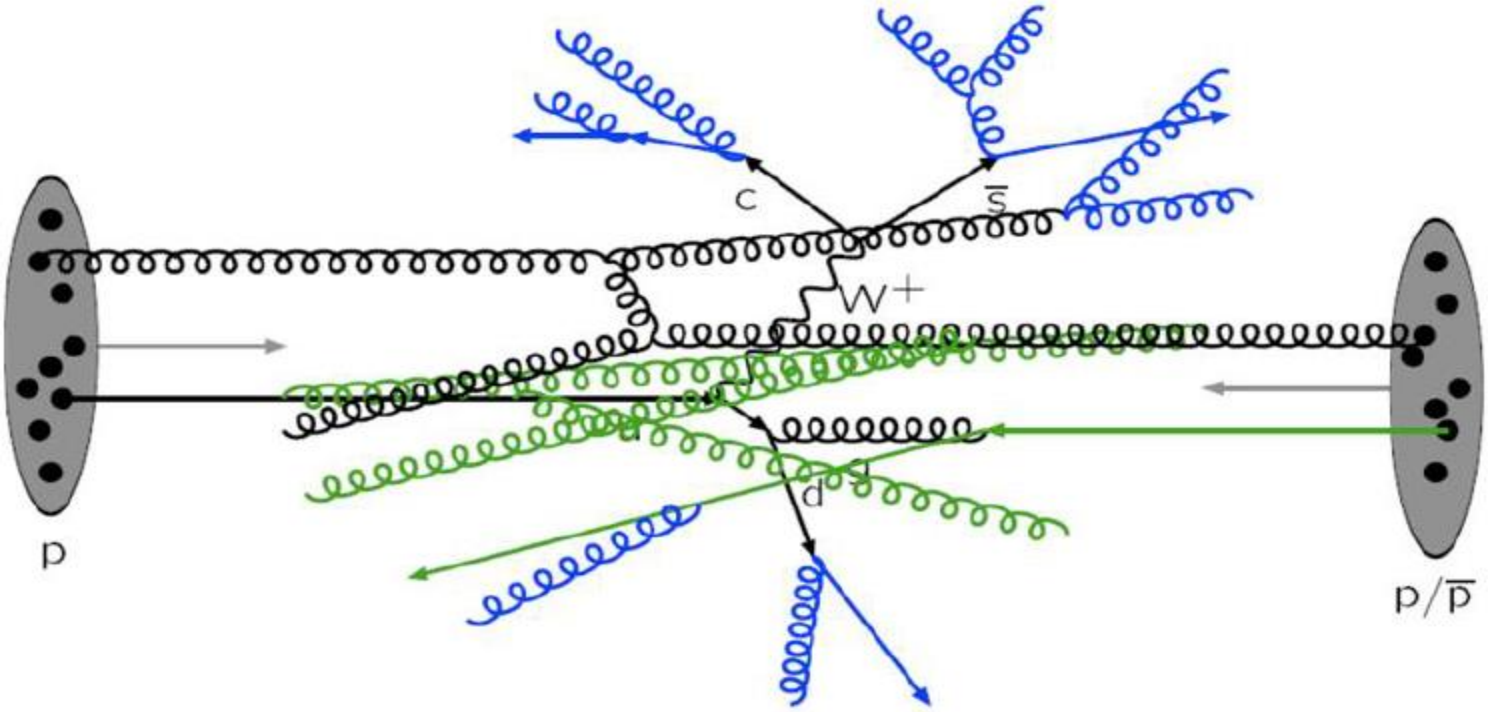
Proton-proton scattering at LHC

$$\sigma = \sum_{ij} \int dx_1 dx_2 \; f_i(x_1, Q^2)\, f_j(x, Q^2)\, \hat{\sigma}(Q^2)$$

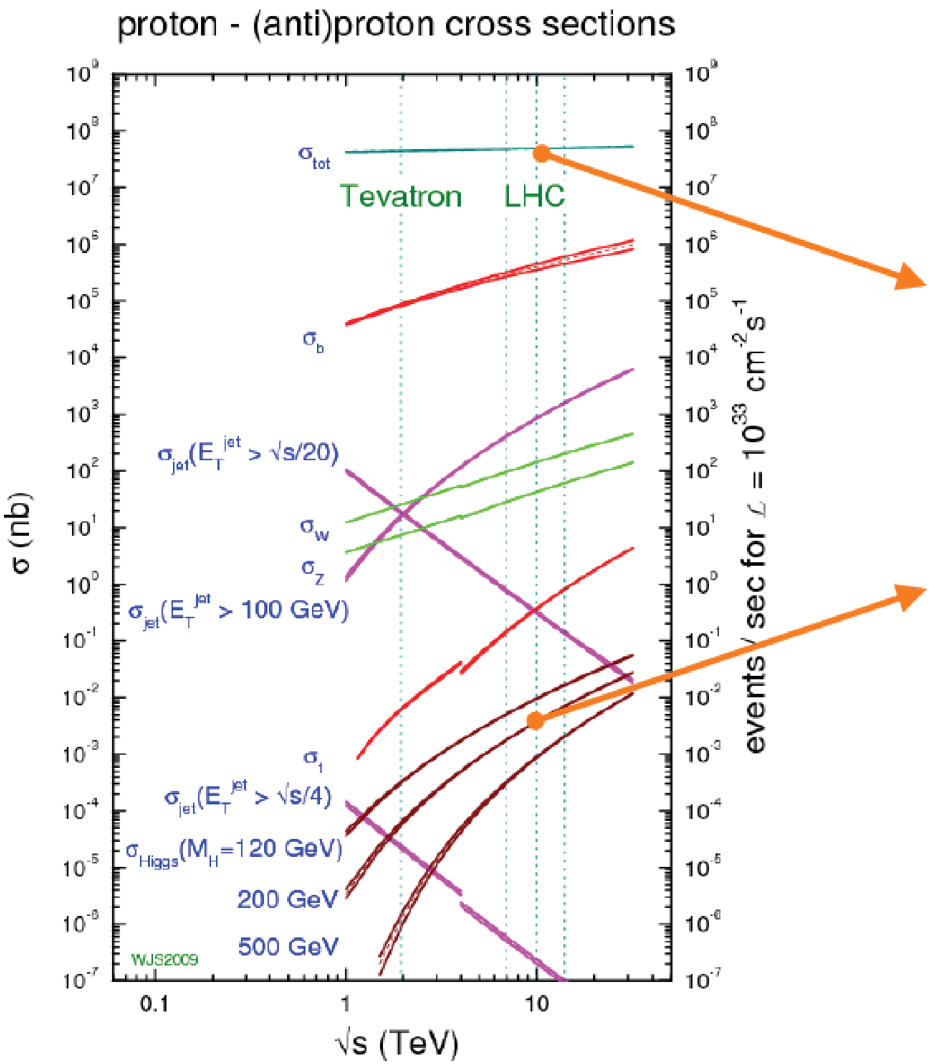


Proton-proton scattering at LHC

- Hard interaction: qq, gg, qg fusion
- Initial and final state radiation (ISR,FSR)
- Secondary interaction ["underlying event"]



Cross-sections at LHC



10^8 events/s

$\sim 10^{10}$

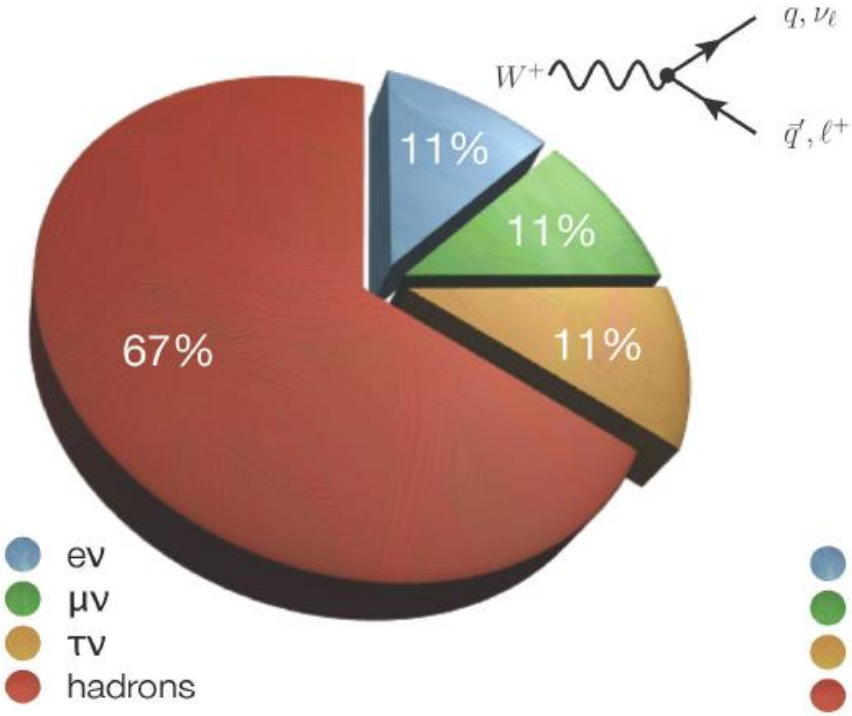
10^{-2} events/s ~

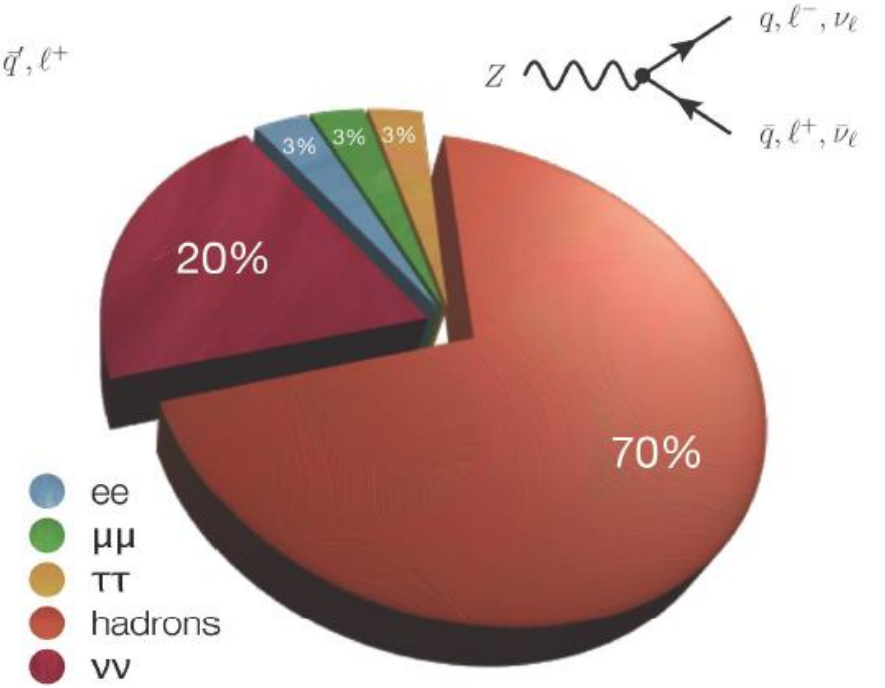
10 events/min

[m_H ~ 120 GeV]

0.2% H → γγ
1.5% H → ZZ

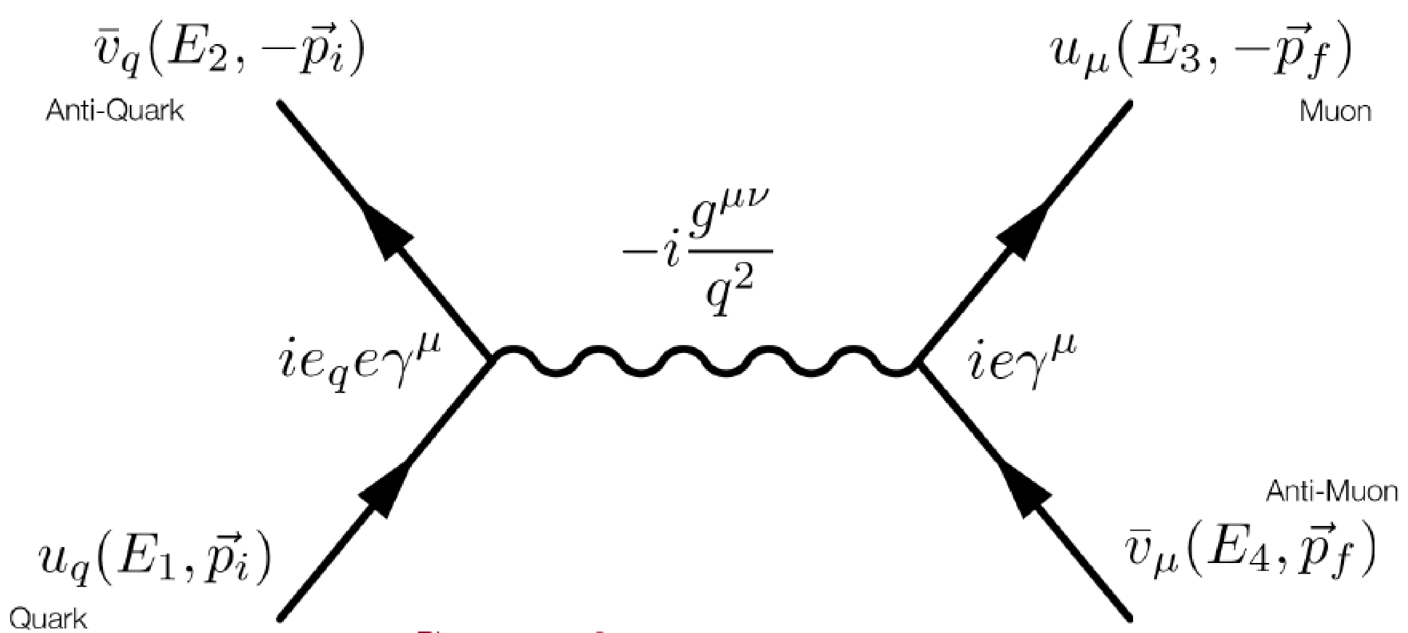
W and Z boson decays





Leptonic decays (e/μ): very clean, but small(ish) branching fractions
Hadronic decays: two-jet final states; large QCD dijet background
Tau decays: somewhere in between…

Example: Drell-Yan process

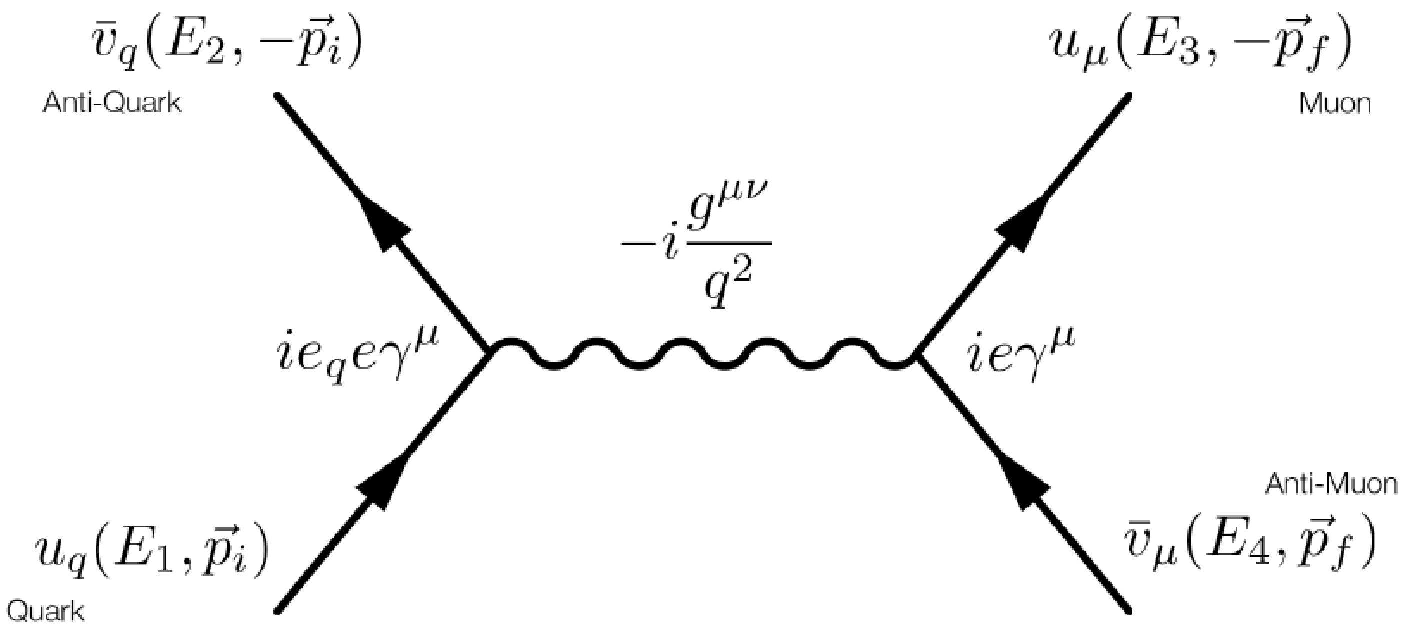


Phase space & delta functions ...

Squared Matrix element

$$\frac{d\sigma}{d\Omega} = \frac{1}{s \cdot 64\pi^2} \cdot \frac{|\vec{p}_f|}{|\vec{p}_i|} \cdot \overline{|M_{fi}|^2}$$

Example: Drell-Yan process

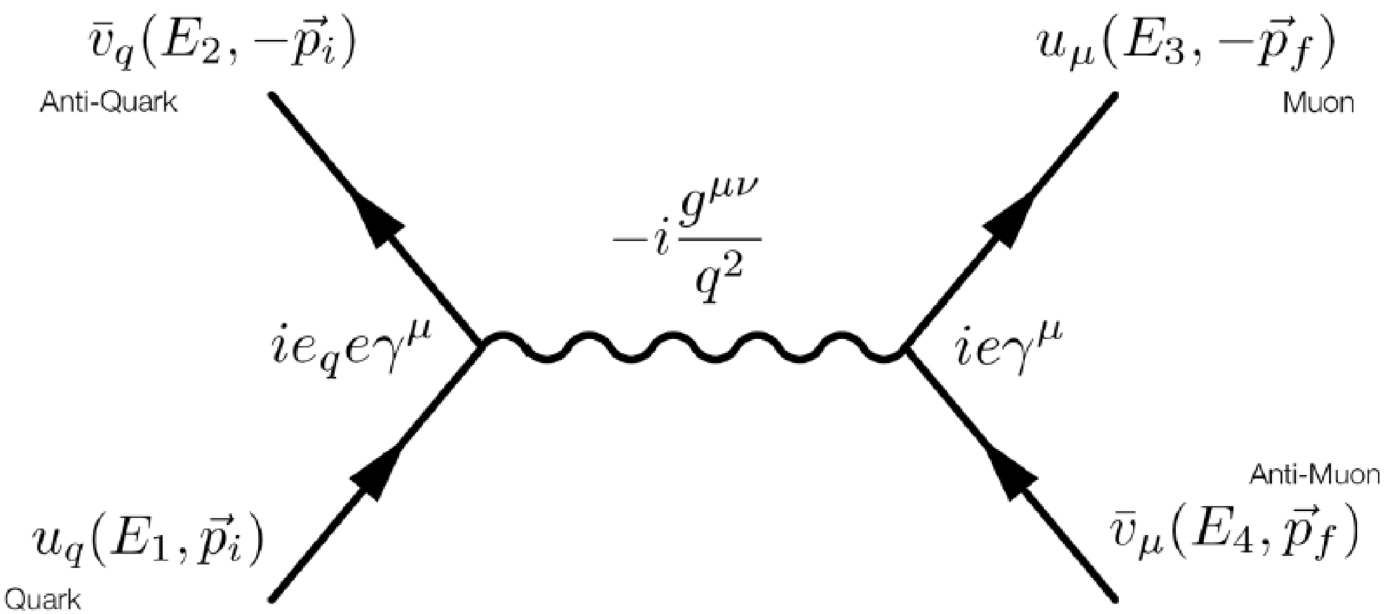


Averaging over initial spins

Summing over initial and final spins

$$\overline{|M_{fi}|^2} = \frac{1}{(2s_q+1)^2} \cdot \sum_{s_q, s'_q} \sum_{s_\mu, s'_\mu} |M_{fi}|^2$$

Example: Drell-Yan process



Couplings; Anti-Quark; Anti-Muon

$$M_{fi} = -\frac{e_q e^2}{q^2} \bar{v}_q \gamma_\mu u_q \cdot \bar{v}_\mu \gamma^\mu u_\mu$$

Propagator; Quark; Muon

Example: Drell-Yan process

$$\overline{|M|^2}_{q\bar{q}\to\mu\mu} = 2e_q^2 e^4 \cdot \frac{t^2+u^2}{s^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{32\pi^2} e_q^2 \cdot \frac{1}{s} \cdot \frac{t^2+u^2}{s^2}$$

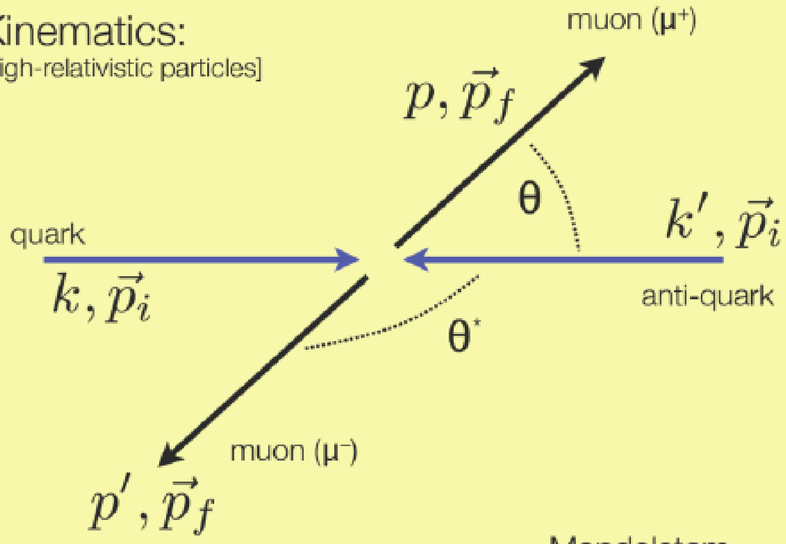
$$= \frac{e^4}{64\pi^2} e_q^2 \cdot \frac{1}{s} \cdot (1 + cos^2\theta)$$

with $e^2 = 4\pi\alpha$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha}{4s} e_q^2 \cdot (1 + cos^2\theta)$$

[θ in CMS frame]

Kinematics:
[high-relativistic particles]



Mandelstam variables

$$s = (k + k')^2 = 4E_i^2$$

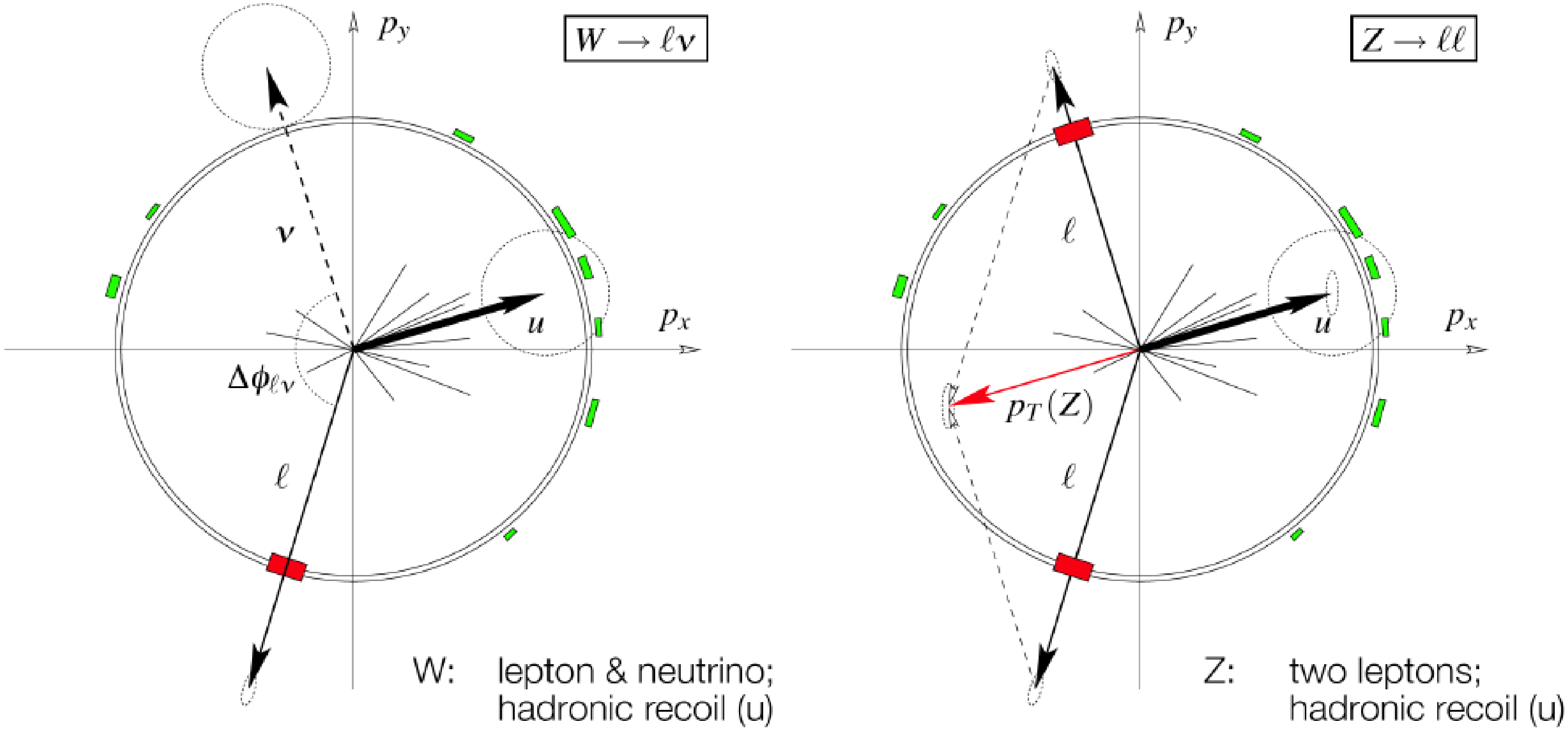
$$t = (k - p)^2 \approx -2kp \approx -2E_i^2(1 - \cos\theta^*)$$

$$\approx -\frac{s}{2}(1 + \cos\theta)$$

$$u = (k - p')^2 \approx -2kp' \approx -2E_i^2(1 - \cos\theta)$$

$$\approx -\frac{s}{2}(1 - \cos\theta)$$

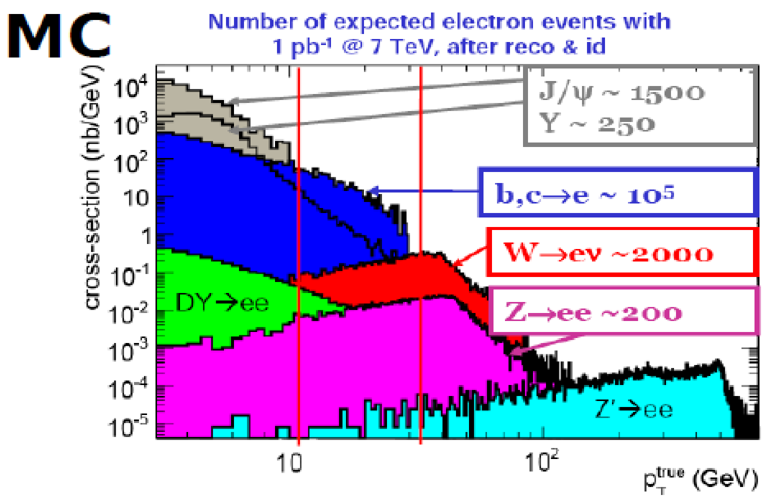
W and Z boson signatures



Additional hadronic activity ➛ recoil, not as clean as e^+e^-
Precision measurements: only leptonic decays

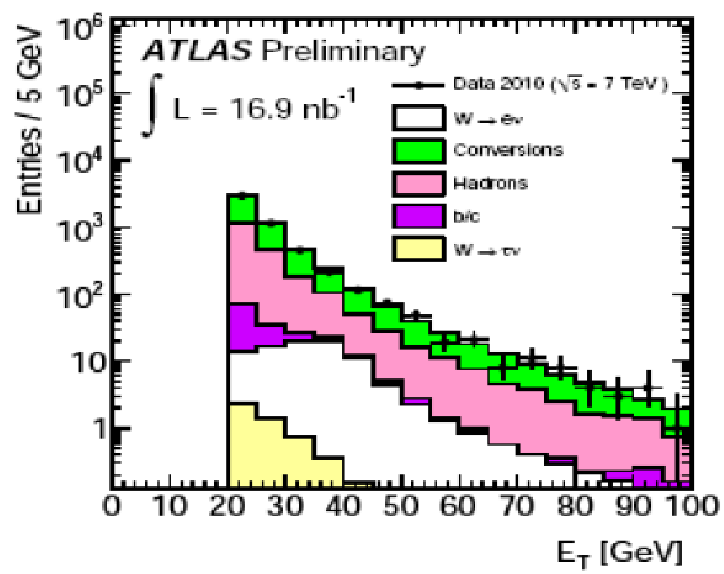
Electrons and jets

MC



- There is also lot of true electrons from semileptonic decays inside jets

DATA: loose electron ID



- Jets can look like electrons
  - Photon conversion from π^0's
  - Early showering charged pions
- And there is lot of jets
- Difficult to model in Monte Carlo
  - Detailed simulation in tracking and calorimeter volume

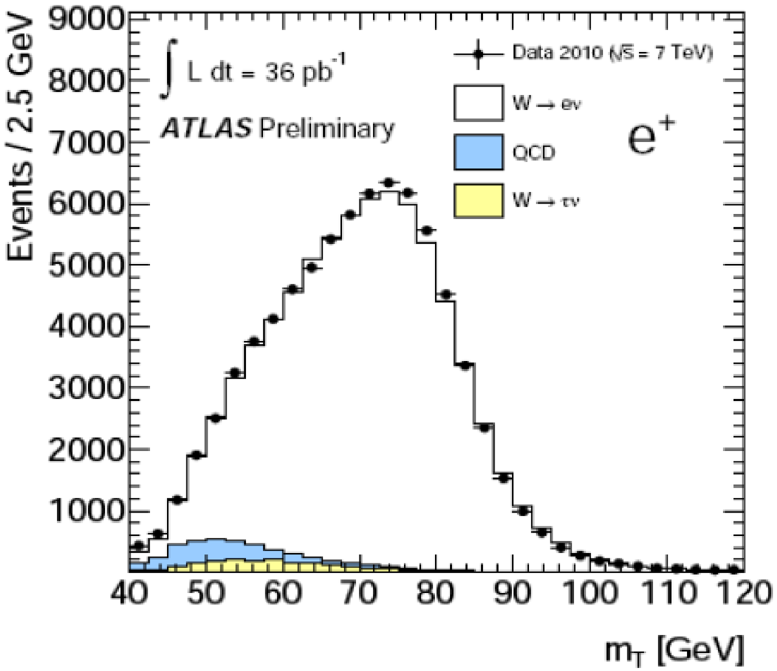
W selection (2010)

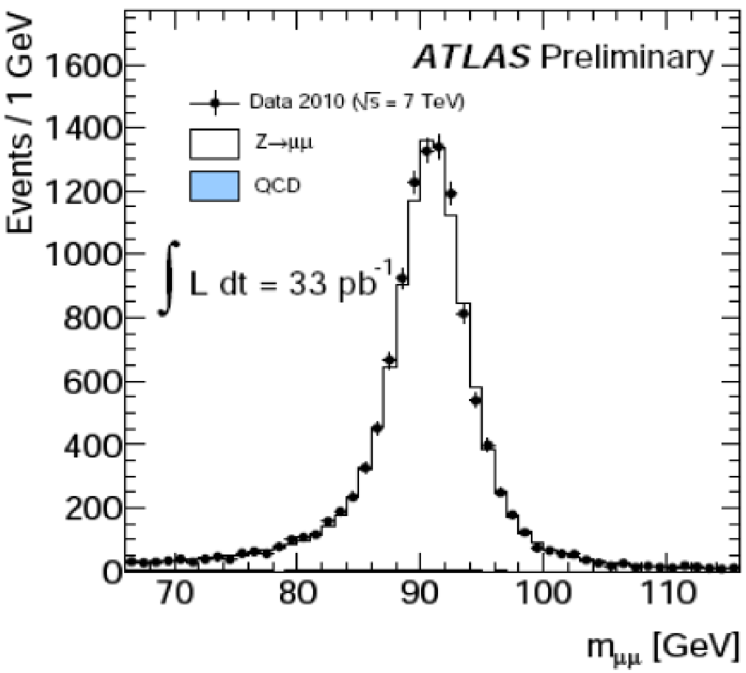
$W \to \ell\nu$

- One e/μ with $p_T > 20$ GeV
- $E_T^{miss} > 25$ GeV
- $m_T(\ell, E_T^{miss}) > 40$ GeV

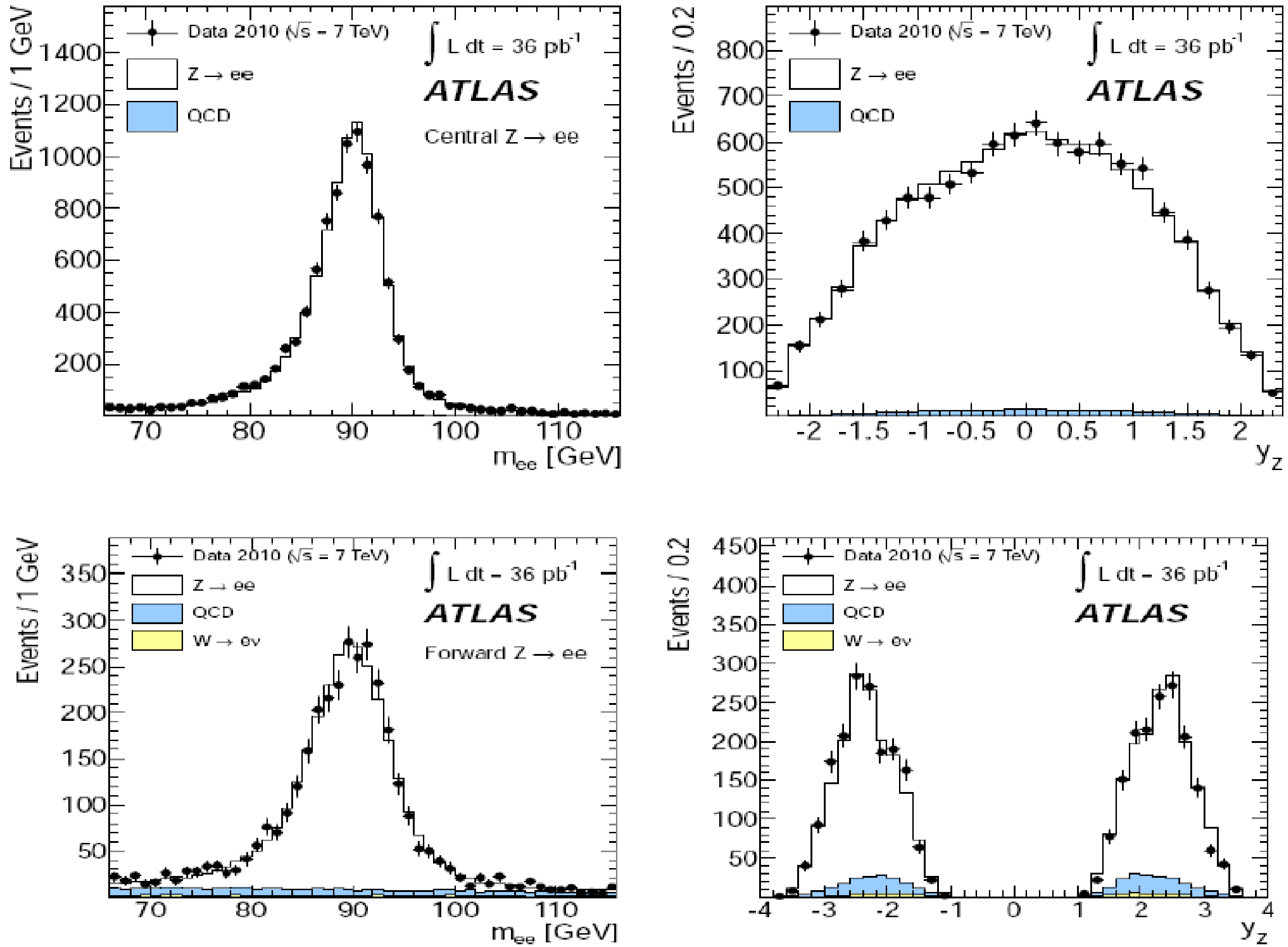
$Z \to \ell\ell$

- Two e/μ with $p_T > 20$ GeV
- $m_{\ell\ell} = 66\text{–}116$ GeV





Z selection (2010)



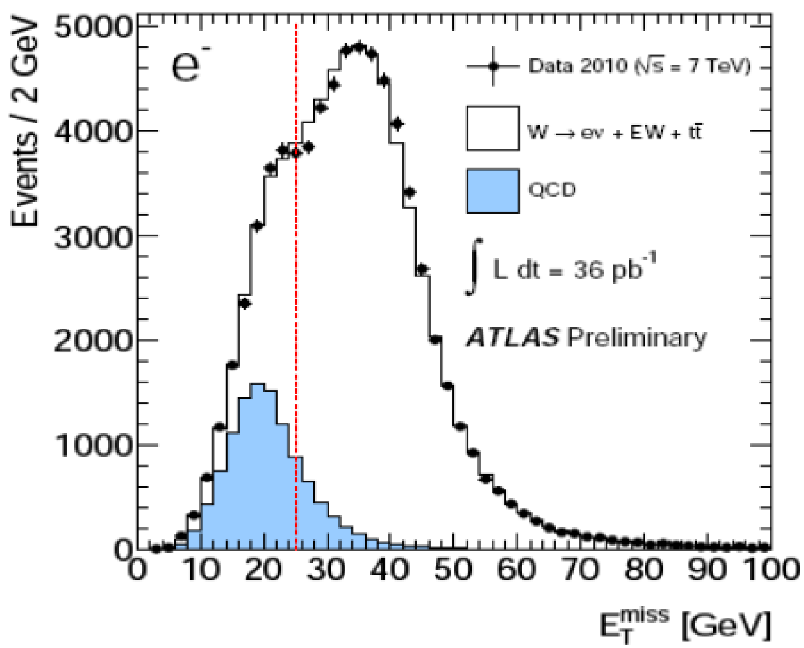
W backgrounds

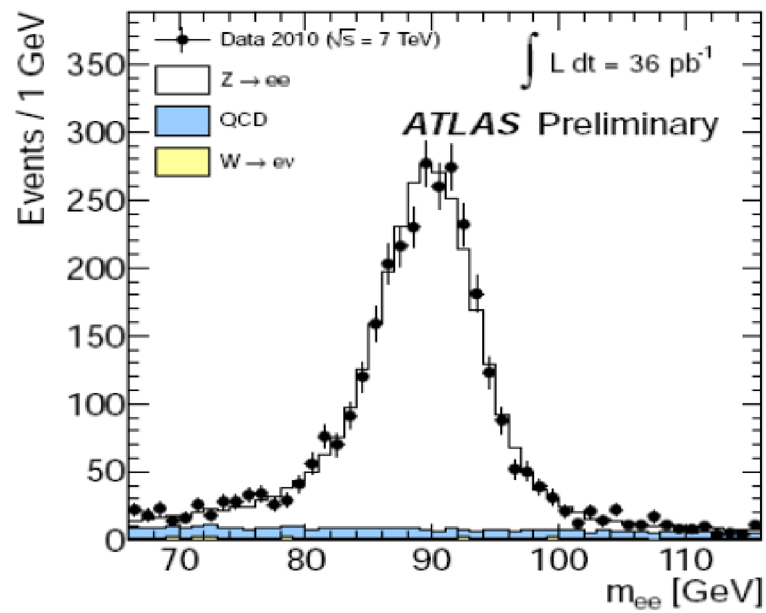
$W \to e\nu$: template fit to E_T^{miss}. Template derived from data with inverted electron ID and isolation.

$Z \to ee$: template fit to $m_{\ell\ell}$ to a sample with looser electron ID, extrapolated to the signal region.

$W \to \mu\nu$: matrix method using track isolation.

$Z \to \mu\mu$: ABCD method with track isolation in $m_{\mu\mu}$ side-band.





Cross-section & Luminosity

$$\sigma = \frac{N_{\mathrm{obs}} - N_{\mathrm{bkg}}}{A \cdot C \cdot \int dt\, \mathcal{L}}$$

N_{obs}: number of observed events in the signal region

N_{bkg}: estimated number of background events

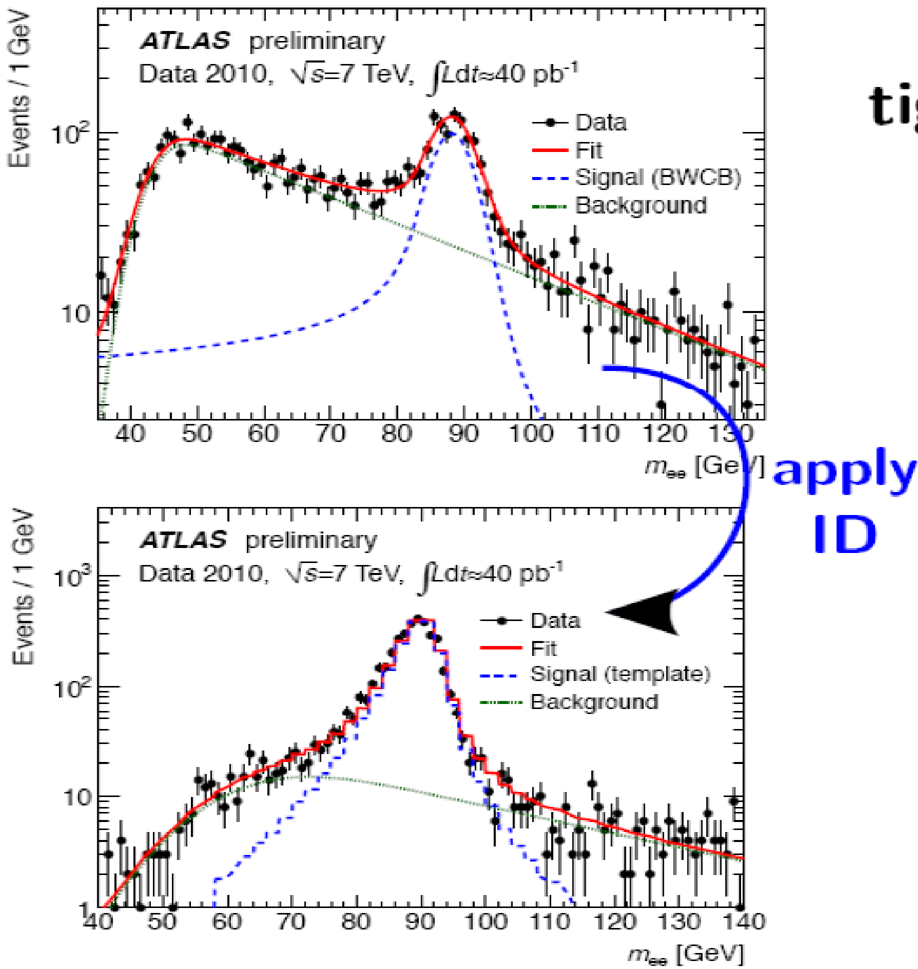
- EW backgrounds are estimated with Monte Carlo, constrained to data with performance scale factors.
- QCD backgrounds are estimated with **data-driven** methods.

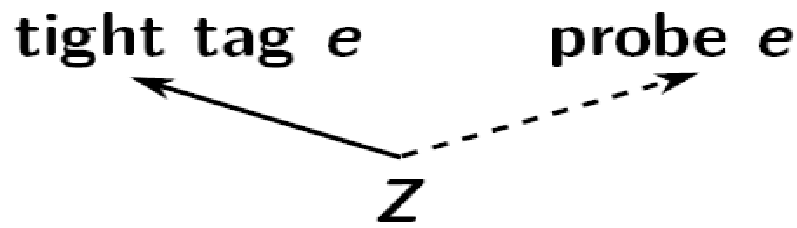
A: kinematic acceptance factor, estimated with generator-level Monte Carlo.

C: summarizes reconstruction efficiency, estimated with reconstructed Monte Carlo, corrected with **scale factors**.

$\int dt\, \mathcal{L}$: integrated luminosity.

Scale factors: tag and probe studies





- **"Tag"** events with sufficient purity, leaving an unbiased **"probe"** object.
- Measure probe ID efficiency *in situ*.
- Constrains the performance of our object identification.
- Derive **scale factors** for correcting our simulation.

[4] ATLAS-PERF-2010-04-001

Systematic error

	$\delta\sigma_{W\pm}$	$\delta\sigma_{W+}$	$\delta\sigma_{W-}$	$\delta\sigma_{Z}$
Trigger	0.4	0.4	0.4	<0.1
Electron reconstruction	0.8	0.8	0.8	1.6
Electron identification	0.9	0.8	1.1	1.8
Electron isolation	0.3	0.3	0.3	—
Electron energy scale and resolution	0.5	0.5	0.5	0.2
Non-operational LAr channels	0.4	0.4	0.4	0.8
Charge misidentification	0.0	0.1	0.1	0.6
QCD background	0.4	0.4	0.4	0.7
Electroweak+$t\bar{t}$ background	0.2	0.2	0.2	<0.1
$E_{\mathrm{T}}^{\mathrm{miss}}$ scale and resolution	0.8	0.7	1.0	—
Pile-up modeling	0.3	0.3	0.3	0.3
Vertex position	0.1	0.1	0.1	0.1
$C_{W/Z}$ theoretical uncertainty	0.6	0.6	0.6	0.3
Total experimental uncertainty	1.8	1.8	2.0	2.7
$A_{W/Z}$ theoretical uncertainty	1.5	1.7	2.0	2.0
Total excluding luminosity	2.3	2.4	2.8	3.3
Luminosity	3.4			

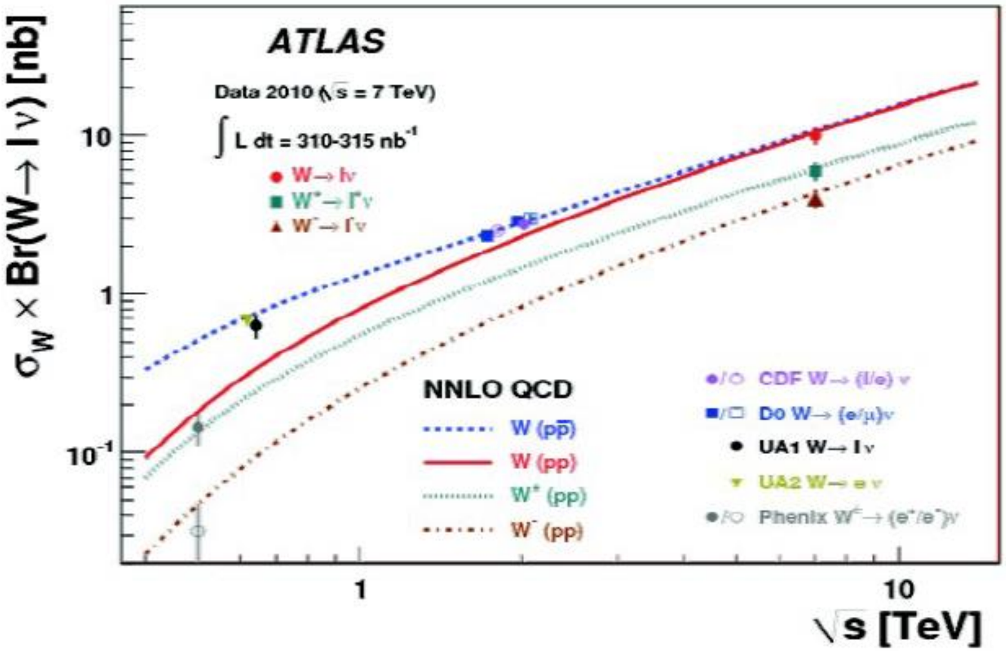
W cross-section measurement

L ≈ 310 - 315 nb^{-1}

$$\text{Theory prediction}: \ 10.46 \pm 0.42 \ \text{nb}$$

$$\sigma_W \times BR(W \to e\nu) = [10.51 \pm 0.34(stat) \pm 0.81(sys) \pm 1.16(lumi)] \, nb$$

$$\sigma_W \times BR(W \to \mu\nu) = [9.58 \pm 0.30(stat) \pm 0.50(sys) \pm 1.05(lumi)] \, nb$$



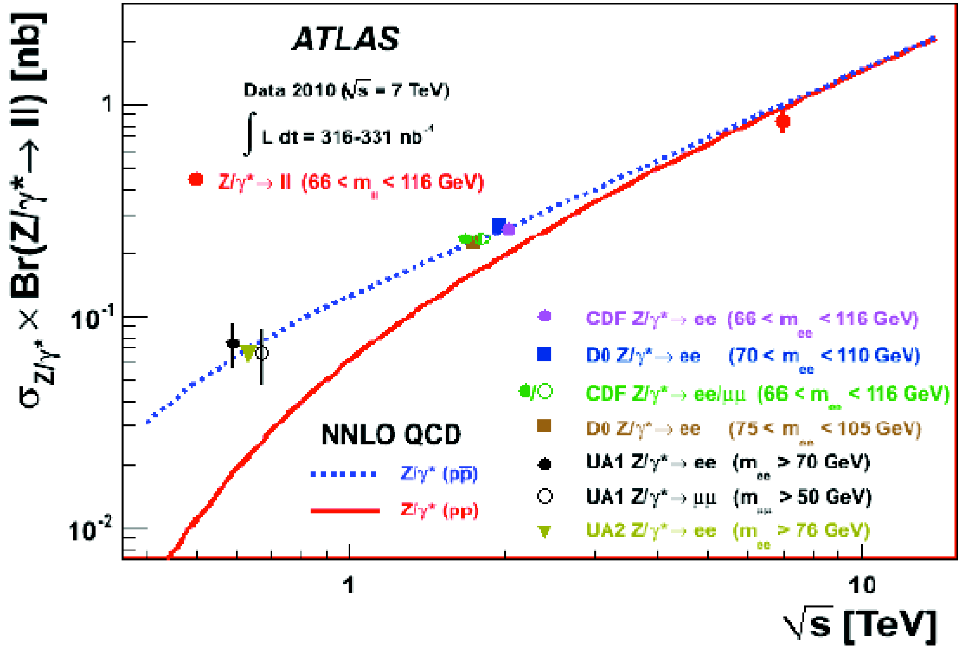
Z cross-section measurement

$L \approx 310$ - $315\ nb^{-1}$

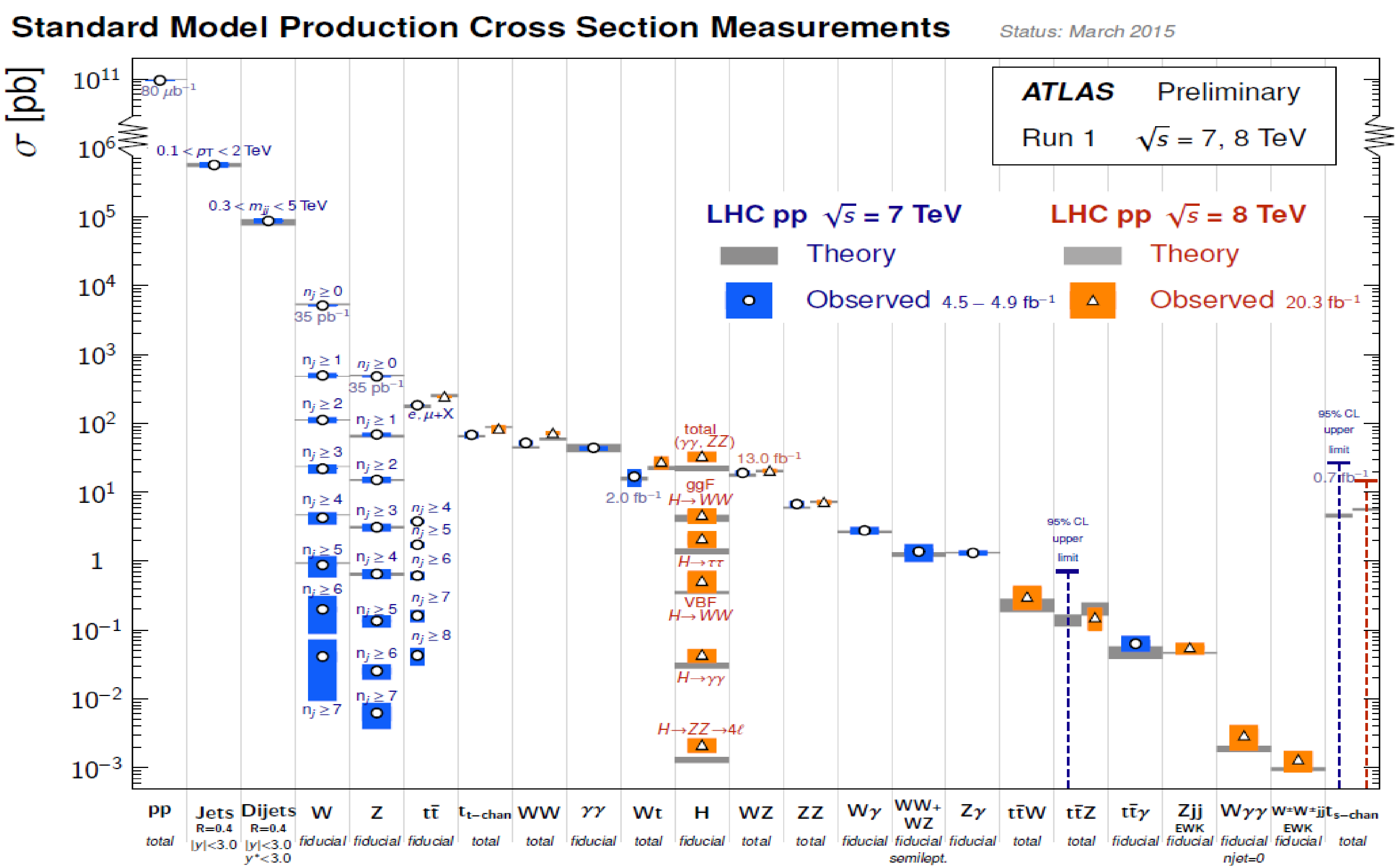
Theory prediction : 0.96 ± 0.04 nb for $[66 - 116]$ GeV mass window

$$\sigma_Z \times BR(Z \to e^+e^-) = [0.75 \pm 0.09(stat) \pm 0.08(sys) \pm 0.08(lumi)]\, nb$$

$$\sigma_Z \times BR(Z \to \mu^+\mu^-) = [0.87 \pm 0.08(stat) \pm 0.06(sys) \pm 0.10(lumi)]\, nb$$

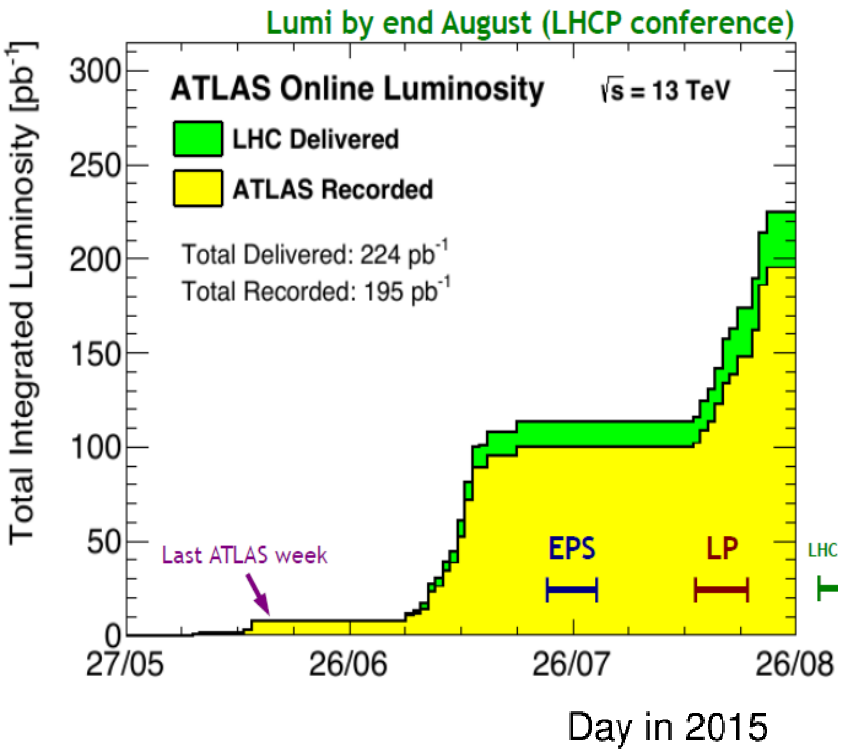


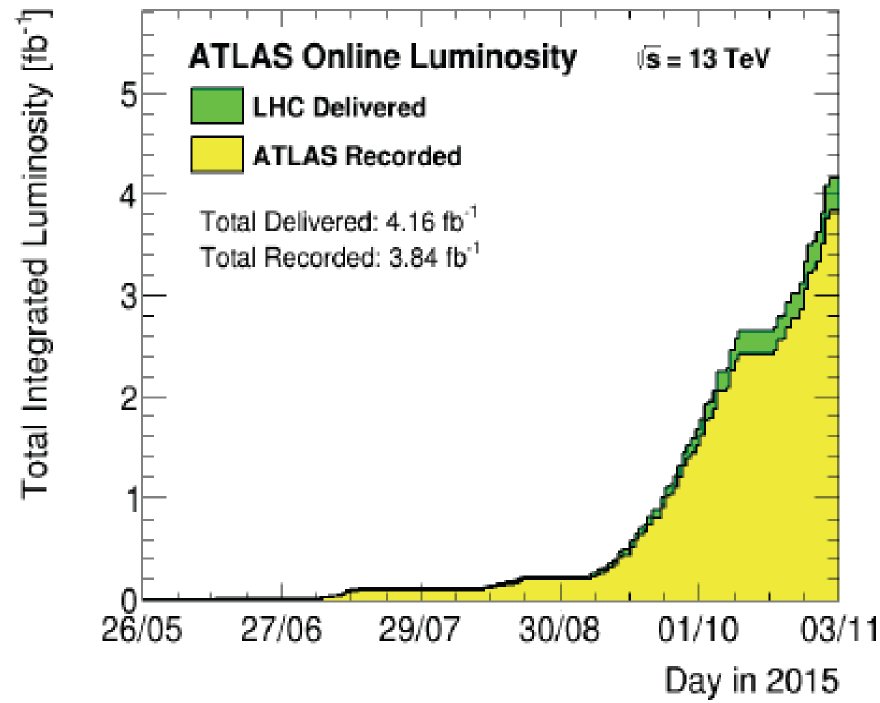
Electroweak measurements at LHC



Data with Run II

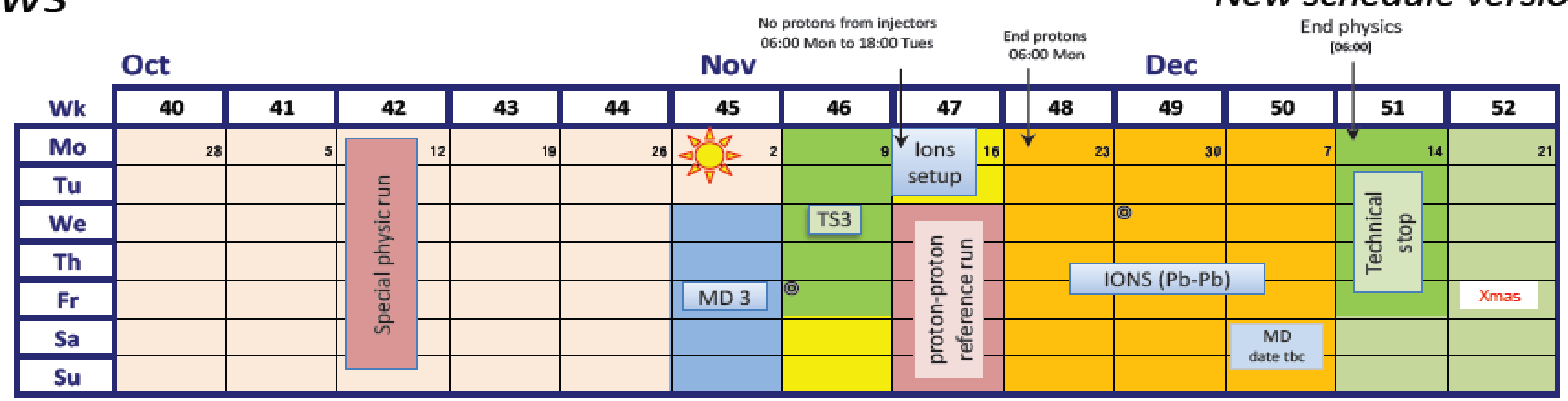
3.84 fb^{-1} Recorded!

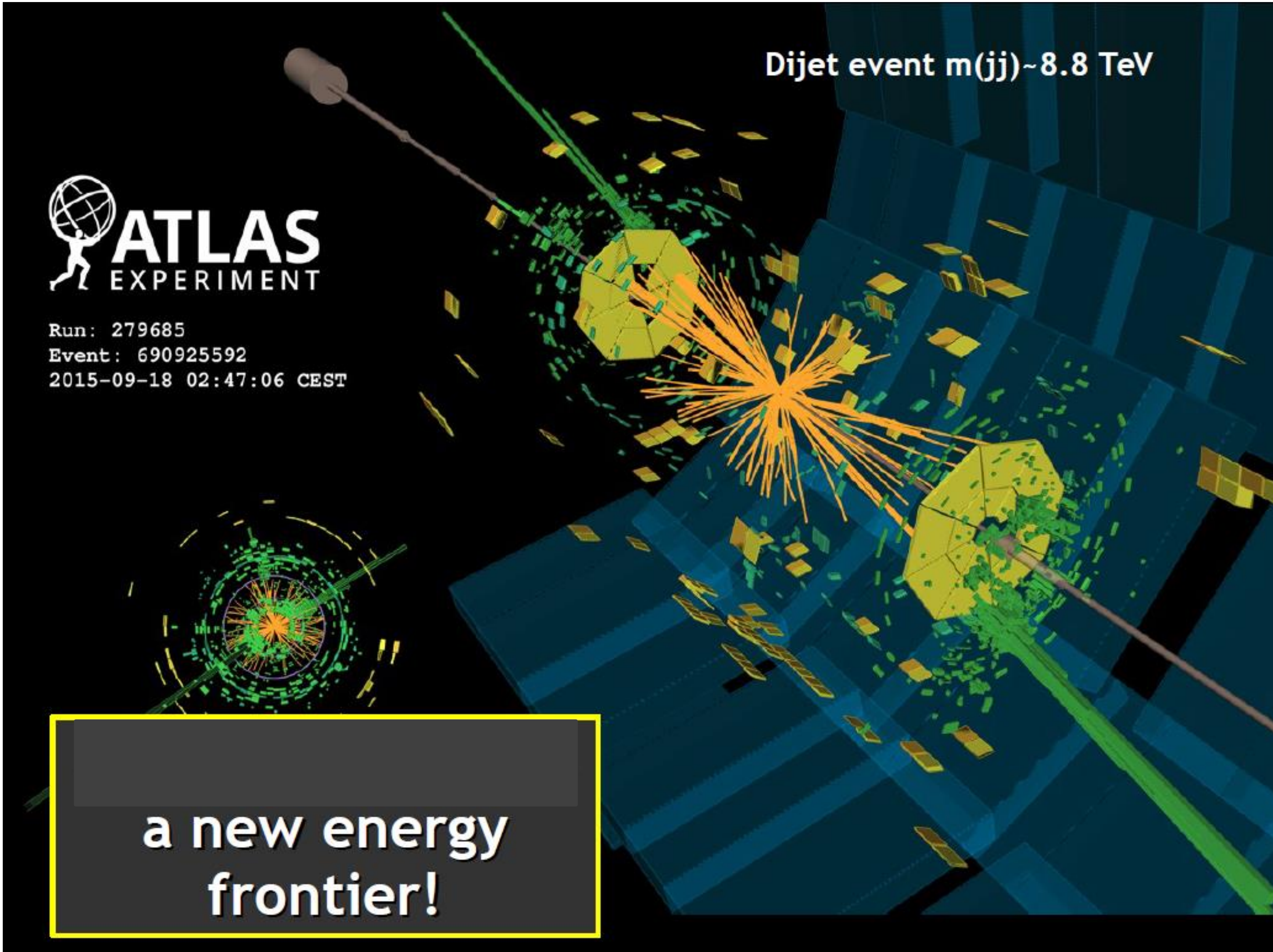




News

New schedule version 1.8

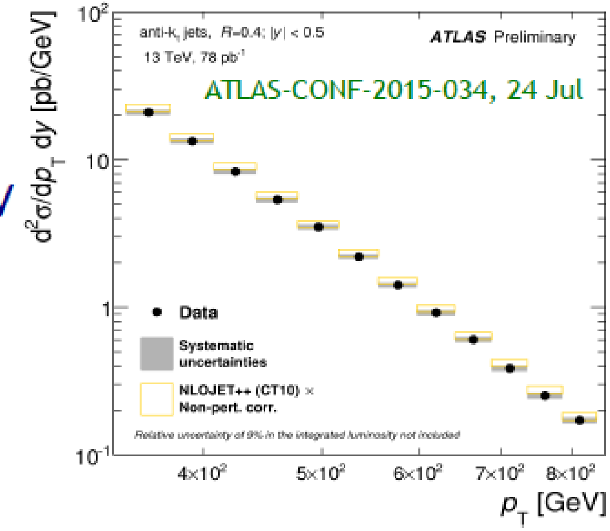


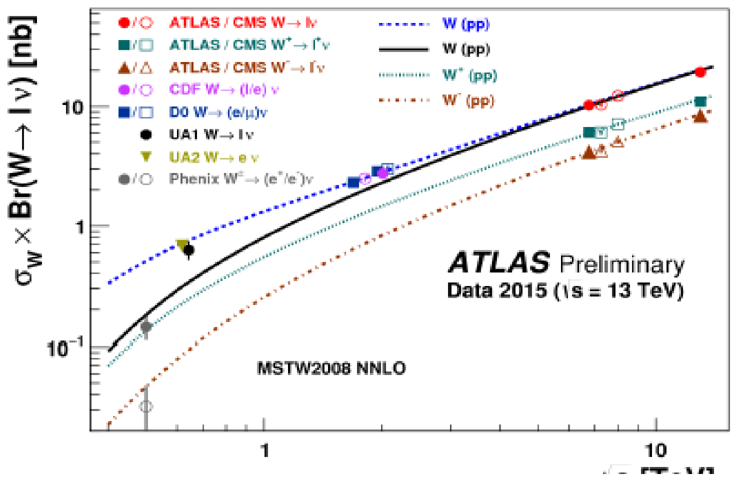

Dijet event m(jj)~8.8 TeV
ATLAS
EXPERIMENT
Run: 279685
Event: 690925592
2015-09-18 02:47:06 CEST
a new energy
frontier!

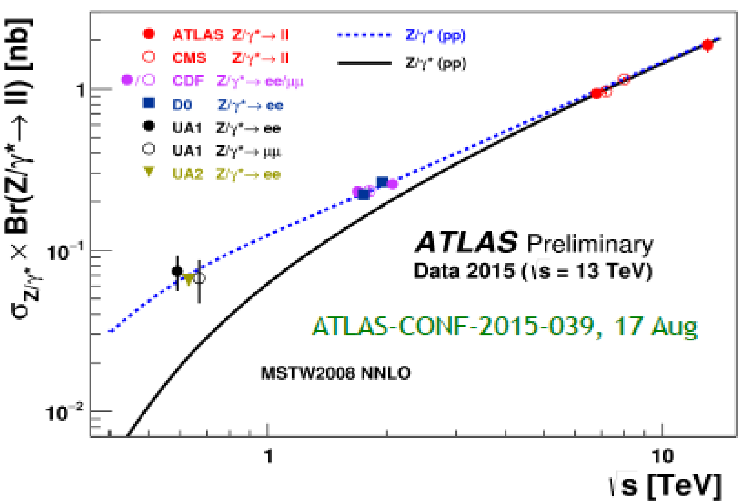
W, Z, $t\bar{t}$, jets

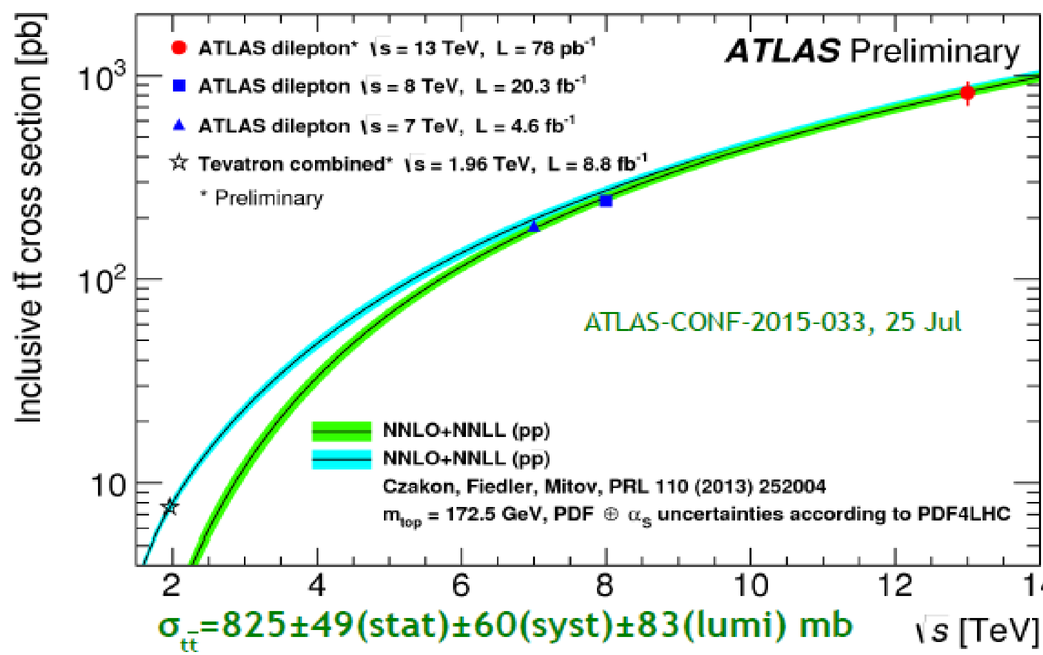
Our first high-p_T cross-section measurements at 13 TeV

Available already for EPS or Lepton-Photon









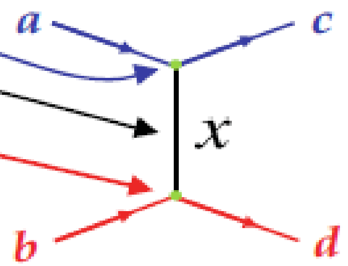
$\sigma_{t\bar{t}}$=825±49(stat)±60(syst)±83(lumi) mb

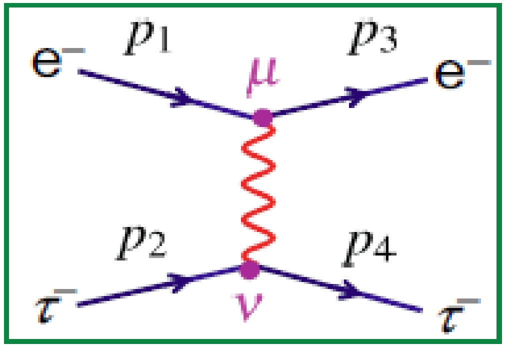
Feynman Rules for QED

★ Interaction by particle exchange naturally gives rise to **Lorentz Invariant Matrix Element** of the form

$$M = \langle \psi_c | V | \psi_a \rangle \frac{1}{q^2 - m_x^2} \langle \psi_d | V | \psi_b \rangle$$

$\langle \psi_c | V | \psi_a \rangle = g_a$, $\langle \psi_d | V | \psi_b \rangle = g_b$





★ In QED we could again go through the procedure of summing the time-orderings using Dirac spinors and the expression for $\hat{V}_D$. If we were to do this, remembering to sum over all photon polarizations, we would obtain:

$$M = [u_e^\dagger(p_3) q_e \gamma^0 \gamma^\mu u_e(p_1)] \sum_\lambda \frac{\varepsilon_\mu^\lambda (\varepsilon_\nu^\lambda)^*}{q^2} [u_\tau^\dagger(p_4) q_\tau \gamma^0 \gamma^\nu u_\tau(p_2)]$$

Interaction of e^- with photon	Massless photon propagator summing over polarizations	Interaction of τ^- with photon

• All the physics of **QED** is in the above expression !

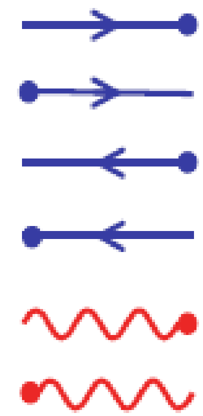
from M. A. Thomson lectures at Cambridge University in 2011

Feynman Rules for QED

- **External Lines**

spin 1/2:
- incoming particle $u(p)$
- outgoing particle $\overline{u}(p)$
- incoming antiparticle $\overline{v}(p)$
- outgoing antiparticle $v(p)$

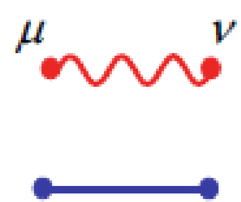
spin 1:
- incoming photon $\varepsilon^{\mu}(p)$
- outgoing photon $\varepsilon^{\mu}(p)^{*}$



- **Internal Lines (propagators)**

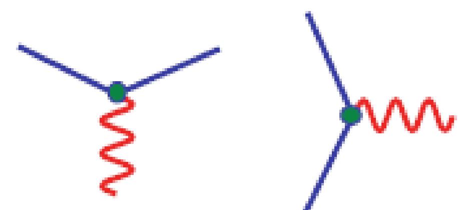
spin 1 photon $-\frac{ig_{\mu\nu}}{q^2}$

spin 1/2 fermion $\frac{i(\gamma^{\mu}q_{\mu}+m)}{q^2-m^2}$



- **Vertex Factors**

spin 1/2 fermion (charge -|e|) $ie\gamma^{\mu}$



- **Matrix Element** $-iM$ = product of all factors

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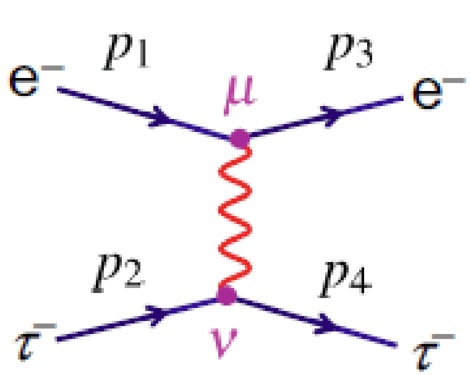
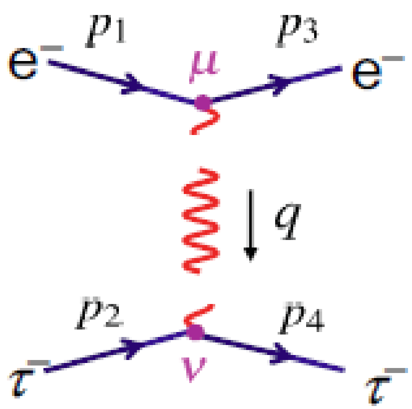
Feynman Rules for QED

e.g.





$$\overline{u}_e(p_3)[ie\gamma^\mu]u_e(p_1)$$

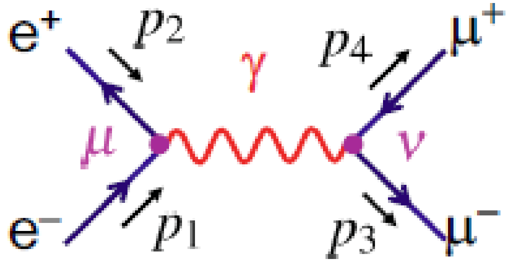
$$\frac{-ig_{\mu\nu}}{q^2}$$

$$\overline{u}_\tau(p_4)[ie\gamma^\nu]u_\tau(p_2)$$

$$iM = [\overline{u}_e(p_3)ie\gamma^\mu u_e(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\overline{u}_\tau(p_4)ie\gamma^\nu u_\tau(p_2)]$$

- Which is the same expression as we obtained previously

e.g.



$$-iM = [\overline{v}(p_2)ie\gamma^\mu u(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\overline{u}(p_3)ie\gamma^\nu v(p_4)]$$

Note:
- At each vertex the adjoint spinor is written first
- Each vertex has a different index
- The $g_{\mu\nu}$ of the propagator connects the indices at the vertices

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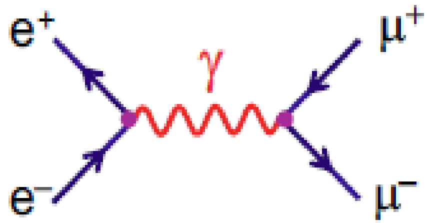
QED calculations

- How to calculate a cross section using QED (e.g. $e^+e^- \rightarrow \mu^+\mu^-$):

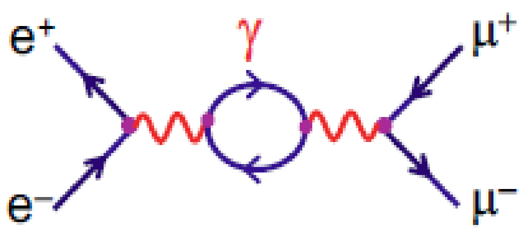
❶ Draw all possible Feynman Diagrams

- For $e^+e^- \rightarrow \mu^+\mu^-$ there is just one lowest order diagram

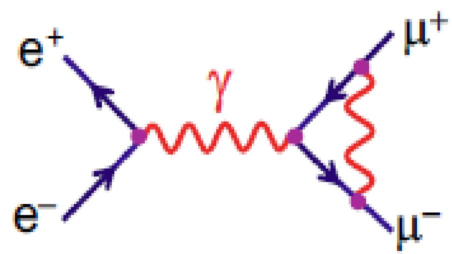


$$M \propto e^2 \propto \alpha_{em}$$

+ many second order diagrams + ...



$+$



$+\ldots$

$$M \propto e^4 \propto \alpha_{em}^2$$

❷ For each diagram calculate the matrix element using Feynman rules

❸ Sum the individual matrix elements (i.e. sum the amplitudes)

$$M_{fi} = M_1 + M_2 + M_3 + \ldots.$$

- Note: summing amplitudes therefore different diagrams for the same final state can interfere either positively or negatively!

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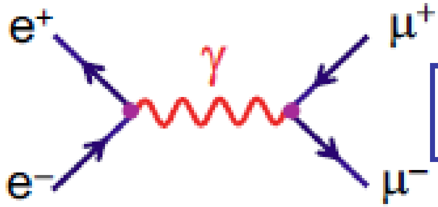
QED calculations

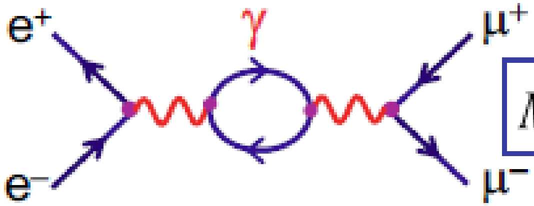
and then square $|M_{fi}|^2 = (M_1 + M_2 + M_3 + \ldots.)(M_1^* + M_2^* + M_3^* + \ldots.)$

➡ this gives the full perturbation expansion in α_{em}

- For QED $\alpha_{em} \sim 1/137$ the lowest order diagram dominates and for most purposes it is sufficient to neglect higher order diagrams.



$M^2 \propto \alpha_{em}^2$



$M^2 \propto \alpha_{em}^4$

❹ Calculate decay rate/cross section

- e.g. for a decay

$$\Gamma = \frac{p^*}{32\pi^2 m_a^2} \int |M_{fi}|^2 \mathrm{d}\Omega$$

- For scattering in the centre-of-mass frame

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^*} = \frac{1}{64\pi^2 s} \frac{|\vec{p}_f^*|}{|\vec{p}_i^*|} |M_{fi}|^2 \qquad (1)$$

- For scattering in lab. frame (neglecting mass of scattered particle)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2} \left(\frac{E_3}{M E_1} \right)^2 |M_{fi}|^2$$

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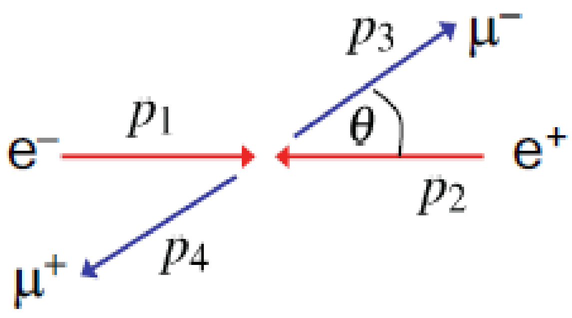
Electron-positron annihilation

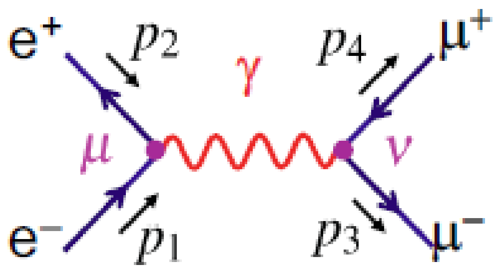
★Consider the process: $e^+e^- \rightarrow \mu^+\mu^-$

- Work in C.o.M. frame (this is appropriate for most e^+e^- colliders).

$$p_1 = (E,0,0,p) \qquad p_2 = (E,0,0,-p)$$
$$p_3 = (E,\vec{p}_f) \qquad p_4 = (E,-\vec{p}_f)$$



- Only consider the lowest order Feynman diagram:



- Feynman rules give:

$$-iM = [\overline{v}(p_2)ie\gamma^\mu u(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\overline{u}(p_3)ie\gamma^\nu v(p_4)]$$

NOTE:
- Incoming anti-particle $\overline{v}$
- Incoming particle u
- Adjoint spinor written first

- In the C.o.M. frame have

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{p}_f|}{|\vec{p}_i|}|M_{fi}|^2 \qquad \text{with} \qquad s = (p_1+p_2)^2 = (E+E)^2 = 4E^2$$

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Electron and muon currents

- Here $q^2 = (p_1 + p_2)^2 = s$ and matrix element

$$-iM = [\overline{v}(p_2)ie\gamma^\mu u(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\overline{u}(p_3)ie\gamma^\nu v(p_4)]$$

$$\Rightarrow \quad M = -\frac{e^2}{s}g_{\mu\nu}[\overline{v}(p_2)\gamma^\mu u(p_1)][\overline{u}(p_3)\gamma^\nu v(p_4)]$$

the four-vector current

$$j^\mu = \overline{\psi}\gamma^\mu\psi$$

which has same form as the two terms in [ ] in the matrix element

- The matrix element can be written in terms of the electron and muon currents

$$(j_e)^\mu = \overline{v}(p_2)\gamma^\mu u(p_1) \quad \text{and} \quad (j_\mu)^\nu = \overline{u}(p_3)\gamma^\nu v(p_4)$$

$$\Rightarrow \quad M = -\frac{e^2}{s}g_{\mu\nu}(j_e)^\mu(j_\mu)^\nu$$

$$\boxed{M = -\frac{e^2}{s}j_e\cdot j_\mu}$$

- Matrix element is a four-vector scalar product – confirming it is Lorentz Invariant

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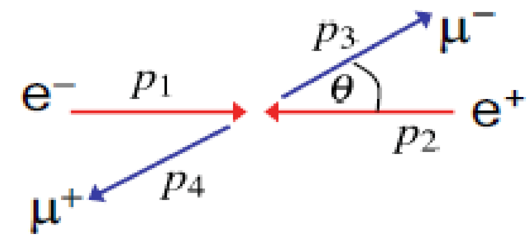
Spin in e$^+$e$^-$ annihilation



- In the C.o.M. frame in the limit $E \gg m$

$$p_1 = (E,0,0,E); \quad p_2 = (E,0,0,-E)$$
$$p_3 = (E, E\sin\theta, 0, E\cos\theta);$$
$$p_4 = (E, -\sin\theta, 0, -E\cos\theta)$$

- Left- and right-handed helicity spinors

$$u_\uparrow = N \begin{pmatrix} c \\ e^{i\phi} s \\ \frac{|\vec{p}|}{E+m} c \\ \frac{|\vec{p}|}{E+m} e^{i\phi} s \end{pmatrix} \quad u_\downarrow = N \begin{pmatrix} -s \\ e^{i\phi} c \\ \frac{|\vec{p}|}{E+m} s \\ -\frac{|\vec{p}|}{E+m} e^{i\phi} c \end{pmatrix} \quad v_\uparrow = N \begin{pmatrix} \frac{|\vec{p}|}{E+m} s \\ -\frac{|\vec{p}|}{E+m} e^{i\phi} c \\ -s \\ e^{i\phi} c \end{pmatrix} \quad v_\downarrow = N \begin{pmatrix} \frac{|\vec{p}|}{E+m} c \\ \frac{|\vec{p}|}{E+m} e^{i\phi} s \\ c \\ e^{i\phi} s \end{pmatrix}$$

where $s = \sin\frac{\theta}{2}$; $c = \cos\frac{\theta}{2}$ and $N = \sqrt{E+m}$

- In the limit $E \gg m$ these become:

$$u_\uparrow = \sqrt{E} \begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}; \; u_\downarrow = \sqrt{E} \begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix}; \; v_\uparrow = \sqrt{E} \begin{pmatrix} s \\ -ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix}; \; v_\downarrow = \sqrt{E} \begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}$$

- The initial-state electron can either be in a left- or right-handed helicity state

$$u_\uparrow(p_1) = \sqrt{E} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \; u_\downarrow(p_1) = \sqrt{E} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix};$$

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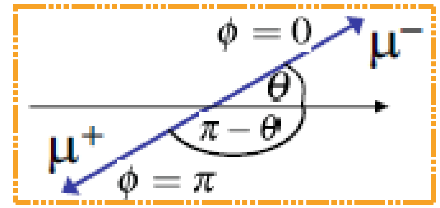
Spin in e^+e^- annihilation

- For the initial state positron $(\theta = \pi)$ can have either:

$$v_\uparrow(p_2) = \sqrt{E}\begin{pmatrix}1\\0\\-1\\0\end{pmatrix};\ v_\downarrow(p_2) = \sqrt{E}\begin{pmatrix}0\\1\\0\\1\end{pmatrix}$$

- Similarly for the final state μ^- which has polar angle θ and choosing $\phi = 0$

$$u_\uparrow(p_3) = \sqrt{E}\begin{pmatrix}c\\s\\c\\s\end{pmatrix};\ u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix}-s\\c\\s\\-c\end{pmatrix};$$



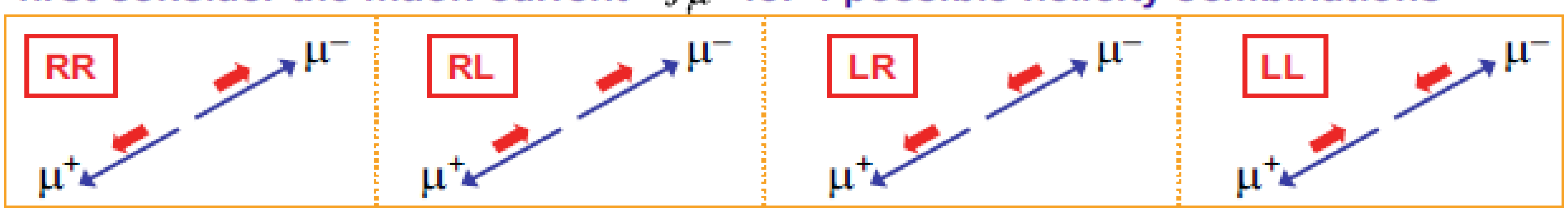
- And for the final state μ^+ replacing $\theta \to \pi - \theta;\quad \phi \to \pi$ obtain

$$v_\uparrow(p_4) = \sqrt{E}\begin{pmatrix}c\\s\\-c\\-s\end{pmatrix};\ v_\downarrow(p_4) = \sqrt{E}\begin{pmatrix}s\\-c\\s\\-c\end{pmatrix};$$

using
$$\sin\left(\tfrac{\pi-\theta}{2}\right) = \cos\tfrac{\theta}{2}$$
$$\cos\left(\tfrac{\pi-\theta}{2}\right) = \sin\tfrac{\theta}{2}$$
$$e^{i\pi} = -1$$

- Wish to calculate the matrix element $M = -\frac{e^2}{s} j_e \cdot j_\mu$

★ first consider the muon current j_μ for 4 possible helicity combinations



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The muon current

- Want to evaluate $(j_\mu)^\nu = \overline{u}(p_3)\gamma^\nu v(p_4)$ for all four helicity combinations
- For arbitrary spinors ψ, ϕ with it is straightforward to show that the components of $\overline{\psi}\gamma^\mu\phi$ are

$$\overline{\psi}\gamma^0\phi = \psi^\dagger\gamma^0\gamma^0\phi = \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4 \quad (3)$$
$$\overline{\psi}\gamma^1\phi = \psi^\dagger\gamma^0\gamma^1\phi = \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1 \quad (4)$$
$$\overline{\psi}\gamma^2\phi = \psi^\dagger\gamma^0\gamma^2\phi = -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1) \quad (5)$$
$$\overline{\psi}\gamma^3\phi = \psi^\dagger\gamma^0\gamma^3\phi = \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2 \quad (6)$$

- Consider the $\mu_R^-\mu_L^+$ combination using $\psi = u_\uparrow \quad \phi = v_\downarrow$

with $v_\downarrow = \sqrt{E}\begin{pmatrix} s \\ -c \\ s \\ -c \end{pmatrix}$; $u_\uparrow = \sqrt{E}\begin{pmatrix} c \\ s \\ c \\ s \end{pmatrix}$;

$$\overline{u}_\uparrow(p_3)\gamma^0 v_\downarrow(p_4) = E(cs - sc + cs - sc) = 0$$
$$\overline{u}_\uparrow(p_3)\gamma^1 v_\downarrow(p_4) = E(-c^2 + s^2 - c^2 + s^2) = 2E(s^2 - c^2) = -2E\cos\theta$$
$$\overline{u}_\uparrow(p_3)\gamma^2 v_\downarrow(p_4) = -iE(-c^2 - s^2 - c^2 - s^2) = 2iE$$
$$\overline{u}_\uparrow(p_3)\gamma^3 v_\downarrow(p_4) = E(cs + sc + cs + sc) = 4Esc = 2E\sin\theta$$

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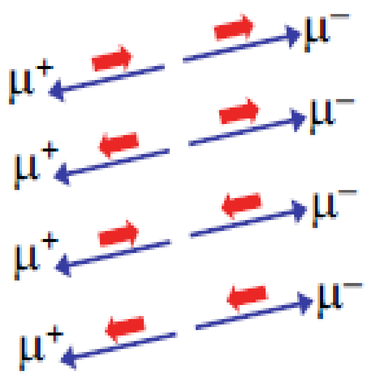
The muon current

- Hence the four-vector muon current for the RL combination is

$$\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4) = 2E(0,-\cos\theta,i,\sin\theta)$$

- The results for the 4 helicity combinations (obtained in the same manner) are:



$$\begin{aligned}
\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4) &= 2E(0,-\cos\theta,i,\sin\theta) && \text{RL}\\
\bar{u}_\uparrow(p_3)\gamma^\nu v_\uparrow(p_4) &= (0,0,0,0) && \text{RR}\\
\bar{u}_\downarrow(p_3)\gamma^\nu v_\downarrow(p_4) &= (0,0,0,0) && \text{LL}\\
\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) &= 2E(0,-\cos\theta,-i,\sin\theta) && \text{LR}
\end{aligned}$$

★ **IN THE LIMIT** $E \gg m$ **only two helicity combinations are non-zero !**

- This is an important feature of QED. It applies equally to QCD.
- In the Weak interaction only one helicity combination contributes.

- But as a consequence of the 16 possible helicity combinations only four given non-zero matrix elements

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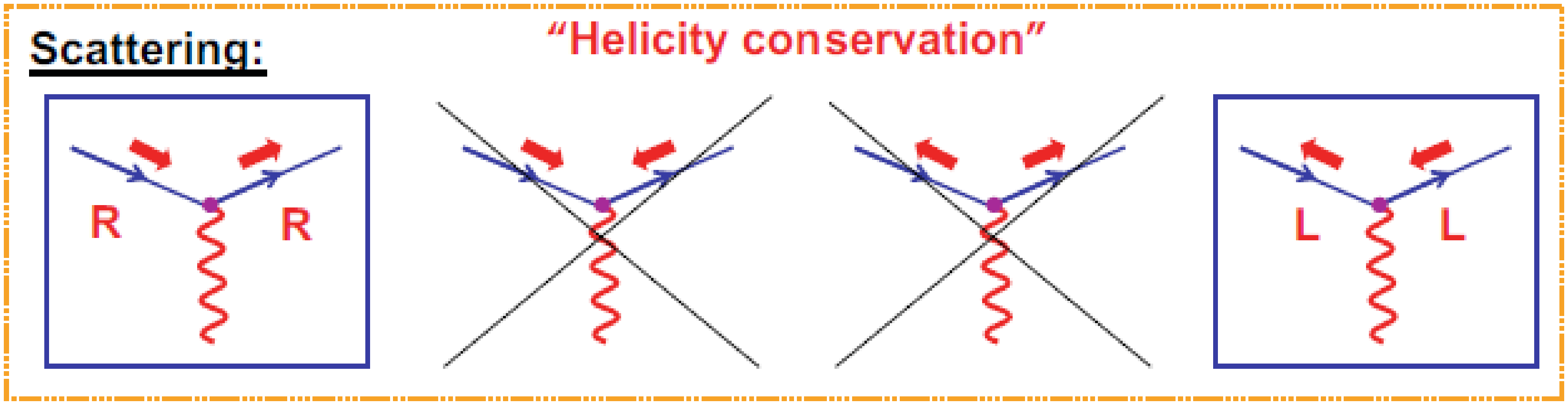
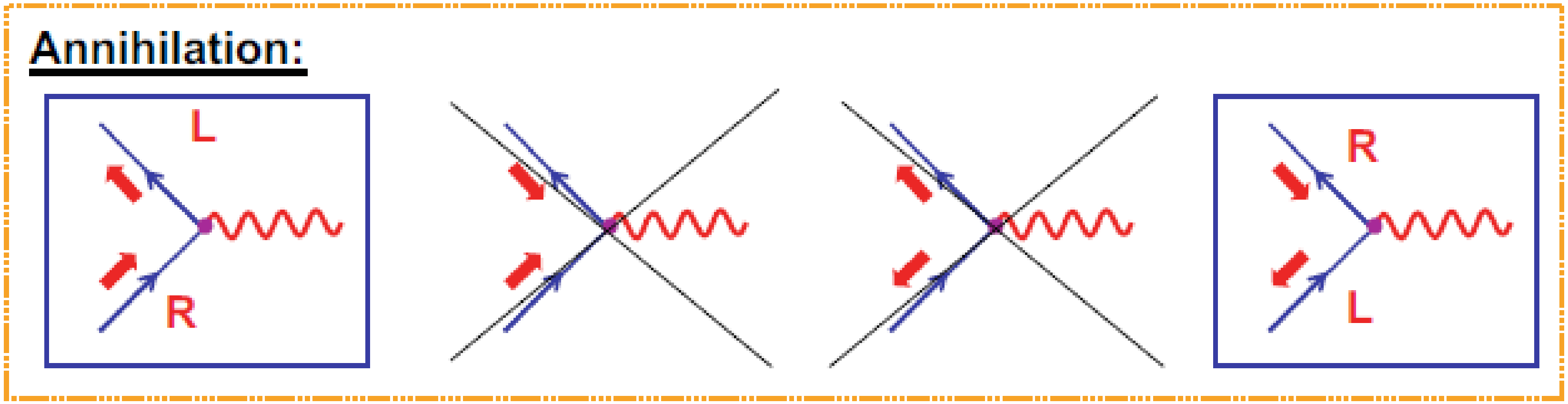
Allowed QED helicity combinations

- In the ultra-relativistic limit the helicity eigenstates ≡ chiral eigenstates
- In this limit, the only non-zero helicity combinations in QED are:





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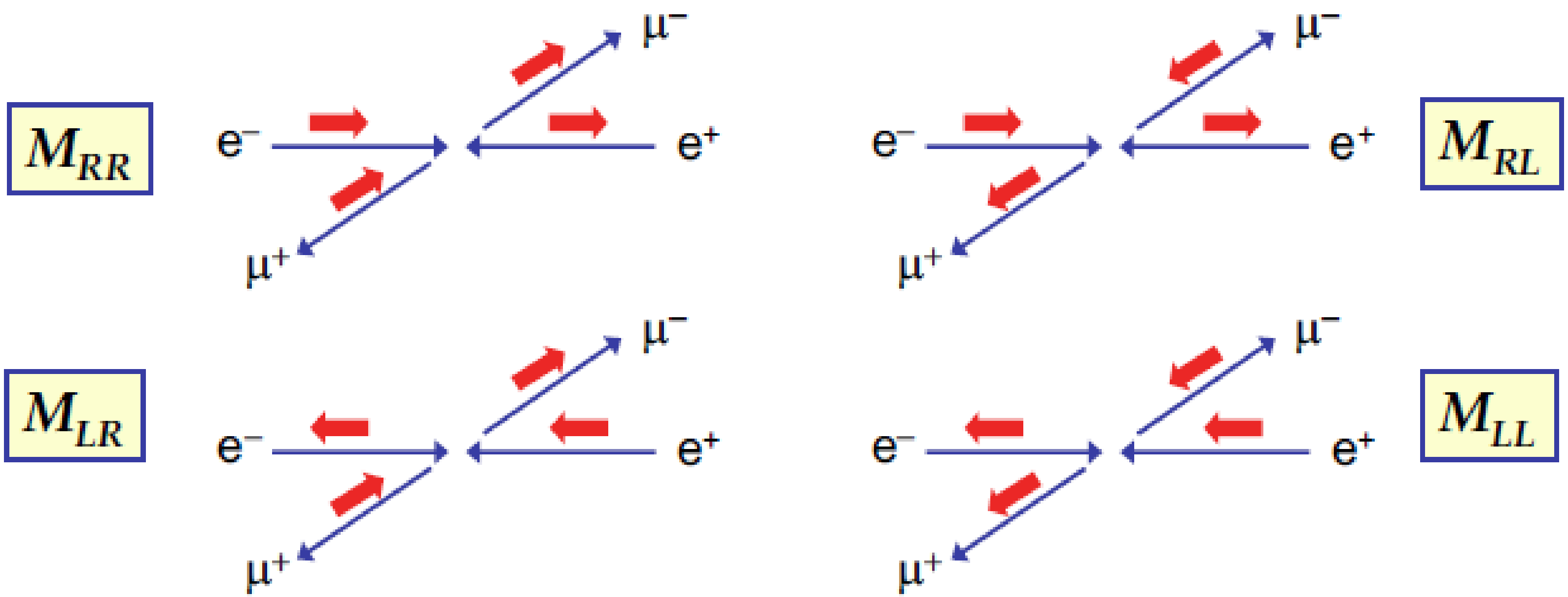
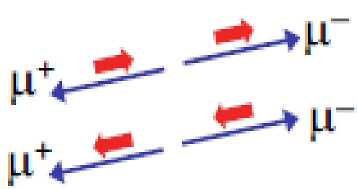
Electron-positron annihilation

★ For $e^+e^- \to \mu^+\mu^-$ now only have to consider the 4 matrix elements:



• Previously we derived the muon currents for the allowed helicities:



$$\mu_R^-\mu_L^+ : \quad \bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4) = 2E(0,-\cos\theta,i,\sin\theta)$$
$$\mu_L^-\mu_R^+ : \quad \bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) = 2E(0,-\cos\theta,-i,\sin\theta)$$

• Now need to consider the electron current

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The electron current

- The incoming electron and positron spinors (L and R helicities) are:

$$u_\uparrow = \sqrt{E}\begin{pmatrix}1\\0\\1\\0\end{pmatrix};\ u_\downarrow = \sqrt{E}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix};\quad v_\uparrow = \sqrt{E}\begin{pmatrix}1\\0\\-1\\0\end{pmatrix};\ v_\downarrow = \sqrt{E}\begin{pmatrix}0\\1\\0\\1\end{pmatrix}$$

- The electron current can either be obtained from equations (3)-(6) as before or it can be obtained directly from the expressions for the muon current.

$$(j_e)^\mu = \overline{v}(p_2)\gamma^\mu u(p_1) \qquad\qquad (j_\mu)^\mu = \overline{u}(p_3)\gamma^\mu v(p_4)$$

- Taking the Hermitian conjugate of the muon current gives

$$\begin{aligned}
[\overline{u}(p_3)\gamma^\mu v(p_4)]^\dagger &= [u(p_3)^\dagger\gamma^0\gamma^\mu v(p_4)]^\dagger \\
&= v(p_4)^\dagger\gamma^{\mu\dagger}\gamma^{0\dagger}u(p_3) && (AB)^\dagger = B^\dagger A^\dagger \\
&= v(p_4)^\dagger\gamma^{\mu\dagger}\gamma^0 u(p_3) && \gamma^{0\dagger} = \gamma^0 \\
&= v(p_4)^\dagger\gamma^0\gamma^\mu u(p_3) && \gamma^{\mu\dagger}\gamma^0 = \gamma^0\gamma^\mu \\
&= \overline{v}(p_4)\gamma^\mu u(p_3)
\end{aligned}$$

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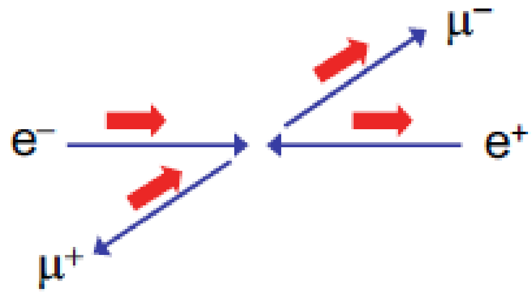
Matrix element calculation

- We can now calculate $M = -\frac{e^2}{s} j_e . j_\mu$ for the four possible helicity combinations.

e.g. the matrix element for $e_R^- e_L^+ \rightarrow \mu_R^- \mu_L^+$ which will denote M_{RR}



Here the first subscript refers to the helicity of the e⁻ and the second to the helicity of the μ⁻. Don't need to specify other helicities due to "helicity conservation", only certain chiral combinations are non-zero.

★Using: $e_R^- e_L^+$: $(j_e)^\mu = \bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1) = 2E(0,-1,-i,0)$

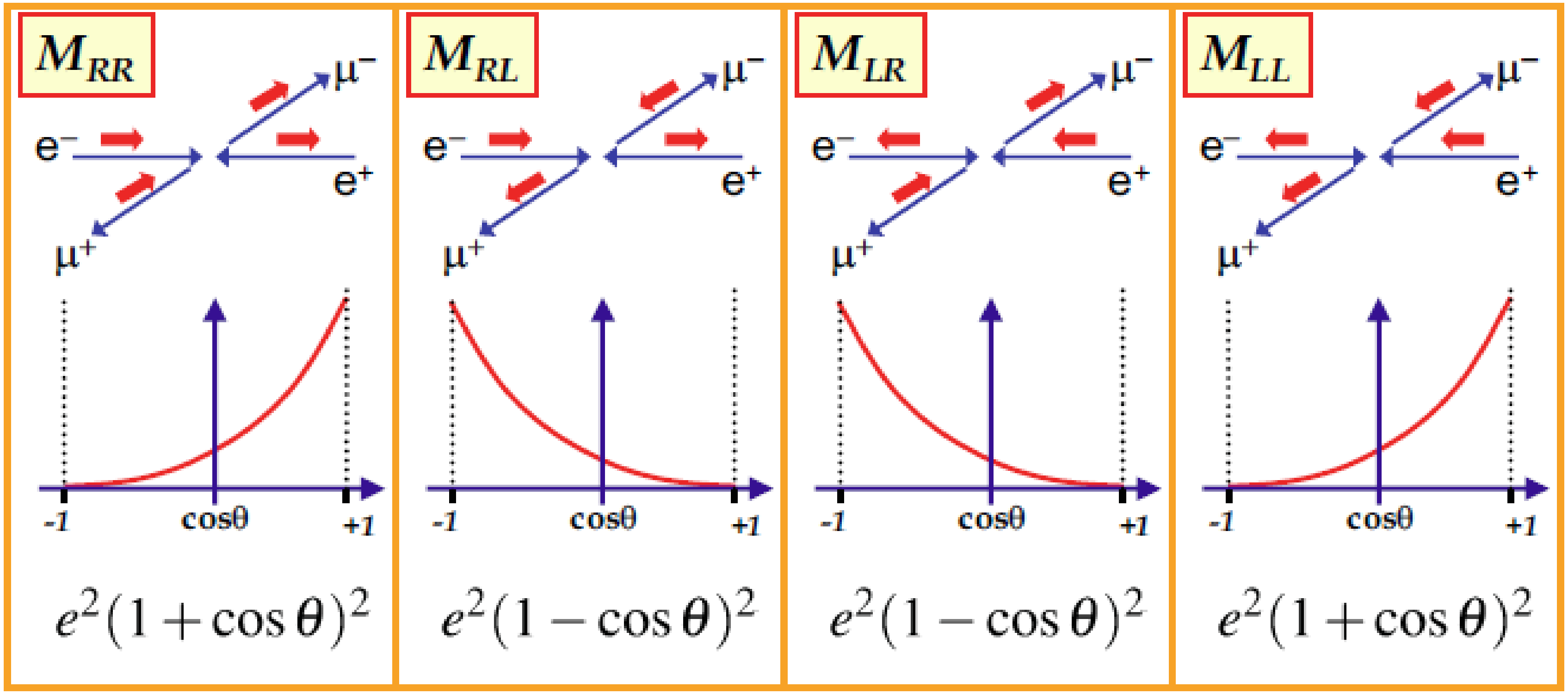
$\mu_R^- \mu_L^+$: $(j_\mu)^\nu = \bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4) = 2E(0,-\cos\theta,i,\sin\theta)$

gives
$$M_{RR} = -\frac{e^2}{s}[2E(0,-1,-i,0)].[2E(0,-\cos\theta,i,\sin\theta)]$$
$$= -e^2(1+\cos\theta)$$
$$= -4\pi\alpha(1+\cos\theta)$$
where $\alpha = e^2/4\pi \approx 1/137$

from M. A. Thomson lectures at Cambridge University in 2011

Matrix element calculation

Similarly
$$|M_{RR}|^2 = |M_{LL}|^2 = (4\pi\alpha)^2(1+\cos\theta)^2$$
$$|M_{RL}|^2 = |M_{LR}|^2 = (4\pi\alpha)^2(1-\cos\theta)^2$$



- Assuming that the incoming electrons and positrons are unpolarized, all 4 possible initial helicity states are equally likely.

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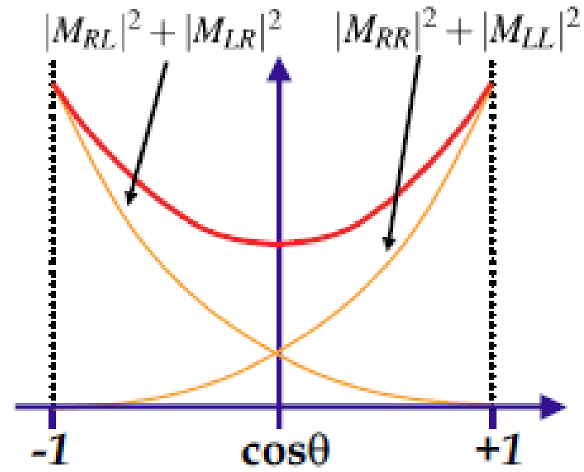
Differential cross-section

- The cross section is obtained by averaging over the initial spin states and summing over the final spin states:

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\times\frac{1}{64\pi^2 s}(|M_{RR}|^2+|M_{RL}|^2+|M_{LR}|^2+|M_{LL}^2|)$$

$$= \frac{(4\pi\alpha)^2}{256\pi^2 s}(2(1+\cos\theta)^2+2(1-\cos\theta)^2)$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta)$$

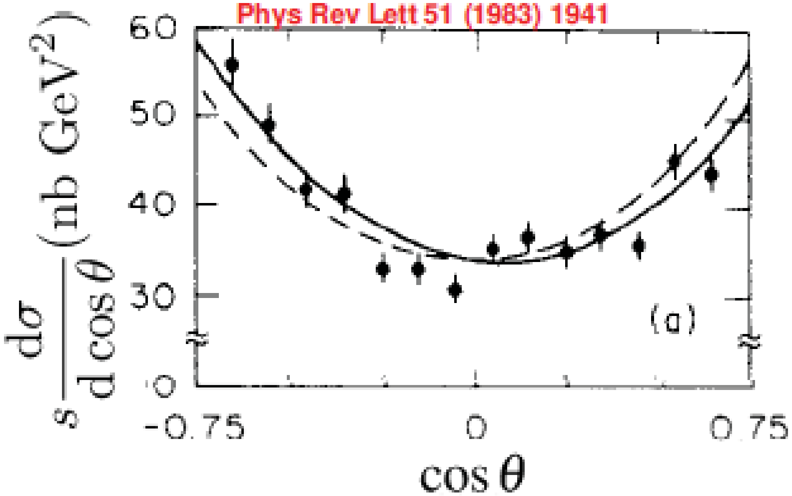


Example:

$e^+e^- \rightarrow \mu^+\mu^-$

$\sqrt{s} = 29$ GeV

Mark II Expt., M.E.Levi et al., Phys Rev Lett 51 (1983) 1941



- - - - - pure QED, O(α^3)

——— QED plus Z contribution

Angular distribution becomes slightly asymmetric in higher order QED or when Z contribution is included

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Differential cross-section

- The total cross section is obtained by integrating over θ, ϕ using

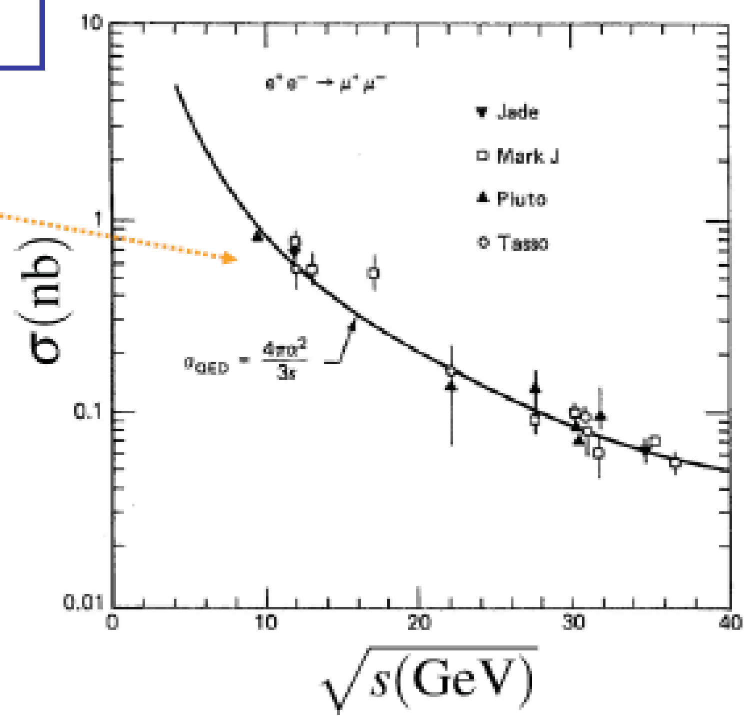
$$\int (1+\cos^2\theta)\mathrm{d}\Omega = 2\pi \int_{-1}^{+1} (1+\cos^2\theta)\mathrm{d}\cos\theta = \frac{16\pi}{3}$$

giving the QED total cross-section for the process $e^+e^- \to \mu^+\mu^-$

$$\sigma = \frac{4\pi\alpha^2}{3s}$$

★ Lowest order cross section calculation provides a good description of the data !

This is an impressive result. From first principles we have arrived at an expression for the electron-positron annihilation cross section which is good to 1%



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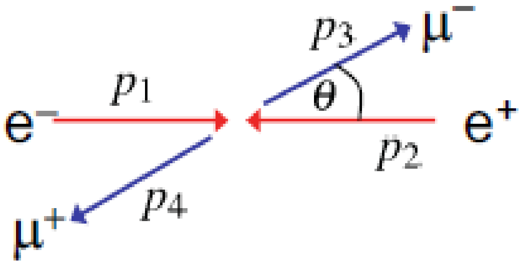
Lorentz Invariant Matrix Element

- Before concluding this discussion, note that the spin-averaged Matrix Element derived above is written in terms of the muon angle in the C.o.M. frame.

$$\begin{aligned}\langle|M_{fi}|^2\rangle &= \frac{1}{4}\times(|M_{RR}|^2+|M_{RL}|^2+|M_{LR}|^2+|M_{LL}^2|)\\ &= \frac{1}{4}e^4(2(1+\cos\theta)^2+2(1-\cos\theta)^2)\\ &= e^4(1+\cos^2\theta)\end{aligned}$$



- The matrix element is Lorentz Invariant (scalar product of 4-vector currents) and it is desirable to write it in a frame-independent form, i.e. express in terms of Lorentz Invariant 4-vector scalar products
- In the C.o.M. $p_1=(E,0,0,E)\quad p_2=(E,0,0,-E)$

$$p_3=(E,E\sin\theta,0,E\cos\theta)\quad p_4=(E,-E\sin\theta,0,-E\cos\theta)$$

giving: $p_1.p_2=2E^2;\quad p_1.p_3=E^2(1-\cos\theta);\quad p_1.p_4=E^2(1+\cos\theta)$

- Hence we can write

$$\langle|M_{fi}|^2\rangle = 2e^4\frac{(p_1.p_3)^2+(p_1.p_4)^2}{(p_1.p_2)^2}$$

$$\equiv 2e^4\left(\frac{t^2+u^2}{s^2}\right)$$

★ Valid in any frame !

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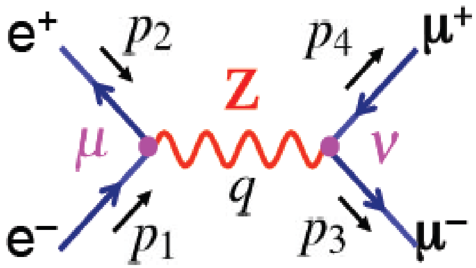
The Z resonance

★ Want to calculate the cross-section for $e^+e^- \to Z \to \mu^+\mu^-$

•Feynman rules for the diagram below give:



e^+e^- vertex: $\overline{v}(p_2)\cdot -ig_Z\gamma^\mu \frac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)$

Z propagator: $\dfrac{-ig_{\mu\nu}}{q^2 - m_Z^2}$

$\mu^+\mu^-$ vertex: $\overline{u}(p_3)\cdot -ig_Z\gamma^\nu \frac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)$

$$\Rightarrow \quad -iM_{fi} = [\overline{v}(p_2)\cdot -ig_Z\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)].\frac{-ig_{\mu\nu}}{q^2 - m_Z^2}.[\overline{u}(p_3)\cdot -ig_Z\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)]$$

$$\Rightarrow \quad \boxed{M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[\overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)].[\overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)]}$$

★ Convenient to work in terms of helicity states by explicitly using the Z coupling to **LH** and **RH** chiral states (ultra-relativistic limit so helicity = chirality)

$$\tfrac{1}{2}(c_V - c_A\gamma^5) = c_L\tfrac{1}{2}(1-\gamma^5) + c_R\tfrac{1}{2}(1+\gamma^5)$$

LH and RH projections operators

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The Z resonance

hence $c_V = (c_L + c_R), \;\; c_A = (c_L - c_R)$

and $\frac{1}{2}(c_V - c_A\gamma^5) \;=\; \frac{1}{2}(c_L + c_R - (c_L - c_R)\gamma^5)$

$\qquad\qquad\qquad\qquad\quad = \; c_L\frac{1}{2}(1-\gamma^5) + c_R\frac{1}{2}(1+\gamma^5)$

with $c_L = \frac{1}{2}(c_V + c_A), \;\; c_R = \frac{1}{2}(c_V - c_A)$

★ Rewriting the matrix element in terms of LH and RH couplings:

$$M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1) + c_R^e \overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(1+\gamma^5)u(p_1)]$$
$$\times[c_L^\mu \overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(1-\gamma^5)v(p_4) + c_R^\mu \overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(1+\gamma^5)v(p_4)]$$

★ Apply projection operators remembering that in the ultra-relativistic limit

$\frac{1}{2}(1-\gamma^5)u = u_\downarrow; \quad \frac{1}{2}(1+\gamma^5)u = u_\uparrow, \quad \frac{1}{2}(1-\gamma^5)v = v_\uparrow, \quad \frac{1}{2}(1+\gamma^5)v = v_\downarrow$

⇨ $$M_{fi} = -\frac{g_Z}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \overline{v}(p_2)\gamma^\mu u_\uparrow(p_1)]$$
$$\times[c_L^\mu \overline{u}(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu \overline{u}(p_3)\gamma^\nu v_\downarrow(p_4)]$$

★ For a combination of V and A currents, $\overline{u}_\uparrow \gamma^\mu v_\uparrow = 0$ etc, gives four orthogonal contributions

⇨ $$-\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)]$$
$$\times[c_L^\mu \overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu \overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$

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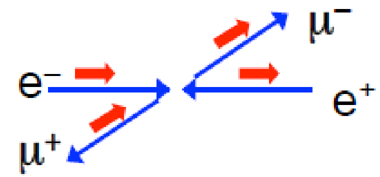
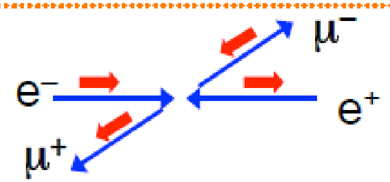
The Z resonance

★ Sum of 4 terms

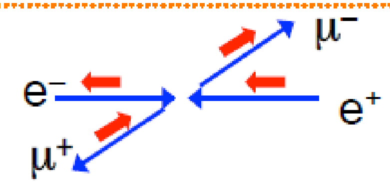
$$M_{RR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_R^\mu g_{\mu\nu} [\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



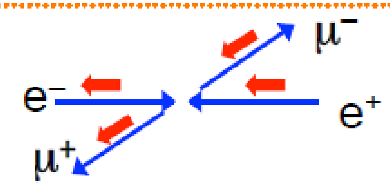
$$M_{RL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_L^\mu g_{\mu\nu} [\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



$$M_{LR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_R^\mu g_{\mu\nu} [\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



$$M_{LL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_L^\mu g_{\mu\nu} [\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



Remember: the L/R refer to the helicities of the initial/final state particles

★ Fortunately we have calculated these terms before when considering $e^+e^- \to \gamma \to \mu^+\mu^-$ giving:

$$[\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)] = s(1+\cos\theta)$$ etc.

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The Z resonance

- Applying the QED results to the Z exchange with

$$\frac{e^2}{q^2} \rightarrow \frac{g_Z^2}{q^2 - m_Z^2} c^e c^\mu$$

where $q^2 = s = 4E_e^2$

gives:

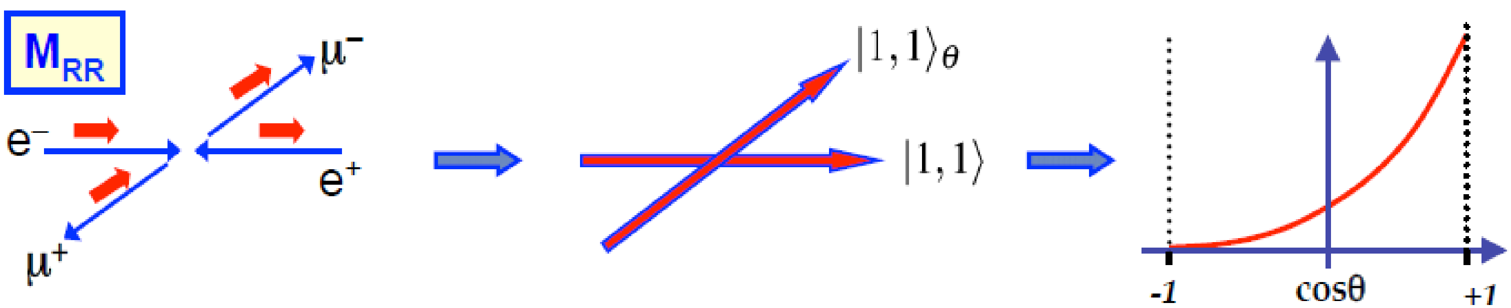
$$|M_{RR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_R^\mu)^2 (1 + \cos\theta)^2$$

$$|M_{RL}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_L^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_R^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_L^\mu)^2 (1 + \cos\theta)^2$$

- As before, the angular dependence of the matrix elements can be understood in terms of the spins of the incoming and outgoing particles e.g.



The Breit-Wigner Resonance

- ★ Need to consider carefully the propagator term $1/(s-m_Z^2)$ which diverges when the C.o.M. energy is equal to the rest mass of the Z boson
- ★ To do this need to account for the fact that the Z boson is an unstable particle
  - For a stable particle at rest the time development of the wave-function is:

$$\psi \sim e^{-imt}$$

  - For an unstable particle this must be modified to

$$\psi \sim e^{-imt} e^{-\Gamma t/2}$$

  so that the particle probability decays away exponentially

$$\psi^* \psi \sim e^{-\Gamma t} = e^{-t/\tau} \qquad \text{with} \qquad \tau = \frac{1}{\Gamma_Z}$$

  - Equivalent to making the replacement

$$m \rightarrow m - i\Gamma/2$$

- ★ In the Z boson propagator make the substitution:

$$m_Z \rightarrow m_Z - i\Gamma_Z/2$$

- ★ Which gives:

$$(s-m_Z^2) \longrightarrow [s-(m_Z - i\Gamma_Z/2)] = s - m_Z^2 + im_Z\Gamma_Z + \tfrac{1}{4}\Gamma_Z^2 \approx s - m_Z^2 + im_Z\Gamma_Z$$

  where it has been assumed that $\Gamma_Z \ll m_Z$

- ★ Which gives

$$\left|\frac{1}{s-m_Z^2}\right|^2 \rightarrow \left|\frac{1}{s-m_Z^2+im_Z\Gamma_Z}\right|^2 = \frac{1}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$

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The Z resonance

- ★ And the Matrix elements become

$$|M_{RR}|^2 = \frac{g_Z^4 s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2 \qquad \text{etc.}$$

- ★ In the limit where initial and final state particle mass can be neglected:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|M_{fi}|^2$$

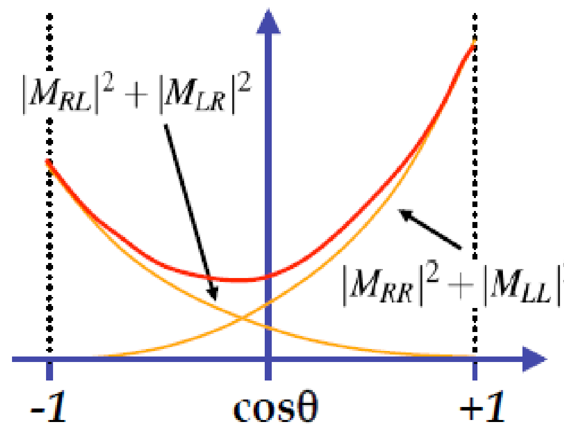
- ★ Giving:

$$\frac{d\sigma_{RR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$

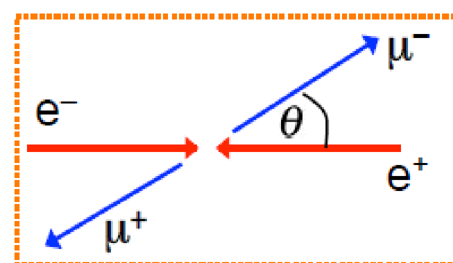
$$\frac{d\sigma_{LL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_L^\mu)^2(1+\cos\theta)^2$$

$$\frac{d\sigma_{LR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

$$\frac{d\sigma_{RL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$



- ★ Because $|M_{LL}|^2 + |M_{RR}|^2 \neq |M_{LR}|^2 + |M_{RL}|^2$, the differential cross section is asymmetric, i.e. parity violation (although not maximal as was the case for the W boson).



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Cross-section for unpolarised beams

★ To calculate the total cross section need to sum over all matrix elements and **average** over the **initial spin states**. Here, assuming unpolarized beams (i.e. both e^+ and both e^- spin states equally likely) there a four combinations of initial electron/positron spins, so

$$\langle |M_{fi}|^2 \rangle = \frac{1}{2}\cdot\frac{1}{2}\cdot(|M_{RR}|^2 + |M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2)$$

$$= \frac{1}{2}\cdot\frac{1}{2}\frac{g_Z^4 s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \times \{[(c_R^e)^2(c_R^\mu)^2 + (c_L^e)^2(c_L^2)^2](1+\cos\theta)^2 + [(c_L^e)^2(c_R^\mu)^2 + (c_R^e)^2(c_L^2)^2](1-\cos\theta)^2\}$$

★ The part of the expression {…} can be rearranged:

$$\{\ldots\} = [(c_R^e)^2 + (c_L^e)^2][(c_R^\mu)^2 + (c_L^\mu)^2](1+\cos^2\theta) + 2[(c_R^e)^2 - (c_L^e)^2][(c_R^\mu)^2 - (c_L^\mu)^2]\cos\theta \qquad (1)$$

and using $c_V^2 + c_A^2 = 2(c_L^2 + c_R^2)$ and $c_V c_A = c_L^2 + c_R^2$

$$\{\ldots\} = \frac{1}{4}[(c_V^e)^2 + (c_A^e)^2][(c_V^\mu)^2 + (c_A^\mu)^2](1+\cos^2\theta) + 2c_V^e c_A^e c_V^\mu c_A^\mu \cos\theta$$

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Cross-section for unpolarised beams

★ Hence the complete expression for the unpolarized differential cross section is:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\langle|M_{fi}|^2\rangle$$

$$= \frac{1}{64\pi^2}\cdot\frac{1}{4}\cdot\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\times$$

$$\left\{\tfrac{1}{4}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2](1+\cos^2\theta)+2c_V^e c_A^e c_V^\mu c_A^\mu\cos\theta\right\}$$

★ Integrating over solid angle $d\Omega = d\phi\, d(\cos\theta) = 2\pi d(\cos\theta)$

$$\int_{-1}^{+1}(1+\cos^2\theta)d(\cos\theta) = \int_{-1}^{+1}(1+x^2)dx = \tfrac{8}{3} \text{ and } \int_{-1}^{+1}\cos\theta\, d(\cos\theta) = 0$$

$$\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2]$$

★ Note: the total cross section is proportional to the sums of the squares of the vector- and axial-vector couplings of the initial and final state fermions

$$(c_V^f)^2+(c_A^f)^2$$

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Connection to the Breit-Wigner formula

★ Can write the total cross section

$$\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^\mu)^2+(c_A^\mu)^2\right]$$

in terms of the Z boson decay rates (partial widths)

$$\Gamma(Z\to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}\left[(c_V^e)^2+(c_A^e)^2\right] \quad \text{and} \quad \Gamma(Z\to\mu^+\mu^-) = \frac{g_Z^2 m_Z}{48\pi}\left[(c_V^\mu)^2+(c_A^\mu)^2\right]$$

$$\Rightarrow \quad \sigma = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma(Z\to e^+e^-)\Gamma(Z\to\mu^+\mu^-)$$

★ Writing the partial widths as $\Gamma_{ee}=\Gamma(Z\to e^+e^-)$ etc., the total cross section can be written

$$\sigma(e^+e^-\to Z\to f\bar{f}) = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff} \qquad (2)$$

where f is the final state fermion flavour:

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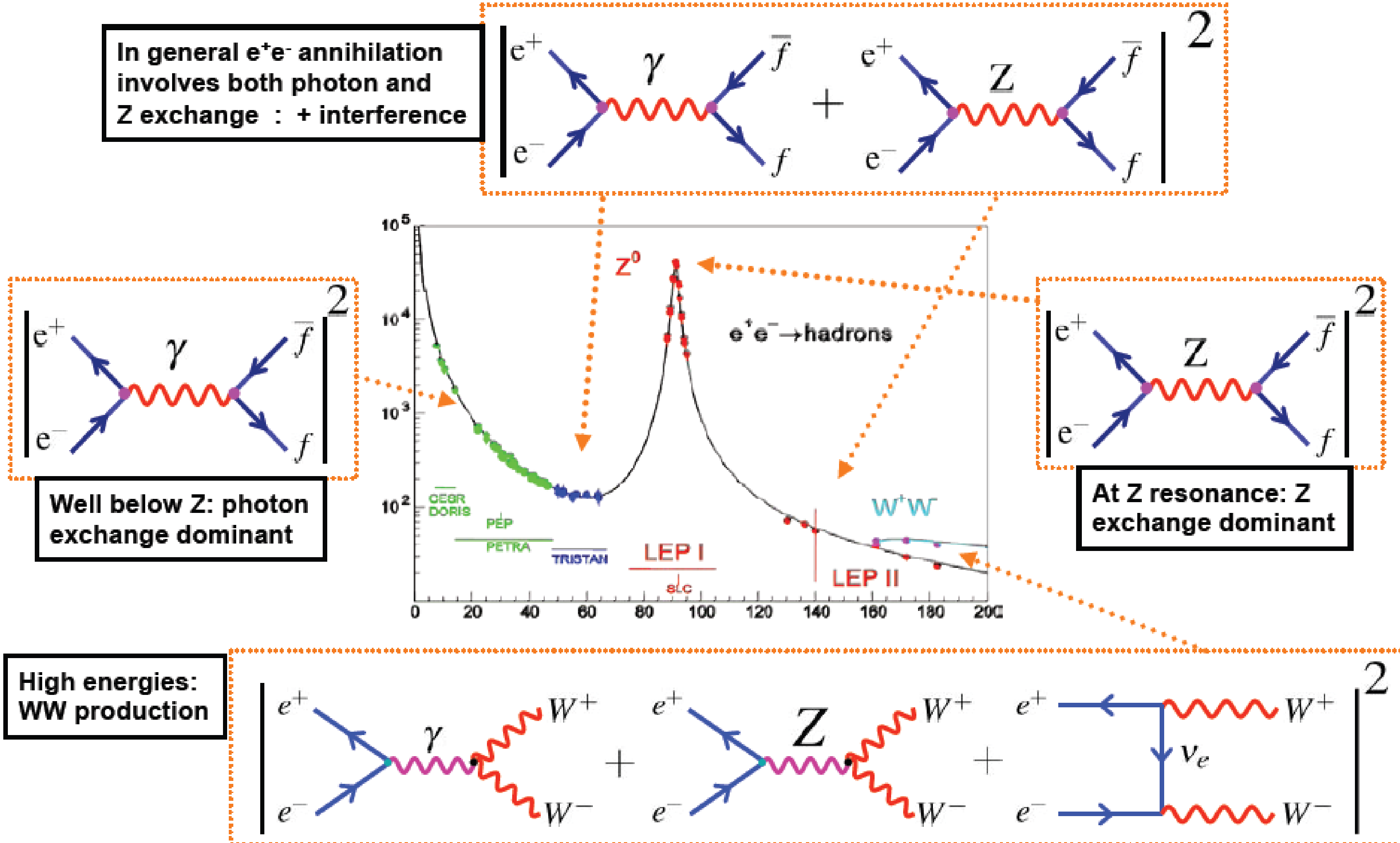
e+ e- annihilation in Feynman diagrams



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Forward-backward asymmetry

★ we obtained the expression for the differential cross section:

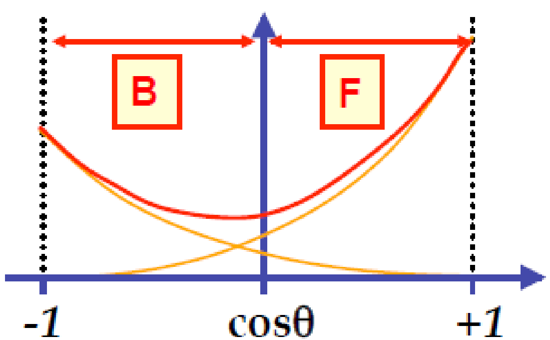
$$\langle |M_{fi}| \rangle^2 \propto [(c_L^e)^2 + (c_R^e)^2][(c_L^\mu)^2 + (c_R^\mu)^2](1+\cos^2\theta) + [(c_L^e)^2 - (c_R^e)^2][(c_L^\mu)^2 - (c_R^\mu)^2]\cos\theta$$

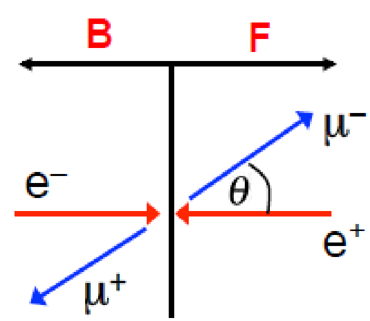
★ The differential cross sections is therefore of the form:

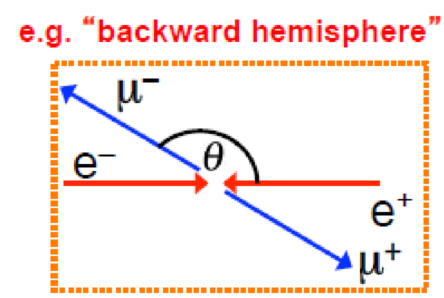
$$\frac{d\sigma}{d\Omega} = \kappa \times [A(1+\cos^2\theta) + B\cos\theta] \quad \begin{cases} A = [(c_L^e)^2 + (c_R^e)^2][(c_L^\mu)^2 + (c_R^\mu)^2] \\ B = [(c_L^e)^2 - (c_R^e)^2][(c_L^\mu)^2 - (c_R^\mu)^2] \end{cases}$$

★ Define the **FORWARD** and **BACKWARD** cross sections in terms of angle incoming electron and out-going particle

$$\sigma_F \equiv \int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta \qquad \sigma_B \equiv \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta$$







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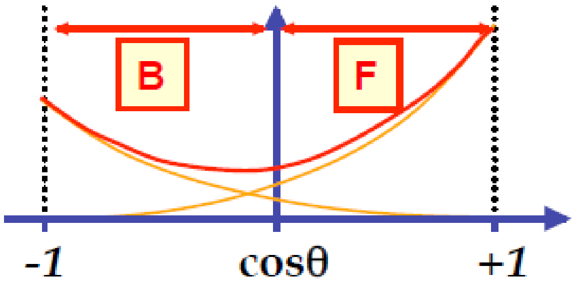
Forward-backward asymmetry

★ The level of asymmetry about cosθ=0 is expressed in terms of the Forward-Backward Asymmetry

$$A_{\mathrm{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$



• Integrating equation (1):

$$\sigma_F = \kappa \int_0^1 [A(1+\cos^2\theta) + B\cos\theta]\,\mathrm{d}\cos\theta = \kappa \int_0^1 [A(1+x^2) + Bx]\mathrm{d}x = \kappa\left(\frac{4}{3}A + \frac{1}{2}B\right)$$

$$\sigma_B = \kappa \int_{-1}^0 [A(1+\cos^2\theta) + B\cos\theta]\,\mathrm{d}\cos\theta = \kappa \int_{-1}^0 [A(1+x^2) + Bx]\mathrm{d}x = \kappa\left(\frac{4}{3}A - \frac{1}{2}B\right)$$

★ Which gives:

$$A_{\mathrm{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{B}{(8/3)A} = \frac{3}{4}\left[\frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2}\right] \cdot \left[\frac{(c_L^\mu)^2 - (c_R^\mu)^2}{(c_L^\mu)^2 + (c_R^\mu)^2}\right]$$

★ This can be written as

$$A_{\mathrm{FB}} = \frac{3}{4}A_e A_\mu$$ with $$A_f \equiv \frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} = \frac{2c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2} \qquad (4)$$

★ Observe a non-zero asymmetry because the couplings of the Z to LH and RH particles are different. Contrast with QED where the couplings to LH and RH particles are the same (parity is conserved) and the interaction is FB symmetric

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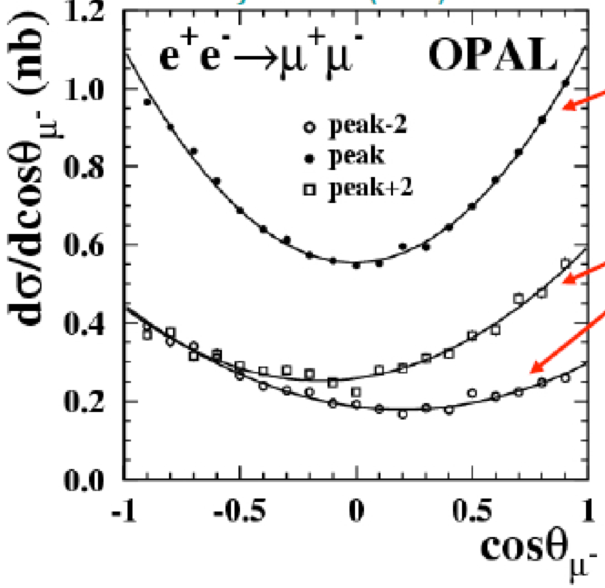
Forward-backward asymmetry

★ Forward-backward asymmetries can only be measured for final states where the charge of the fermion can be determined, e.g. $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$

OPAL Collaboration,
Eur. Phys. J. C19 (2001) 587-651.



Because $\sin^2\theta_W \approx 0.25$, the value of A_{FB} for leptons is almost zero

For data above and below the peak of the Z resonance interference with $e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$ leads to a larger asymmetry

★ LEP data combined:

$$A_{FB}^{0,e} = 0.0145 \pm 0.0025$$
$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$$
$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$$

★ To relate these measurements to the couplings uses $A_{FB} = \frac{3}{4} A_e A_\mu$

★ In all cases asymmetries depend on A_e

★ To obtain A_e could use $A_{FB}^{0,e} = \frac{3}{4} A_e^2$

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