TTP92–12
March 18, 1992

ANGULAR DECAY DISTRIBUTION OF LEPTONS FROM W-BOSONS AT NLO IN HADRONIC COLLISIONS

E. Mirkes

Institut für theoretische Teilchenphysik
Universität Karlsruhe
Kaiserstr. 12, Postfach 6980
7500 Karlsruhe 1, Germany

Abstract

We propose the measurement of the angular decay distribution of leptons from W-production at high-q_T in $p\bar{p}$ collisions. The polar and azimuthal angles in the W rest frame can be reconstructed modulo a sign ambiguity in $\cos\theta$ enabling the measurement of six out of nine helicity cross sections σ^α describing the most general decay distribution. We review the LO results of the nine helicity cross sections and present the results of a complete $O(\alpha_s^2)$ next-to-leading (NLO) calculation of four parity conserving structure functions. We show that large deviations from the well-known $(1+\cos\theta)^2$ distribution valid at low-q_T are expected already at moderate transverse W-momentum ($q_T > 20\,\mathrm{GeV}$). We present detailed analytical formulas for all parton level cross sections and discuss the size of the NLO corrections for W^+ production at the Tevatron collider. It is shown that the measurement of σ^A (originating from the p.v. part of the hadron tensor) provide clear constraints on the gluon structure function since it receives sizable contributions only from the qG initiated process.

1 Introduction

The physics of high transverse momentum q_T gauge boson production at hadron colliders is a rich source of information on many aspects of the physics of the standard model. From the fact that the gauge boson is produced at high-q_T one can define an event plane spanned by the beam and the gauge boson's momentum direction which provides a reference plane for a detailed study of lepton-hadron correlation effects using the decay leptons from the decay of the gauge bosons. In fact, these correlations are described by a set of nine hadronic structure functions which can be calculated within the context of the parton model using perturbative QCD. In this way one can test the standard model in a much more detailed way as can be done by rate measurements alone.

In this paper we concentrate on the production and decay of W-bosons. Charged W's have the advantage that they are produced more copiously than Z-bosons (approximately 6 : 1 at Tevatron conditions).

In refs. [1,2] the complete next-to-leading order $O(\alpha_s^2)$ corrections to the production rate of high q_T gauge bosons at hadron colliders have been calculated. In this paper, we extend the calculation of [1,2] to the polarization of the produced high-q_T gauge boson to $O(\alpha_s^2)$.

The polarization of the gauge boson determines the shape of the spectra of the decay leptons or jets in the laboratory frame which has important implications for the study of the production of the top quark.

Also the polarization state of the W determines the angular distribution of the decay leptons or jets in the W rest frame. The general structure of the angular distribution is given by nine helicity cross sections corresponding to nine density matrix elements of the gauge boson. Since the NLO corrections to the rate are sizable it is important to have available the complete NLO corrections to the polarization of high q_T gauge bosons produced in hadronic collisions.

Concerning the lepton decay modes the W's have the disadvantage that the decay kinematics cannot be completely reconstructed due to the unobservability of the neutrino. The transverse components of the neutrino's momentum can be approximated from transverse momentum balancing in calorimetric experiments. However, the longitudinal momentum of the neutrino can only be obtained up to a twofold ambiguity. Choosing the Collins-Soper frame [3] we show that the polar and azimuthal angles of the leptons θ and ϕ in the W rest frame can be reconstructed modulo a sign ambiguity in $\cos\theta$ enabling the measurement of six out of nine helicity cross sections.

We review the LO results for six dispersive [8] (time-reversal-(T)-even, $O(\alpha_s)$) and three absorptive [6] (T-odd, $O(\alpha_s^2)$) helicity cross sections and show that large deviations from the well-known $(1+\cos\theta)^2$ distribution valid at low q_T are expected already at moderate transverse-W-momenta. We calculate the $O(\alpha_s^2)$ corrections to the four parity conserving (T-even) helicity cross sections and show that there are sizable corrections to the born results at high q_T. It is clear that the NLO results are more reliable than the $O(\alpha_s)$ results as they depend less on the renormalization and factorization scales. Once the asymmetries are established for standard W, we can use the method to determine the couplings of possible heavier weak bosons.

On the other hand, measurements of the angular decay distribution of leptons (quarks) provide additional constraints on the parton distribution functions, besides the parton distribution function determintation from deep inelastic scattering experiments. It is shown that the angular coefficient A_3 coming from the parity violating part of the hadron tensor receives sizable contributions only from the qG partonic subprocess. We will also show relative contributions for different $O(\alpha_s^2)$ parton processes to various helicity cross sections at $\sqrt{S} = 1.8$ TeV. We find that the $O(\alpha_s^2)$ contribution of the qG process is even more important than the $O(\alpha_s)$ $q\bar{q}$ contributions for moderate values of q_T at these energies. However, the GG contribution is still neglegible at Tevatron energies. First numerical results for the dominant $O(\alpha_s^2)$ contributions are given in [4,5].

The paper is organized as follows:

In sec. 2, we discuss the general form of the angular decay distribution of leptons (quarks) in the gauge boson rest frame and discuss the advantage of the Collins-Soper frame for the reconstruction of the decay angles. In sec. 3, we review the LO results and present explicit tree level expressions for all six T-even helicity cross sections and LO ($O(\alpha_s^2)$) results for the three T-odd helicity cross sections. In sec. 4 we present the $O(\alpha_s^2)$ tree plus one-loop corrections to four parity conserving helicity cross sections. In sec. 5, we give numerical predictions for the $O(\alpha_s^2)$ helicity cross sections and discuss the size of the NLO corrections. Sec. 5 deals with some details of the mass factorization for the different helicity cross sections. All technical details and detailed analytical formulas for all parton level cross sections at NLO are relegated to the appendices.

2 Angular decay distribution

We consider the angular decay distribution of leptons from gauge bosons produced in high energy proton-antiproton collisions. For definiteness we take

$$p(P_1) + \bar{p}(P_2) \to W^+(Q) + X \to l^+(l_1) + \nu_l(l_2) + X \tag{1}$$

We denote the transverse momentum of the W boson in the laboratory frame by q_T, its rapidity by y, and the squared cms energy by $S = (P_1 + P_2)^2$. At low q_T the angular distribution of the decay lepton in (1) is given by the well-know formula

$$\frac{d\sigma}{dy\, d\cos\theta} = \text{valence quarks} \otimes (1+\cos\theta)^2 + \text{sea quarks} \otimes (1+\cos\theta^2) \tag{2}$$

where θ is the polar angle of the electron (positron) in the gauge boson rest frame with respect to the proton (antiproton) direction.

In this paper we are interested in the case of high-q_T W production. In the parton model the hadronic cross section is obtained by folding the hard parton level cross section with the respective parton densities. One has

$$\frac{d\sigma^{h_1h_2}}{dq_T^2\, dy\, d\Omega^*} = \sum_{ab} \int dx_1 dx_2 f_a^{h_1}(x_1, M^2) f_b^{h_2}(x_2, M^2) \frac{s\, d\tilde{\sigma}_{ab}}{dt\, du\, d\Omega^*} \left(x_1P_1, x_2P_2, \alpha_s(\mu^2)\right) \tag{3}$$

where one sums over $a, b = q, \bar{q}, G$. $f_a^h(x, M^2)$ is the probability density to find parton a with momentum fraction x in hadron h if it is probed at scale M^2. The parton cross section for the process

$$a(p_1) + b(p_2) \to W(Q) + X \tag{4}$$

is denoted by $d\tilde{\sigma}_{ab}$. We define the Mandelstam variables to the hadron and parton level as follows:

$$\begin{aligned} s &= (p_1+p_2)^2 &= x_1 x_2 S \\ t &= (p_1-q)^2 &= x_1(T-Q^2)+Q^2 \\ u &= (p_2-q)^2 &= x_2(U-Q^2)+Q^2 \end{aligned} \tag{5}$$

The rapidity y is defined by

$$y = \frac{1}{2}\log\left(\frac{Q^2-U}{Q^2-T}\right) \tag{6}$$

Note that the transverse momentum squared

$$q_T^2 = \frac{(Q^2-U)(Q^2-T)}{S} - Q^2 \tag{7}$$

is invariant under boosts along the beam direction.

The angles θ and ϕ in $d\Omega^* = d\cos\theta\, d\phi$ are the polar and azimuthal decay angles of the leptons in the gauge boson rest frame with respect to a coordinate system described below. The angular dependence in (3) can be extracted by introducing nine helicity cross sections corresponding to the nine polarization density matrix elements defined in (81). The general form of the angular distribution of the (charged) decay lepton at the parton level is derived in appendix C. At the hadronic level, the angular distribution in the W rest frame is then given by the following nine hadronic helicity cross sections $d\sigma^\alpha$

$$\frac{d\sigma}{dq_T^2\, dy\, d\Omega^*} = \sum_{\alpha\in\mathcal{M}} g_\alpha(\theta,\phi)\,\frac{3}{16\pi}\,\frac{d\sigma^\alpha}{dq_T^2\, dy} \qquad \mathcal{M} := \{U{+}L, L, T, I, P, A, 7, 8, 9\} \tag{8}$$

where the coefficients $g_\alpha(\theta,\phi)$ are defined by:

$$\begin{aligned} g_{U+L}(\theta,\phi) &= 1+\cos^2\theta & g_A(\theta,\phi) &= 4\sqrt{2}\sin\theta\,\cos\phi \\ g_L(\theta,\phi) &= 1-3\cos^2\theta & g_7(\theta,\phi) &= 2\sin^2\theta\,\sin 2\phi \\ g_T(\theta,\phi) &= 2\sin^2\theta\,\cos 2\phi & g_8(\theta,\phi) &= 2\sqrt{2}\sin 2\theta\,\sin\phi \\ g_I(\theta,\phi) &= 2\sqrt{2}\sin 2\theta\,\cos\phi & g_9(\theta,\phi) &= 4\sqrt{2}\sin\theta\,\sin\phi \\ g_P(\theta,\phi) &= 2\cos\theta & & \end{aligned} \tag{9}$$

For $\alpha = U+L$, $d\sigma^\alpha$ denotes the production cross section (i.e. the cross section for unpolarized bosons), whereas for all other values of $\alpha \in \mathcal{M}$ $d\sigma^\alpha$ denotes the different contributions to the cross section for polarized gauge bosons. These helicity cross sections $d\sigma^\alpha$ give the angular distribution for one of the leptons. Hereafter we take the charged lepton.

The helicity cross sections $\sigma_{U+L,L,T,I,9}$ receive contributions from the parity conserving (p.c.) part of the hadron tensors $H_{\mu\nu}$. The remaining four $\sigma_{P,A,7,8}$ are proportional to the parity violating (p.v.) part of $H_{\mu\nu}$, i.e. they change sign under parity (P) transformation. Since the angular coefficients $g_{P,A,9}$ change sign too, angular distributions involving $\sigma_{U+L,L,T,I,P,A}$ are parity conserving. Distributions such as charge asymmetry in W-boson events can thus not discriminate between $V-A$ and $V+A$ couplings. The relevant coupling coefficients are:

$$\sigma_{U+L,L,T,I} \approx (v_l^2+a_l^2)(v_q^2+a_q^2) \tag{10}$$

$$\sigma_{P,A} \approx v_l a_l v_q a_q \tag{11}$$

Here $v_q(v_l)$ and $a_q(a_l)$ denotes the vector and axial vector coupling of the gauge boson to the quark (lepton) respectively. On the other hand, $\sigma_{7,8,9}$ are proportional to the parity (P)-odd coupling of the W-boson to the lepton current times the P-even coupling to the quark current or vice versa:

$$\sigma_9 \approx v_l a_l (v_q^2+a_q^2) \qquad \sigma_{7,8} \approx (v_l^2+a_l^2) v_q a_q \tag{12}$$

Switching from $V-A$ theory to $V+A$ theory is hence equivalent to reversing sign of these terms in (8). These angular distributions of the lepton momentum with respect to the W-boson production plane are not only P-odd but also T-odd [6].

The helicity cross sections $d\sigma^\alpha$ in (8) are obtained by convoluting the partonic cross section of partons with momenta $p_1 = x_1 P_1$ and $p_2 = x_2 P_2$ with the respective parton distributions

$$\frac{d\sigma^\alpha}{dq_T^2\, dy} = \sum_{a,b} \int dx_1 dx_2 f_a^{h_1}(x_1, M^2) f_b^{h_2}(x_2, M^2) \frac{s\, d\hat{\sigma}^\alpha_{ab}}{dt\, du} \left(p_1, p_2, \alpha_s(\mu^2)\right) \tag{13}$$

The choice of the mass factorization scale M and the renormalization scale μ in (13) will be discussed below.

Introducing standard angular coefficients [3,8],

$$A_0 = \frac{2\, d\sigma^L}{d\sigma^{U+L}} \qquad A_1 = \frac{2\sqrt{2}\, d\sigma^I}{d\sigma^{U+L}} \qquad A_2 = \frac{4\, d\sigma^T}{d\sigma^{U+L}} \qquad A_3 = \frac{4\sqrt{2}\, d\sigma^A}{d\sigma^{U+L}} \tag{14}$$

$$A_4 = \frac{2\, d\sigma^P}{d\sigma^{U+L}} \qquad A_5 = \frac{2\, d\sigma^7}{d\sigma^{U+L}} \qquad A_6 = \frac{2\sqrt{2}\, d\sigma^8}{d\sigma^{U+L}} \qquad A_7 = \frac{4\sqrt{2}\, d\sigma^9}{d\sigma^{U+L}}$$

the angular distribution of eq. (8) is conveniently written as:

$$\begin{aligned} \frac{d\sigma}{dq_T^2\, dy\, d\cos\theta\, d\phi} = \frac{3}{16\pi} \frac{d\sigma^{U+L}}{dq_T^2\, dy} \Bigg[& (1+\cos^2\theta) \\ & + \frac{1}{2}A_0 \quad (1-3\cos^2\theta) \\ & + A_1 \quad \sin 2\theta \cos\phi \\ & + \frac{1}{2}A_2 \quad \sin^2\theta \cos 2\phi \\ & + A_3 \quad \sin\theta \cos\phi \end{aligned} \tag{15}$$

$$
\begin{aligned}
+ \quad & A_4 \quad \cos\theta \\
+ \quad & A_5 \quad \sin^2\theta \sin 2\phi \\
+ \quad & A_6 \quad \sin 2\theta \sin\phi \\
+ \quad & A_7 \quad \sin\theta \sin\phi \quad \Big]
\end{aligned}
$$

Note that the total (angle integrated) rate σ^{U+L} is factored out from the r.h.s. of eq. (15). Integrating the angular distribution in eq. (15) over the azimuthal angle ϕ we get:

$$\frac{d\sigma}{dq_T^2\, dy\, d\cos\theta} = C\,(1 + \alpha_1 \cos\theta + \alpha_2 \cos^2\theta) \tag{16}$$

where

$$c = \frac{3}{8}\frac{d\sigma^{U+L}}{dq_T^2\, dy}\left[1 + \frac{A_0}{2}\right] \qquad \alpha_1 = \frac{2\,A_4}{2 + A_0} \qquad \alpha_2 = \frac{2 - 3A_0}{2 + A_0} \tag{17}$$

And integration over θ gives

$$\frac{d\sigma}{dq_T^2\, dy\, d\phi} = \frac{1}{2\pi}\frac{d\sigma^{U+L}}{dq_T^2\, dy}(1 + \beta_1 \cos\phi + \beta_2 \cos 2\phi + \beta_3 \sin\phi + \beta_4 \sin 2\phi) \tag{18}$$

where

$$\beta_1 = \frac{3}{4}A_3 \qquad \beta_2 = \frac{A_2}{4} \qquad \beta_3 = \frac{3}{4}A_7 \qquad \beta_4 = \frac{A_5}{2} \tag{19}$$

At this point it is necessary to discusss the choice of the z-axis in the rest frame of the lepton pair. In general it will not be possible to reconstruct the boost to the W rest frame from the measured three-momentum of the charged lepton and the missing transverse momentum approximating the neutrino p_T. However, there is a frame where one can obtain θ and ϕ in (8) uniquely modulo a sign ambiguity in $\cos\theta$ unaffected by the unknown longitudinal momentum of the neutrino. This is the Collins-Soper (CS) frame [3] where the initial $P\bar{P}$-pair lies in the xz-plane and the z-axis bisects the angle between the proton and negative antiproton momenta $\vec{P}_1$ and $-\vec{P}_2$.

$$
\begin{aligned}
CS: \quad \vec{P}_1 &= E_1\,(\sin\gamma_{cs}, 0, \cos\gamma_{cs}) \\
\vec{P}_2 &= E_2\,(\sin\gamma_{cs}, 0, -\cos\gamma_{cs})
\end{aligned} \tag{20}
$$

with:

$$\sin\gamma_{cs} = \frac{-r}{\sqrt{1 + r^2}} \qquad \cos\gamma_{cs} = \frac{1}{\sqrt{1 + r^2}} \tag{21}$$

where $r = q_T/M_W, E_1 = (\sqrt{S}/2)\sqrt{1 + r^2}e^{-y}$ and $E_2 = (\sqrt{S}/2)\sqrt{1 + r^2}e^{+y}$. The z direction of the lab frame is defined by the proton momentum, whereas the x-direction is fixed by the transverse momentum of the gauge boson. We briefly show how θ and ϕ can be obtained from measured quantities. Denote the lepton's (electron or muon) four-momentum in the lab frame by $p_l = (E_l, p_{lx}, p_{ly}, p_{lz})$ where p_{lz} and p_{lx} are the lepton's momentum components parallel and perpendicular to the proton in the event plane, i.e. the plane spanned by the proton and the W boson with $q_T > 0$. The lepton's four momentum in the CS frame, p_l^{CS}, can be obtained by

Lorentz-boosting p_l from the lab frame to the CS frame. One obtains

$$\begin{aligned} p_{lx}^{CS} &= \frac{1}{2}\frac{M_W}{\sqrt{M_W^2+q_T^2}}\,(2\,p_{lx}-q_T) \\ p_{ly}^{CS} &= p_{ly} \\ p_{lz}^{CS} &= \pm\frac{M_W}{2}\sqrt{1-\frac{(p_{lx}^{CS})^2+\left(p_{ly}^{CS}\right)^2}{M_W^2/4}} \end{aligned} \tag{22}$$

where we have imposed the W-mass constraint on the lepton-neutrino system. The $\pm$ correspond to the two solutions for the longitudinal component of the neutrino. The transverse component p_{lx}^{CS} is seen to be uniquely determined by the measurable lab frame quantities p_{lx} and $q_T = \sqrt{p_{lT}^2 + p_{\nu T}^2}$. The CS frame is thus unique in the sence that in this frame the lepton's transverse momentum is independent of the unmeasured longitudinal momentum of the neutrino. On the other hand note that the longitudinal component p_{lz}^{CS} is determined only up to a sign. From the charged lepton momentum in (22) the angles are built up via $\phi = \arctan\left(p_{ly}^{CS}/p_{lx}^{CS}\right)$ and $\theta = \arctan\sqrt{(p_{lx}^{CS})^2+(p_{ly}^{CS})^2}/p_{lz}^{CS}$. The twofold ambiguity in the reconstruction of the lepton's momenta in the CS frame translates in an ambiguity $\theta \leftrightarrow \pi-\theta$ in the polar angle while ϕ is determined. This implies that only the helicity cross sections $U+L, L, T, A, 7$ and 9 in (8) can be determined in a W production experiment.

3 LO results

In leading order the following parton subprocesses

$$\begin{aligned} q+\bar{q} &\to W+g \\ q+g &\to W+q \end{aligned} \tag{23}$$

contribute to large q_T W production. The corresponding diagrams for the gauge boson production are shown in fig. 1. The partonic cross sections contain the delta function $\delta(s+t+u-Q^2)$ and the helicity cross sections $d\sigma^\alpha$ in eq. (8) are given by:

$$\begin{aligned} d\sigma^\alpha \equiv \frac{d\sigma^\alpha}{dq_T^2\,dy} &= \alpha_s \int_B^1 \frac{dx_1}{X_1}\left\{w_1^{q\bar{q}}(x_1,\bar{x}_2)\,T_{q\bar{q}}^\alpha + w_1^{\bar{q}q}(x_1,\bar{x}_2)\,T_{\bar{q}q}^\alpha + w^{qG}(x_1,\bar{x}_2)\,T_{qG}^\alpha\right. \\ &+ \left. w^{\bar{q}G}(x_1,\bar{x}_2)\,T_{\bar{q}G}^\alpha + w^{Gq}(x_1,\bar{x}_2)\,T_{Gq}^\alpha + w^{G\bar{q}}(x_1,\bar{x}_2)\,T_{G\bar{q}}^\alpha\right\} \end{aligned} \tag{24}$$

where

$$\bar{x}_2 = \frac{-Q^2 - x_1(T-Q^2)}{x_1 S + U - Q^2} \qquad B = \frac{-U}{S+T-Q^2} \qquad X_1 = x_1 S + U - Q^2 \tag{25}$$

$T_{ab}^\alpha = T_{ab}^\alpha(Q^2,t,u) = T_{ab}^\alpha(q_T,y,x_1)$ are the partonic matrix elements squared for the partonic subprocesses in (23). All information about the electroweak coupling and the parton distributions is contained in the functions $w_i(x_1,x_2)$. For the particular case of W^+ production they are given in appendix A.

At $O(\alpha_s)$ only $d\sigma^{U+L,L,T,I,P,A}$ receives contributions from the tree level parton subprocesses in eq. (23). The corresponding $O(\alpha_s)$ tree level functions T^α have been first calculated in [7] for the production of a virtual photon and in [8] for the case of Z-production. To make the paper self-contained, we reproduce all LO functions T^α valid in the Collins Soper frame. The results may be expressed in terms of q_T^2, y and x_1 by noting that

$$\begin{aligned} t &= Q^2 - \sqrt{(Q^2+q_T^2)S}\; x_1 e^{-y} \\ u &= Q^2 - \sqrt{(Q^2+q_T^2)S}\; \bar{x}_2 e^{y} \\ s &= x_1 \bar{x}_2 S \end{aligned} \tag{26}$$

so that e.g.

$$\frac{(Q^2-t)(Q^2-u)}{s} = q_T^2 + Q^2 \qquad \text{and also} \qquad \frac{ut}{s} = q_T^2 \tag{27}$$

One has:

$$\begin{aligned}
T_{q\bar{q}}^{U+L} &= \frac{(Q^2-u)^2+(Q^2-t)^2}{ut} \\
T_{q\bar{q}}^{L} &= \frac{1}{2}\left(\frac{Q^2-u}{Q^2-t}+\frac{Q^2-t}{Q^2-u}\right) \\
T_{q\bar{q}}^{I} &= \frac{1}{2}\sqrt{\frac{Q^2 s}{2ut}}\left(\frac{Q^2-u}{Q^2-t}-\frac{Q^2-t}{Q^2-u}\right) \\
T_{q\bar{q}}^{T} &= \frac{1}{2}T_{q\bar{q}}^{L} \\
T_{q\bar{q}}^{P} &= \sqrt{\frac{Q^2 s}{(Q^2-u)(Q^2-t)}}\;\frac{(Q^2-u)^2+(Q^2-t)^2}{ut} \\
T_{q\bar{q}}^{A} &= \frac{1}{2}\sqrt{\frac{(Q^2-u)(Q^2-t)}{2ut}}\left(\frac{Q^2-u}{Q^2-t}-\frac{Q^2-t}{Q^2-u}\right) \\
T_{qG}^{U+L} &= -\frac{(Q^2-s)^2+(Q^2-t)^2}{st} \\
T_{qG}^{L} &= -u\frac{(Q^2+s)^2+(Q^2-t)^2}{2s(Q^2-u)(Q^2-t)} \\
T_{qG}^{I} &= \frac{1}{2}\sqrt{\frac{Q^2 u}{2st}}\,\frac{(Q^2-u)^2-2(Q^2-t)^2}{(Q^2-u)(Q^2-t)} \\
T_{qG}^{T} &= \frac{1}{2}T_{qG}^{L} \\
T_{qG}^{P} &= -\sqrt{\frac{Q^2 s}{(Q^2-u)(Q^2-t)}}\;\frac{Q^4+u^2-2Q^2 t}{st}
\end{aligned}$$

$$T^A_{qG} = \frac{1}{2}\sqrt{\frac{u}{2t(Q^2-u)(Q^2-t)}}\,\frac{Q^4+u^2+2su}{s}$$

The T-odd distributions $T^{7,8,9}$ are still zero at $O(\alpha_s)$. The reason is that they obtain contributions from the absorptive part of the the one-loop amplitudes for the annihilation and the compton processes. As a result, the $O(\alpha_s^2)$ terms of T-odd observables are the lowest order contributions. They have been first calculated for the W-production in [6][1]. We reproduce the explicit $O(\alpha_s^2)$ results for the T-odd functions in Appendix B.

All other subprocess cross sections are obtained from the above as follows:

$$\begin{array}{lllllll}
T^\alpha_{\bar{q}q} & = & T^\alpha_{q\bar{q}} & \text{if} \quad \alpha\in\mathcal{M}_1 & T^\alpha_{\bar{q}q} & = & -T^\alpha_{q\bar{q}} \quad \text{else} \\
T^\alpha_{Gq} & = & T^\alpha_{qG}(u\leftrightarrow t) & \text{if} \quad \alpha\in\mathcal{M}_2 & T^\alpha_{Gq} & = & -T^\alpha_{qG}(u\leftrightarrow t) \quad \text{else} \\
T^\alpha_{\bar{q}G} & = & T^\alpha_{qG} & \text{if} \quad \alpha\in\mathcal{M}_1 & T^\alpha_{\bar{q}G} & = & -T^\alpha_{qG} \quad \text{else} \\
T^\alpha_{G\bar{q}} & = & T^\alpha_{Gq} & \text{if} \quad \alpha\in\mathcal{M}_1 & T^\alpha_{G\bar{q}} & = & -T^\alpha_{Gq} \quad \text{else}
\end{array}$$

where $\mathcal{M}_1 = \{U+L, L, T, I, 9\}$ and $\mathcal{M}_2 = \{U+L, L, T, A, 8\}$.

As mentioned before only the angular coefficients A_0, A_2, A_3, A_5 and A_7 can be determined in a W production experiment.

We will now present some numerical results for the y integrated angular coefficients A_0, A_2 and A_3 at $O(\alpha_s)$ in the Collins-Soper frame.

To be specific we shall evaluate the q_T distribution for the angular coefficients A_0, A_2 and A_3 for W^+ production at $\sqrt{S} = 1.8\,\text{TeV}$ for the contributing parton subprocesses in eq. (23). We use the parton density parametrization set 2 of DFLM [9] with $\Lambda_{QCD} = 175\,\text{MeV}$ for five flavours. We identify the scales used in the coupling constant and in the parton distribution function and set them equal to $\mu^2 = (m_W^2 + q_T^2)/2$.

Figs. 9a,b show the q_T distribution of A_0, A_2 and A_3 at $O(\alpha_s)$ for the $q\bar{q}$ and qG initiatel states separately. A_0 and A_2 are increasing functions of q_T reaching about 0.9 at high q_T. As has been emphasized by Tung et. al. [10] these coefficients are simply related by $A_0 = A_2$ at the Born level. The result $A_0 = A_2$ at $O(\alpha_s)$ is valid for any choice of axis in the lepton-pair rest frame, as long the z-axis lies in the hadronic event plane. We will show in the next section that this relation is no longer true at $O(\alpha_s^2)$. The relation $A_0 = A_2$ has been used in W decays by UA1 [11] to determine the spin of the gluon. In a scalar gluon theory one finds $A_0 \neq A_2$ [12] even at the born level.

As it is shown in fig. 9b the angular coefficient A_3 receives sizable contributions only from the qG initiated process. The reason is that the $q\bar{q}$-contribution to A_3 is antisymmetric for the interchange of x_1 and x_2, which is seen from $T^A_{q\bar{q}}$ using eq. (26) and (14) [2]. Therefore $A_3^{q\bar{q}}$ is expected to be very small in $P\bar{P}$, which is explicitly

[1] Note that there is a misprint in eq. (8) of Ref. [6]

[2] Note that the rapidity range is symmetric for equal proton and antiproton momenta. It is given by: $|y| < \ln[(\sqrt{(S-Q^2)^2 - 4\,S\,q_T^2} + S + Q^2)/(2\sqrt{(Q^2+q_T^2)S})]$.

shown in fig. 9a. $A_3^{q\bar{q}}$ vanishes exactly for collisions where the product of parton distributions is symmetric in x_1 and x_2. The measurement of A_3 provide therefore clear constraints on the gluon distribution function.

Fig. 10 shows the q_T distribution of A_0, A_2 and A_3 at $O(\alpha_s)$ for all $O(\alpha_s)$ contributions. One observes that the coefficient A_3 reaches about 0.2 at $q_T = 100$ GeV.

We plot the coefficients α_2, β_1 and β_2 defined in eqs. (17,19) in fig. 11a,b. Again there are large deviations from the $(1 + \cos\theta)^2$ (i.e. $\alpha_2 = 1$) distribution valid at low q_T even at $O(\alpha_s)$. Note that in the limit $q_T \to 0$ all coefficients A_i vanish, implying that the polar angle distribution reduce to the prediction of the Drell Yan mechanism.

We do not show numerical results for $A_5 - A_7$ in this section, as they should be normalized to the NLO order rate $d\sigma^{U+L}$, which is calculated in the next section. Results for A_5 and A_7 are presented in fig. 15.

4 NLO analytical results

At $O(\alpha_s^2)$ we have the following partonic tree $(a(p_1)+b(p_2)\to V(q)+c(k_1)+d(k_2))$ and loop $(a(p_1)+b(p_2)\to V(q)+c(k_1))$ processes that contribute to high q_T gauge boson production:

$$\begin{array}{lccc} \text{tree contributions}: & q+\bar{q} & \to & V+G+G \\ & q+\bar{q} & \to & V+q+\bar{q} \\ & q+G & \to & V+q+G \\ & q+q & \to & V+q+q \\ & G+G & \to & V+q+\bar{q} \end{array} \tag{28}$$

$$\begin{array}{lccc} \text{loop-contributions}: & q+\bar{q} & \to & V+G \\ & q+G & \to & V+q \end{array} \tag{29}$$

The diagrams giving the $O(\alpha_s^2)$ tree and loop corrections in (28,29) are shown in figs. 2-8. Note that there are two $q\bar{q}$ initiated tree contributions in $O(\alpha_s^2)$. The second-order contribution of (29) comes from the interference of the corresponding one-loop diagrams of figs. 7 and 8 with the leading-order diagrams in fig. 1. The Ultraviolet-(UV)-divergencies are removed by UV renormalization using the $\overline{\text{MS}}$-scheme which introduces a renormalization scale dependence into the strong coupling constant $\alpha_s(\mu^2)$.

We found it convenient to calculate certain covariant projection cross sections $d\sigma^\beta$ ($\beta \in \{U+L, L_1, L_2, L_{12}\}$) defined in eq. (38) ff. and appendix C. They are conveniently written as ($d\sigma \equiv d\sigma/dq_T^2 dy$):

$$d\sigma^\beta = d\sigma^\beta_{q\bar{q}} + d\sigma^\beta_{qG} + d\sigma^\beta_{qq} + d\sigma^\beta_{GG} \tag{30}$$

The covariant projection cross sections $d\sigma^\beta$ are related to the hadronic helicity cross-sections $\dfrac{d\sigma^\alpha}{dq_T^2\,dy}$ for $\alpha \in \{U+L, L, T, I\}$ defined in eq. (8) in the CS frame by a transformation matrix $\left(M^{CS}\right)_{\alpha\beta}$

$$\frac{d\sigma^\alpha}{dq_T^2\,dy} = \left(M^{cs}\right)_{\alpha\beta}\frac{d\sigma^\beta}{dq_T^2\,dy} \tag{31}$$

where $(M^{CS})_{\alpha\beta}$ is given in eq. (90) of appendix C. Similar transformation matrices lead to the corresponding helicity cross sections in other gauge boson rest frames like the Gottfried-Jackson frame or the Helicity frame.

Before specifying the different hadronic projection cross sections $d\sigma^\beta$ in more detail, we briefly discuss some technical details of the NLO calculation. For the $O(\alpha_s^2)$ tree contributions in eq. (28) we introduce the variable

$$s_2 = (k_1+k_2)^2 = (p_1+p_2-q)^2 = s+t+u-Q^2 \tag{32}$$

in addition to s, t, u defined in eq. (5). s_2 is the invariant mass of the system recoiling against the Boson.

To obtain the q_T distribution of the W, the $O(\alpha_s^2)$ tree diagrams have to be integrated over the phase space of the two final state partons with the gauge boson is held fixed at a given q_T. The integration over the recoiling partons is most easily performed in the (k_1k_2) CM system over the solid angle $d\Omega_{k_1k_2}$. $d\Omega_{k_1k_2}$ is defined in eq. (110) [3]. The correponding phase space and the necessary set of integrals are listed in Appendix D. Since all partons are massles, collinear divergencies appear after integrating over $d\Omega_{k_1k_2}$. Soft gluon singularities show up as poles in the variable s_2. We extract the divergent part of the $s_2 \to 0$ singularity by using the identity ($s_2^{-\varepsilon}$ is a phase space factor) [13]:

$$\begin{aligned}\frac{1}{(s_2)^{1+\varepsilon}} &= -\frac{1}{\varepsilon}\delta(s_2)\left[1-\varepsilon\ln A+\frac{1}{2}\varepsilon^2\ln^2 A\right] \\ &\quad +\frac{1}{(s_2)_{A+}}-\varepsilon\left(\frac{\ln s_2}{s_2}\right)_{A+}+O(\varepsilon^2)\end{aligned} \tag{33}$$

A is the upper limit of the s_2 integration and is given in eq. (37). The distributions denoted by the subscript $A+$ in eq. (33) are defined as

$$\int_0^A ds_2\,\frac{f(s_2)}{(s_2)_{A+}} = \int_0^A \frac{ds_2}{s_2}\;[f(s_2)-f(0)] \tag{34}$$

and

$$\int_0^A ds_2\left(\frac{\ln s_2}{s_2}\right)_{A+} f(s_2) = \int_0^A \frac{ds_2}{s_2}\;[f(s_2)-f(0)]\,\ln s_2 \tag{35}$$

Infrared and collinear divergencies associated with final partons cancel among loop and tree diagrams in accordance to the Kinoshita-Lee-Nauenberg theorem. The remaining collinear initial state divergencies are absorbed into the parton densities introducing a factorization scale dependence into the parton densities $f(x_i,M^2)$. The explicit mass factorization terms and a brief discussion of different factorization schemes for the different projection cross-sections is given in sec. 6.

It is useful to make a change of the integration variables to deal with the singular $s_2 \to 0$ behaviour of the partonic cross sections. Following [13] we use the following transformation:

$$\int_0^1 dx_1 dx_2 \Theta(s_2) = \int_B^1 \frac{dx_1}{X_1}\int_0^A ds_2 \tag{36}$$

where X_1 and B are defined in (25) and

$$A = U + x_1(S+T-Q^2) \tag{37}$$

After integration over $d\Omega_{k_1k_2}$ the hadronic projection cross sections up to $O(\alpha_s^2)$ can be written as

$$\frac{d\sigma^\beta}{dq_T^2\,dy} = \sum_{ab}\int \frac{dx_1}{X_1} ds_2 f_a(x_1,M^2) f_b(x_2,M^2)\frac{s\,d\tilde{\sigma}^\beta_{ab}}{dt\,du}\left(x_1P_1,x_2P_2,\alpha_s(\mu^2)\right) \tag{38}$$

[3]Note that the phase space integration commutes with the projectors defined in appendix C and used to obtain the projection cross sections.

with

$$x_2 = \frac{s_2 - Q^2 - x_1(T - Q^2)}{X_1} \tag{39}$$

$d\tilde{\sigma}^{\beta}_{ab}$ are the partonic projection cross sections from which initial state collinear singularities in the $O(\alpha_s^2)$ tree graph contributions have been factorized out at a scale M^2.

We introduce the following notation to present the partonic cross sections (see appendix C):

$O(\alpha_s)$-Born contributions:

$$\frac{s\, d\tilde{\sigma}^{\beta}_{ab}}{dt\, du} = \frac{s\, d\hat{\sigma}^{\beta\,(1)}_{ab}}{dt\, du} = \frac{K^W_{ab}}{s}\, \alpha_s\, \delta(s+t+u-Q^2)\, T^{\beta}_{ab}(\mathrm{B}) \tag{40}$$

$O(\alpha_s^2)$-virtual corrections:

$$\frac{s\, d\tilde{\sigma}^{\beta}_{ab}}{dt\, du} = \frac{s\, d\hat{\sigma}^{\beta,virt}_{ab}}{dt\, du} = \frac{K^W_{ab}}{s}\, \frac{\alpha_s^2}{2\pi}\, \delta(s+t+u-Q^2)\, V(\varepsilon)\, T^{\beta}_{ab}(\mathrm{V}) \tag{41}$$

$O(\alpha_s^2)$-treegraph corrections:

$$\frac{s\, d\tilde{\sigma}^{\beta\,(2)}_{ab}}{dt\, du} = \frac{K^W_{ab}}{s}\, \frac{\alpha_s^2}{2\pi}\, V(\varepsilon)\, T^{\beta}_{ab}(\mathrm{T}) \tag{42}$$

T^{β}_{ab}(B,V,T) are the covariant projections of the hadron tensor for the born, virtual and tree contributions. They are defined in eq. (85). The superscripts (1)(2) indicates the order in α_s at which the tree contributions appear. The constant K^W_{ab} and $V(\varepsilon)$ are given in eqs. (70) and (76).

We next specify the different hadronic projection cross sections defined in eq. (30) and (38). The hadronic cross sections are obtained by folding the parton level cross sections given in eqs. (40,41,42) with respective parton densities. We will use eq. (33) to split the $O(\alpha_s^2)$-treegraph contributions into a soft and hard part.

$$T^{\beta}_{ab}(\mathrm{T}) = T^{\beta}_{ab}(\mathrm{S}) + T^{\beta}_{ab}(\mathrm{H}) \tag{43}$$

We denote the terms proportional to $\delta(s_2)$ as the soft part. Note that this decomposition is not an exact splitting of soft and hard contributions since $T_{ab}(\mathrm{H})$ contains also some finite soft terms due to the $A+$-distribution occuring in eq. (33).

4.1 $q\bar{q}$-Annihilation and Scattering

These processes receive contributions from the diagrams of figs. (1, 7,2,4). and can be written as:

$$d\sigma^{\beta}_{q\bar{q}} = d\sigma^{\beta}_{q\bar{q}}(\mathrm{B}) + d\sigma^{\beta}_{q\bar{q}}(\mathrm{V}) + d\sigma^{\beta}_{q\bar{q}}(\mathrm{S}) + d\sigma^{\beta}_{q\bar{q}}(\mathrm{H}) \tag{44}$$

According to eqs. (38,40,41,42) the hadronic cross sections can be written as

$$
\begin{aligned}
d\sigma^{\beta}_{q\bar{q}}(\mathrm{B}) &= \alpha_s \int \frac{dx_1}{X_1} w_1^{q\bar{q}+\bar{q}q}(x_1,\bar{x}_2)\, T^{\beta}_{q\bar{q}}(\mathrm{B}) && (45)\\
d\sigma^{\beta}_{q\bar{q}}(\mathrm{V,S}) &= \frac{\alpha_s^2}{2\pi} V(\varepsilon) \int \frac{dx_1}{X_1} w_1^{q\bar{q}+\bar{q}q}(x_1,\bar{x}_2)\, T^{\beta}_{q\bar{q}}(\mathrm{V,S}) && (46)\\
d\sigma^{\beta}_{q\bar{q}}(\mathrm{H}) &= \frac{\alpha_s^2}{2\pi} \int \frac{dx_1}{X_1} ds_2 \sum_{i=1}^{5} w_i^{q\bar{q}+\bar{q}q}(x_1,x_2)\, T^{\beta}_{q\bar{q}}(\mathrm{H}_i) && (47)
\end{aligned}
$$

$\bar{x}_2$ is defined in eq. (25). The fuctions $w_i^{q\bar{q}}$ are given in appendix A.

$$
\begin{aligned}
T^{\beta}_{q\bar{q}}(\mathrm{B}) &= N_\beta\, A^{\beta}_{q\bar{q}}\\
T^{\beta}_{q\bar{q}}(\mathrm{V}) &= N_\beta\, B^{\beta}_{q\bar{q}}\\
T^{\beta}_{q\bar{q}}(\mathrm{S}) &= N_\beta\, (C^{\beta}_{1,q\bar{q}} + D^{\beta}_{1,aa})\\
T^{\beta}_{q\bar{q}}(\mathrm{H}_1) &= N_\beta\, (C^{\beta}_{2,q\bar{q}} + D^{\beta}_{2,aa} + D^{\beta}_{ac} + D^{\beta}_{ad})\\
T^{\beta}_{q\bar{q}}(\mathrm{H}_2) &= N_\beta\, (D^{\beta}_{bb} + D^{\beta}_{bc})\\
T^{\beta}_{q\bar{q}}(\mathrm{H}_3) &= N_\beta\, (D^{\beta}_{bb} + D^{\beta}_{bd})\\
T^{\beta}_{q\bar{q}}(\mathrm{H}_4) &= N_\beta\, D^{\beta}_{cc}\\
T^{\beta}_{q\bar{q}}(\mathrm{H}_5) &= N_\beta\, D^{\beta}_{dd}
\end{aligned}
$$

The normalization factors N_β are defined in eq. (86). The partonic matrix elements squared $A^{\beta}_{q\bar{q}}$, $B^{\beta}_{q\bar{q}}$, $C^{\beta}_{q\bar{q}}$ and D^{β} are given in appendix E.1, G.1, H.1 and H.2 respectively. The contributions of the $q\bar{q} \to Vq\bar{q}$ (fig. 4) tree graph processes have to be further subdivided into the subclasses D_{xy} according to their $Wq\bar{q}$ coupling structure. The index doublet (xy) in D_{xy} with $x, y \in \{a,b,c,d\}$ takes the values $a \in \{1,2\}, b \in \{3,4\}, c \in \{5,6\}, d \in \{7,8\}$. The correspondence of the various D_{xy} terms with their graphical counterparts can be readily read from fig. 4.

4.2 The Compton process

This process receives contributions from the diagrams of figs. (1, 8, 3) and can be written as:

$$
d\sigma^{\beta}_{qG} = d\sigma^{\beta}_{qG}(\mathrm{B}) + d\sigma^{\beta}_{qG}(\mathrm{V}) + d\sigma^{\beta}_{qG}(\mathrm{S}) + d\sigma^{\beta}_{qG}(\mathrm{H}) \qquad (48)
$$

The hadronic cross sections are given by

$$
\begin{aligned}
d\sigma^{\beta}_{qG}(\mathrm{B}) &= \alpha_s \int \frac{dx_1}{X_1} w^{qG}(x_1,\bar{x}_2)\, T^{\beta}_{qG}(\mathrm{B})\\
d\sigma^{\beta}_{qG}(\mathrm{V,S}) &= \frac{\alpha_s^2}{2\pi} V(\varepsilon) \int \frac{dx_1}{X_1} w^{qG}(x_1,\bar{x}_2)\, T^{\beta}_{qG}(\mathrm{V,S})\\
d\sigma^{\beta}_{qG}(\mathrm{H}) &= \frac{\alpha_s^2}{2\pi} \int \frac{dx_1}{X_1} ds_2\, w^{qG}(x_1,x_2)\, T^{\beta}_{qG}(\mathrm{H})
\end{aligned}
$$

$$
\begin{aligned}
T^{\beta}_{qG}(\mathrm{B}) &= N_\beta \, A^{\beta}_{qG} \\
T^{\beta}_{qG}(\mathrm{V}) &= N_\beta \, B^{\beta}_{qG} \\
T^{\beta}_{qG}(\mathrm{S}) &= N_\beta \, C^{\beta}_{1,qG} \\
T^{\beta}_{qG}(\mathrm{H}_1) &= N_\beta \, C^{\beta}_{2,qG}
\end{aligned}
$$

The partonic matrix elements squared A^{β}_{qG}, A^{β}_{Gq} B^{β}_{qG}, B^{β}_{Gq} C^{β}_{qG}, C^{β}_{Gq} are given in appendix (E.2, E.3, G.2, G.3, H.4, H.5) respectively. The cross sections for the closely related Gq initiated process is obtained from the above formulas by replacing $qG \to Gq$.

4.3 $qq + \bar{q}\bar{q}$-scattering.

These processes receive contributions from the diagrams of figs. (5) and can be written as:

$$
d\sigma^{\beta}_{qq+\bar{q}\bar{q}} = d\sigma^{\beta}_{qq+\bar{q}\bar{q}}(\mathrm{H}) \tag{49}
$$

$$
d\sigma^{\beta}_{qq+\bar{q}\bar{q}}(\mathrm{H}) = \frac{\alpha_s^2}{2\pi} \int \frac{dx_1}{X_1} ds_2 \sum_{i=1}^{5} w_i^{qq+\bar{q}\bar{q}}(x_1, x_2) \, T^{\beta}_{qq+\bar{q}\bar{q}}(\mathrm{H}_i)
$$

$$
\begin{aligned}
T^{\beta}_{qq+\bar{q}\bar{q}}(\mathrm{H}_1) &= N_\beta \, (E^{\beta}_{aa} + E^{\beta}_{cc}) \\
T^{\beta}_{qq+\bar{q}\bar{q}}(\mathrm{H}_2) &= N_\beta \, (E^{\beta}_{bb} + E^{\beta}_{dd}) \\
T^{\beta}_{qq+\bar{q}\bar{q}}(\mathrm{H}_3) &= N_\beta \, E^{\beta}_{ac} \\
T^{\beta}_{qq+\bar{q}\bar{q}}(\mathrm{H}_4) &= N_\beta \, E^{\beta}_{bd} \\
T^{\beta}_{qq+\bar{q}\bar{q}}(\mathrm{H}_5) &= N_\beta \, (E^{\beta}_{ad} + E^{\beta}_{bc})
\end{aligned}
$$

The partonic matrix elements squared E^{β} are given in appendix H.3. The results for E^{β} are divided into subclasses E^{β}_{xy}. The index doublet (xy) with $x, y \in \{a, b, c, d\}$ takes the values $a \in \{1,2\}, b \in \{3,4\}, c \in \{5,6\}, d \in \{7,8\}$. The correspondence of the various E_{xy} terms with their graphical counterparts can be readily read from fig. 5.

4.4 GG-Fusion

This process receives contributions from the diagrams of figs. (6) and can be written as:

$$
d\sigma^{\beta}_{GG} = d\sigma^{\beta}_{GG}(\mathrm{H}) \tag{50}
$$

$$
d\sigma^{\beta}_{GG}(\mathrm{H}) = \frac{\alpha_s^2}{2\pi} \int \frac{dx_1}{X_1} ds_2 \, w^{GG}(x_1, x_2) \, T^{\beta}_{GG}(\mathrm{H})
$$

$$
T^{\beta}_{GG}(\mathrm{H}) = N_\beta \, C^{\beta}_{GG}
$$

The partonic matrix elements squared C^{β}_{GG} are given in appendix H.6.

5 NLO numerical results

In this section we will show numerical results for the angular coefficients A_0, A_2, A_5 and A_7 at $O(\alpha_s^2)$ in the Collins Soper frame.

To be specific we shall evaluate the q_T distribution for W^+ production at $\sqrt{S} = 1.8\,\mathrm{TeV}$ for all contributing parton subprocesses in eq. (28,29). We use the parton density parametrization set 2 of DFLM [9] with $\Lambda_{QCD} = 175\,\mathrm{MeV}$ for five flavours. We work in the DIS factorization scheme, where one substracts the non-pole terms found in the parton structure functions in DIS (see eq. 58 and the discussion in sec. 6). We identify the scales used in the coupling constant and in the parton distribution function and set them equal to $\mu^2 = (m_W^2 + q_T^2)/2$.

Turning first to the NLO corrections to A_0 and A_2 we find that $A_0 = A_2$ is violated at $O(\alpha_s^2)$. For the $q\bar{q}$ initial state (fig. 12a), the dominant corrections affect A_0 and are positive, whereas the $O(\alpha_s^2)$ contributions for the qG initial state (fig. 12b) leads mainly to negative corrections to A_2. In fig. 13 we show the q_T dependence of A_0 and A_2 up to $O(\alpha_s^2)$ including all partonic subprocesses in (28) and (29). It appears that the NLO corrections to the angular coefficients A_0 and A_2 are not large when they are normalized to the NLO rate σ^{U+L}. However, the NLO corrections to the individual helicity cross sections are much larger. It turns out that most of the corrections cancel in the ratios A_i.

To give a feeling for the numerical contributions of different processes, we show their relative contribution to $d\sigma^{U+L}$ and $d\sigma^L$ in the CS frame at $\sqrt{S} = 1.8$ TeV in fig. 14a,b. One observes that the GG contribution (curve G) is neglegible at Tevatron energies. However, the qG contribution (curve D) to the helicity cross sections is large. For the logitudinal cross section σ^L, the second order contribution of the qG process (curve D in fig. 14b) is even larger than the $O(\alpha_s)$ contribution of the $q\bar{q}$ process (curve A) at $20 < q_T < 50$ GeV.

Turning now to the $O(\alpha_s^2)$ contributions to the T-odd angular asymmetries A_5 and A_7 shown in fig. 15, one observes that the contributions are relativly small even at large q_T. The largest asymmetry is expected for A_5 where the magnitude of the asymmetry exceeds -0.017 at Q_T=100 GeV. As mentioned before, these angular distributions reverse the sign, when switching from $V - A$ to $V + A$ theory.

In fig. 16a,b we plot the angular coefficients $\alpha_2, \beta_2, \beta_3$ and β_4 defined in (17) and (19) at $O(\alpha_s^2)$.

We have checked, that the theoretical uncertainties due to the choice of the factorization and renormalization scales are very small for the cross section ratios A_i. Also, most of the uncertainties of the structure functions and of the choice of the factorization scheme cancels in the ratio. This is different for the curves in fig. 14a,b. Switching for example from the DIS factorization scheme to the $\overline{\mathrm{MS}}$ factorization scheme (see sec. 6), we find up to 20% deviations from the curves presented in fig 14 in kinematical regions, where the initial state singularities becomes important.

6 Mass factorization

As has been mentioned in the previous section the renormalized virtual plus soft and hard cross sections still contain initial state collinear divergencies. These divergencies have to be removed using a factorization prescription. The following discussion is an extension of the factorization description proposed e.g. in [13] to different helicity and projection cross sections $d\sigma^{\beta}$.

In this section we will demonstrate how the mass singularities cancel among similar contributions in the bare parton densities. In order to be explicit we rewrite the parton model formula (38):

$$\begin{aligned} \frac{d\sigma^{\beta}}{dq_T^2\,dy} &= \sum_{ab}\int dx_1 dx_2 f_a^0(x_1) f_b^0(x_2) \qquad (51) \\ &\times \left[\frac{s\,d\hat{\sigma}_{ab}^{\beta\,(1)}}{dt}\delta(s+t+u-Q^2) + \frac{s\,d\hat{\sigma}_{ab}^{\beta\,(2)}}{dt\,du}\right](p_1 = x_1 P_1, p_2 = x_2 P_2) \end{aligned}$$

$d\hat{\sigma}_{ab}^{\beta\,(1)}$ and $d\hat{\sigma}_{ab}^{\beta\,(2)}$ are the $O(\alpha_s)$and $O(\alpha_s^2)$contributions for the partonic cross sections given in the form of eqs. (40) and (42). The superscripts (1)(2) indicates the order in α_s at which the terms appear. In contrast to $d\tilde{\sigma}_{ab}^{(2)}$ in eq. (42) $d\hat{\sigma}_{ab}^{\beta\,(2)}$ still contains the collinear initial state singularities. $f_a^0(x_1)$ and $f_b^0(x_2)$ are the bare parton densities. Besides the subleading terms in $d\hat{\sigma}_{ab}^{\beta\,(2)}$ there are $O(\alpha_s^2)$ contributions due to the evolution of the structure functions up to $O(\alpha_s)$. The relationship between the renormalized and the bare structure functions is given by [13]):

$$\begin{aligned} f_i(x,M^2) &= \int_x^1 \frac{dz}{z}\left[\delta(1-\frac{x}{z}) + \frac{\alpha_s}{2\pi}R_{i\leftarrow j}\left(\frac{x}{z},M^2\right)\right] f_j^0(z) \qquad (52) \\ &= f_i^0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z} R_{i\leftarrow j}\left(\frac{x}{z},M^2\right) f_j^0(z) \end{aligned}$$

The function R contain the mass singularities and has the form

$$R_{i\leftarrow j}(z,M^2) = -\frac{1}{\varepsilon}P_{i\leftarrow j}(z)\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{4\pi\mu^2}{M^2}\right)^{\varepsilon} + C_{i\leftarrow j}(z) \qquad (53)$$

The coefficients of the $\frac{1}{\varepsilon}$-poles are the Altarelli-Parisi-fuctions:

$$\begin{aligned} P_{q\leftarrow q}(z) &= C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right] \\ P_{G\leftarrow q}(z) &= C_F\,\frac{1+(1-z)^2}{z} \\ P_{G\leftarrow G}(z) &= 2N_C\left[\frac{1}{(1-z)_+} + \frac{1}{z} + z(1-z) - 2\right] \qquad (54) \\ &\quad + \left(\frac{11}{6}N_C - \frac{2}{3}T_R\right)\delta(1-z) \\ P_{q\leftarrow G}(z) &= \frac{1}{2}\left(z^2 + (1-z)^2\right) \end{aligned}$$

where the "'Plus"'-distribution in 54 is defined as:

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{1-z} \tag{55}$$

Using eq. (52) in (51), the partonic mass factorized cross section is given at $O(\alpha_s^2)$ by:

$$\begin{aligned}
\frac{s\, d\tilde{\sigma}^{\beta}_{ab}}{dt\, du} &= \frac{s\, d\hat{\sigma}^{\beta\,(1)}_{ab}}{dt}\delta(s+t+u-Q^2) + \frac{s\, d\hat{\sigma}^{\beta\,(2)}_{ab}}{dt\, du} \\
&- \frac{\alpha_s}{2\pi}\frac{1}{s+t-Q^2} R_{i\leftarrow j}\left(y_1, M^2\right) \frac{s\, d\hat{\sigma}^{\beta\,(1)}_{ab}}{dt}(y_1 p_1, p_2) \\
&- \frac{\alpha_s}{2\pi}\frac{1}{s+u-Q^2} R_{i\leftarrow j}\left(y_2, M^2\right) \frac{s\, d\hat{\sigma}^{\beta\,(1)}_{ab}}{dt}(p_1, y_2 p_2)
\end{aligned} \tag{56}$$

where:

$$y_1 = \frac{-u}{s+t-Q^2} \qquad\qquad y_2 = \frac{-t}{s+u-Q^2} \tag{57}$$

In eq. (56) we cancel the collinear initial state pole terms appearing in $d\hat{\sigma}^{(2)}_{ab}$ and the subtraction terms. One then has to replace the bare parton distributions by the renormalized, scale dependent parton distributions as has been done in eq. (38).

The functions $C_{i\leftarrow j}$ in $R_{i\leftarrow j}$ do not contain collinear singularities. The choice of these functions determines the factorization scheme: We use:

- **$\overline{\text{MS}}$-factorization scheme:**
  The function $R_{i\leftarrow j}$ contains only the pole term. ($C_{i\leftarrow j} = 0$):

- **DIS-factorization scheme:**
  One substract the none pole terms found in the parton structure functions in deep inelastic lepton quark scattering by choosing: (see e.g [14][15]):

$$\begin{aligned}
C_{q\leftarrow q}(z) &= C_F\left[(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - \frac{3}{2}\frac{1}{(1-z)_+} - \frac{1+z^2}{1-z}\ln z \right. \\
&\qquad \left. +3+2z-\left(\frac{9}{2}+\frac{1}{3}\pi^2\right)\delta(1-z)\right] \\
C_{q\leftarrow G} &= \frac{1}{2}\left[\left(z^2+(1-z)^2\right)\ln\left(\frac{1-z}{z}\right)+6z(1-z)\right] \\
C_{G\leftarrow q}(z) &= -C_{q\leftarrow q}(z) \\
C_{G\leftarrow G}(z) &= -4\,T_R\, C_{q\leftarrow G}(z)
\end{aligned} \tag{58}$$

In our analytical calculations presented in the appendices, we used the $\overline{\text{MS}}$-scheme. A different factorization scheme may be used by adding the correction terms determined by the last equation.

In the following we list the explicit subtractions for the different partonic processes.

6.1 Factorization terms in the $q\bar{q}$-Annihilation and Scattering

The diagrams of fig. 2 have collinear singularities that arise from the radiation of a gluon with momentum $y_1 p_1$ ($y_2 p_2$) from the initial quark (antiquark). The quark (antiquark) lose a fraction $1 - y_1$ $(1 - y_2)$ of its momentum. The factorization terms are then given by:

$$\begin{aligned} &-\frac{\alpha_s}{2\pi}\frac{1}{s+t-Q^2} R_{q\leftarrow q}\left(y_1, M^2\right) \frac{s\, d\hat{\sigma}^{\beta\,(1)}_{q\bar{q}}}{dt}(y_1 p_1, p_2) \\ &-\frac{\alpha_s}{2\pi}\frac{1}{s+u-Q^2} R_{\bar{q}\leftarrow\bar{q}}\left(y_2, M^2\right) \frac{s\, d\hat{\sigma}^{\beta\,(1)}_{q\bar{q}}}{dt}(p_1, y_2 p_2) \end{aligned} \tag{59}$$

with:

$$R_{\bar{q}\leftarrow\bar{q}} = R_{q\leftarrow q} \tag{60}$$

Collinear singularities arise from gluon radiation in the diagrams $|F_5+F_6|^2$ and $|F_7+F_8|^2$ shown in fig. 4 This gluon then scatters with another parton. The factorization terms are

$$\begin{aligned} &-\frac{\alpha_s}{2\pi}\frac{1}{s+u-Q^2} R_{G\leftarrow\bar{q}}\left(y_2, M^2\right) \frac{s\, d\hat{\sigma}^{\beta\,(1)}_{qG}}{dt}(p_1, y_2 p_2) \\ &-\frac{\alpha_s}{2\pi}\frac{1}{s+t-Q^2} R_{G\leftarrow q}\left(y_1, M^2\right) \frac{s\, d\hat{\sigma}^{\beta\,(1)}_{G\bar{q}}}{dt}(y_1 p_1, p_2) \end{aligned} \tag{61}$$

with

$$R_{G\leftarrow\bar{q}} = R_{G\leftarrow q} \tag{62}$$

6.2 Factorization terms in the Compton-Prozess

We have the following factorization subtractions in fig. 3.

1) The initial quark in the diagrams $|I_1 + I_5|^2$ can radiate a collinear gluon and loses a fractions $1 - y_1$ of its momentum. The factorization terms are:

$$-\frac{\alpha_s}{2\pi}\frac{1}{s+t-Q^2} R_{q\leftarrow q}\left(y_1, M^2\right) \frac{s\, d\hat{\sigma}^{\beta\,(1)}_{qG}}{dt}(y_1 p_1, p_2) \tag{63}$$

2) The gluon in the diagrams $|I_1 + I_2|^2$ can decay into a $q\bar{q}$ pair and the antiquark with fraction $y_2 p_2$ can annihilate with the initial state quark. The factorization terms are:

$$-\frac{\alpha_s}{2\pi}\frac{1}{s+u-Q^2} R_{\bar{q}\leftarrow G}\left(y_2, M^2\right) \frac{s\, d\hat{\sigma}^{\beta\,(1)}_{q\bar{q}}}{dt}(p_1, y_2 p_2) \tag{64}$$

3) The gluon in the diagrams $|I_4 + I_8|^2$ can radiate a collinear gluon and scatters with the reduced momentum $y_2 p_2$ with the initial state quark. The factorization substractions are:

$$-\frac{\alpha_s}{2\pi}\frac{1}{s+u-Q^2} R_{G\leftarrow G}\left(y_2, M^2\right) \frac{s\, d\hat{\sigma}^{\beta\,(1)}_{qG}}{dt}(p_1, y_2 p_2) \tag{65}$$

6.3 Factorization terms in the Gluon-Gluon-Process

There are four factorization contributions to the diagrams shown in fig. 6. One of the gluons decays to a $q\bar{q}$-pair and the quark or antiquark then scatters from the other gluon. The factorization subtractions arising in the diagrams $|J_1 + J_2|^2, |J_1 + J_3|^2, |J_4 + J_5|^2, |J_4 + J_6|^2$ are:

$$-\frac{\alpha_s}{2\pi}\frac{1}{s+t-Q^2}R_{\bar{q}\leftarrow G}\left(y_1, M^2\right)\frac{s\,d\hat{\sigma}^{\beta\,(1)}_{\bar{q}G}}{dt}(y_1p_1, p_2) \tag{66}$$

$$-\frac{\alpha_s}{2\pi}\frac{1}{s+u-Q^2}R_{q\leftarrow G}\left(y_2, M^2\right)\frac{s\,d\hat{\sigma}^{\beta\,(1)}_{Gq}}{dt}(p_1, y_2p_2) \tag{67}$$

$$-\frac{\alpha_s}{2\pi}\frac{1}{s+u-Q^2}R_{\bar{q}\leftarrow G}\left(y_2, M^2\right)\frac{s\,d\hat{\sigma}^{\beta\,(1)}_{G\bar{q}}}{dt}(p_1, y_2p_2) \tag{68}$$

$$-\frac{\alpha_s}{2\pi}\frac{1}{s+t-Q^2}R_{q\leftarrow G}\left(y_1, M^2\right)\frac{s\,d\hat{\sigma}^{\beta\,(1)}_{qG}}{dt}(y_1p_1, p_2) \tag{69}$$

A Couplings

In this appendix we list the couplings and combinations of parton distribution functions as they are needed for W^+ production in $p\bar{p}$ collisions.

$$
\begin{aligned}
w_1^{q\bar{q}}(x_1,x_2) &= \frac{1}{s}K_{qq}^W\Big\{\Big[u(x_1)\bar{d}(x_2)+c(x_1)\bar{s}(x_2)\Big]\cos^2\theta_c \\
&+ \quad \Big[u(x_1)\bar{s}(x_2)+c(x_1)\bar{d}(x_2)\Big]\sin^2\theta_c\Big\} \\
w_1^{\bar{q}q}(x_1,x_2) &= w_1^{q\bar{q}}(x_1\leftrightarrow x_2) \\
w_1^{q\bar{q}+\bar{q}q}(x_1,x_2) &= w_1^{q\bar{q}}(x_1,x_2)+w_1^{\bar{q}q}(x_1,x_2) \\
w_2^{q\bar{q}+\bar{q}q}(x_1,x_2) &= \frac{2}{s}K_{qq}^W\Big\{u(x_1)\bar{u}(x_2)+c(x_1)\bar{c}(x_2)+\bar{d}(x_1)d(x_2)+\bar{s}(x_1)s(x_2)\Big\} \\
w_3^{q\bar{q}+\bar{q}q}(x_1,x_2) &= w_2^{q\bar{q}}(x_2,x_1) \\
w_4^{q\bar{q}+\bar{q}q}(x_1,x_2) &= \frac{1}{s}K_{qq}^W\Big\{\Big[u(x_1)+c(x_1)\Big]\Big[\bar{u}(x_2)+\bar{d}(x_2)+\bar{s}(x_2)+\bar{c}(x_2)\Big] \\
&+ \quad \Big[\bar{d}(x_1)+\bar{s}(x_1)\Big]\Big[u(x_2)+d(x_2)+s(x_2)+c(x_2)\Big]\Big\} \\
w_5^{q\bar{q}+\bar{q}q}(x_1,x_2) &= w_4^{q\bar{q}}(x_2,x_1) \\
w^{qG}(x_1,x_2) &= \frac{1}{s}K_{qG}^W\left[u(x_1)+c(x_1)\right]g(x_2) \\
w^{\bar{q}G}(x_1,x_2) &= \frac{1}{s}K_{qG}^W\Big[\bar{d}(x_1)+\bar{s}(x_1)\Big]g(x_2) \\
w^{Gq}(x_1,x_2) &= w^{qG}(x_2,x_1) \\
w^{G\bar{q}}(x_1,x_2) &= w^{\bar{q}G}(x_2,x_1) \\
w^{qG+\bar{q}G}(x_1,x_2) &= w^{qG}(x_1,x_2)+w^{\bar{q}G}(x_1,x_2) \\
w^{Gq+G\bar{q}}(x_1,x_2) &= w^{Gq}(x_1,x_2)+w^{G\bar{q}}(x_1,x_2) \\
w_1^{qq+\bar{q}\bar{q}}(x_1,x_2) &= \frac{1}{2s}K_{qq}^W\Big\{\Big[u(x_1)+c(x_1)\Big]\Big[u(x_2)+d(x_2)+s(x_2)+c(x_2)\Big] \\
&+ \quad \Big[\bar{d}(x_1)+\bar{s}(x_1)\Big]\Big[\bar{u}(x_2)+\bar{d}(x_2)+\bar{s}(x_2)+\bar{c}(x_2)\Big]\Big\} \\
w_2^{qq+\bar{q}\bar{q}}(x_1,x_2) &= w_1^{qq+\bar{q}\bar{q}}(x_2,x_1) \\
w_3^{qq+\bar{q}\bar{q}}(x_1,x_2) &= \frac{1}{2s}K_{qq}^W\Big\{\Big[u(x_1)d(x_2)+c(x_1)s(x_2)+\bar{d}(x_1)\bar{u}(x_2)+\bar{s}(x_1)\bar{c}(x_2)\Big]\cos^2\theta_c \\
&+ \quad \Big[u(x_1)s(x_2)+c(x_1)d(x_2)+\bar{d}(x_1)\bar{c}(x_2)+\bar{s}(x_1)\bar{u}(x_2)\Big]\sin^2\theta_c\Big\} \\
w_4^{qq+\bar{q}\bar{q}}(x_1,x_2) &= w_3^{qq+\bar{q}\bar{q}}(x_2,x_1)
\end{aligned}
$$

$$w_5^{qq+\bar{q}\bar{q}}(x_1,x_2) = \frac{1}{2s}K_{qq}^W\left\{u(x_1)u(x_2)+c(x_1)c(x_2)+\bar{d}(x_1)\bar{d}(x_2)+\bar{s}(x_1)\bar{s}(x_2)\right\}$$

$$w_{GG}(x_1,x_2) = \frac{2}{s}K_{GG}^W\, g(x_1)g(x_2)$$

$u(x_i), d(x_i), s(x_i), c(x_i), \bar{u}(x_i), \bar{d}(x_i), \bar{s}(x_i), \bar{c}(x_i), g(x_i)$ are the parton distributions in the Proton ($i=1$) and Antiproton ($i=2$). The functions $K_{q\bar{q}}^W$, K_{qG}^W, K_{qq}^W and K_{GG}^W contains all the electroweak couplings, phase space factors, spin average factors and the gauge boson propagator for the corresponding $O(\alpha_s)$contributions for W-boson production. They are given by ($D_W = (Q^2 - M_W^2 + iM_W\Gamma_W)^{-1}$):

$$K_{qq}^W = \frac{16\pi}{3}\frac{\alpha^2 Q^2}{128\pi\sin^4\Theta_W}\frac{C_F}{N_C}\frac{(1-\varepsilon)}{\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left(\frac{sQ^2}{ut}\right)^{\varepsilon}|D_W|^2 \tag{70}$$

$$K_{qG}^W = \frac{K_{qq}^W}{2\,C_F\,(1-\varepsilon)} \tag{71}$$

$$K_{GG}^W = \frac{K_{qq}^W}{4C_F^2\,(1-\varepsilon)^2}$$

The factors $(1-\varepsilon)^2$ in K_{GG}^W and $(1-\varepsilon)$ in K_{qG}^W occur because we use the convention that the average over initial gluon spins in $n = 4-2\epsilon$ dimensions gives a factor $1/(n-2)$. The group structure of QCD is handled generally with N_C the dimension of a single fermion representaion and the Casimir C_F defined by $\sum T_a T_a = C_F$. For QCD, these are

$$N_C = 3, \qquad C_F = \frac{4}{3} \tag{72}$$

B $O(\alpha_s^2)$ T-odd distributions in the Collins-Soper Frame

In this appendix, we present the three T-odd functions T^7, T^8, T^9 in eq. (24) for completeness. The Color factor C_1 stands for $C_F - \frac{N_C}{2}$.

$$\begin{aligned} T_{q\bar{q}}^7 &= \frac{\alpha_s}{4}\sqrt{\frac{Q^2 s}{2(Q^2-u)(Q^2-t)}}\left\{-\frac{C_F}{2}\left(\frac{Q^2-t}{Q^2-u}+\frac{Q^2-u}{Q^2-t}\right)\right. \\ &+ \left. C_1\left[\frac{Q^2-t}{t}\left(1-\frac{s}{t}\log\left(\frac{Q^2-u}{s}\right)\right)+\frac{Q^2-u}{u}\left(1-\frac{s}{u}\log\left(\frac{Q^2-t}{s}\right)\right)\right]\right\} \end{aligned}$$

$$\begin{aligned} T_{q\bar{q}}^8 &= \frac{\alpha_s}{2}\frac{Q^2 s}{\sqrt{2(Q^2-u)(Q^2-t)ut}}\left\{-C_F\left(\frac{Q^2-t}{Q^2-u}-\frac{Q^2-u}{Q^2-t}\right)\right. \\ &+ \left. C_1\left[\frac{Q^2-t}{t}\log\left(\frac{Q^2-u}{s}\right)-\frac{Q^2-u}{u}\log\left(\frac{Q^2-t}{s}\right)\right]\right\} \end{aligned}$$

$$
\begin{aligned}
T^9_{q\bar{q}} &= \alpha_s\sqrt{\frac{Q^2 s}{ut}}\Bigg\{\ C_F\left(1-\frac{ut}{2(Q^2-t)(Q^2-u)}\right)\left(\frac{Q^2-t}{Q^2-u}-\frac{Q^2-u}{Q^2-t}\right)\\
&+\ \mathrm{C}_1\left[\frac{Q^2}{Q^2-t}-\frac{Q^2}{Q^2-u}-\frac{s}{t}\log\left(\frac{Q^2-u}{s}\right)+\frac{s}{u}\log\left(\frac{Q^2-t}{s}\right)\right]\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
T^7_{qG} &= -\frac{\alpha_s Q^2 u}{4}\sqrt{\frac{1}{2(Q^2-t)(Q^2-u)Q^2 s}}\Bigg\{-\ C_F\frac{s+2Q^2}{2(Q^2-t)}\\
&+\ \mathrm{C}_1\left[\frac{Q^2}{Q^2-u}+\frac{Q^2-u}{s}\log\left(\frac{(Q^2-u)(Q^2-t)}{ut}\right)\right.\\
&+\ \left.\frac{Q^2-t}{t}\left(1-\frac{Q^2-u}{t}\log\left(\frac{Q^2-u}{s}\right)\right)\right]\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
T^8_{qG} &= -\frac{\alpha_s Q^2 u}{2\sqrt{2}}\sqrt{\frac{1}{(Q^2-t)(Q^2-u)ut}}\Bigg\{-\ C_F\left(\frac{Q^2-u}{Q^2-t}-\frac{2Q^2-t}{2(Q^2-t)}\right)\\
&+\ \mathrm{C}_1\left[\frac{u}{Q^2-u}+\frac{Q^2-u}{s}\left(\frac{Q^2}{Q^2-u}-\frac{u}{s}\log\left(\frac{(Q^2-u)(Q^2-t)}{ut}\right)\right)\right]\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
T^9_{qG} &= -\alpha_s\frac{u}{\sqrt{Q^2 sut}}\Bigg\{-\ C_F\frac{Q^2(ts+2uQ^2)}{2(Q^2-t)^2}\\
&-\ \mathrm{C}_1 Q^2\left[\frac{2Q^2 s}{(Q^2-u)^2}-\frac{Q^2}{Q^2-t}-\frac{u}{Q^2-u}\left(1+\frac{Q^2-u}{s}\log\left(\frac{(Q^2-u)(Q^2-t)}{ut}\right)\right.\right.\\
&\qquad\left.\left.-\ \frac{Q^2-u}{t}\log\left(\frac{Q^2-u}{s}\right)\right)\right]\Bigg\}
\end{aligned}
$$

C Angular distributions and Projection Cross Sections

In this section we will derive the general structure of the angular distribution of the lepton pair in the cross section given in eq. (3). Using the results for the phase space given in eq. (101) and (111) the partonic cross sections may be written as:

$$\frac{s d\hat{\sigma}^{(1)}_{ab}}{dt\, du\, dQ^2\, d\Omega^*} = \frac{K^W_{ab}}{s}\, \alpha_s\, C^{(1)}_{ab}\, L_{\mu\nu} H^{\mu\nu}_{ab}(\mathrm{B})\; \delta(s+t+u-Q^2) \tag{73}$$

$$\frac{s d\hat{\sigma}^{virt}_{ab}}{dt\, du\, dQ^2\, d\Omega^*} = \frac{K^W_{ab}}{s}\, \frac{\alpha_s^2}{2\pi} V(\varepsilon)\, C^{(1)}_{ab}\, L_{\mu\nu} H^{\mu\nu}_{ab}(\mathrm{V})\; \delta(s+t+u-Q^2) \tag{74}$$

$$\frac{s d\hat{\sigma}^{(2)}_{ab}}{dt\, du\, dQ^2\, d\Omega^*} = \frac{K^W_{ab}}{s}\, \frac{\alpha_s^2}{2\pi} V(\varepsilon) \int C^{(2)}_{ab}\, L_{\mu\nu} H^{\mu\nu}_{ab}(\mathrm{T})\; d\Omega_{k_1 k_2} \tag{75}$$

$L_{\mu\nu}$ is the well-known lepton tensor and $H^{\mu\nu}$ is the hadron tensor at the partonic level for parton a, b in the initial state. K^W_{ab} is given in eq. (70).

$$V(\varepsilon) = \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \tag{76}$$

$$C^{(1)}_{ab} = \frac{1}{8(1-\varepsilon)} \tag{77}$$

$$C^{(2)}_{ab} = \frac{C^R_{ab}}{16(1-\varepsilon)} \left(\frac{Q^2 ut}{s_2(ut - Q^2 s_2)}\right)^{\varepsilon} \frac{1}{2\pi} \tag{78}$$

C^R_{ab} is the $O(\alpha_s^2)$-color factor divided by the corresponding $O(\alpha_s)$-color factor. For notational convenience we introduce the following definition:

$$\hat{H}^{\mu\nu}_{ab} = \begin{cases} C^{(1)}_{ab}\, H^{\mu\nu}_{ab}(\mathrm{B}) & \text{in } O(\alpha_s) \\ C^{(1)}_{ab}\, H^{\mu\nu}_{ab}(\mathrm{V}) & \text{in } O(\alpha_s^2) \\ \int C^{(2)}_{ab}\, H^{\mu\nu}_{ab}(\mathrm{T})\; d\Omega_{k_1 k_2} & \text{in } O(\alpha_s^2) \end{cases} \tag{79}$$

It is understood that $\hat{H}^{\mu\nu}_{ab}$ stands for one of the three possibilities (born,virtual,tree) depending on the case under consideration. The angular dependence in eqs. (73-75) can be extracted by introducing nine helicity cross sections[4] H^α

$$\frac{1}{Q^2} L_{\mu\nu} \hat{H}^{\mu\nu}_{ab} =: \sum_{\alpha \in \mathcal{M}} g_\alpha(\theta,\phi)\, H^\alpha \qquad \mathcal{M} := \{U+L, L, T, I, P, A, 7, 8, 9\} \tag{80}$$

The θ and ϕ dependent factors g_α are defined in eq. (9). H^α_{ab} are linearly combinations of density matrix elements

$$H^{\sigma\sigma'}_{ab} = \epsilon_\mu(\sigma) \hat{H}^{\mu\nu}_{ab} \epsilon^*_\nu(\sigma') \tag{81}$$

[4]Equivalently we can decompose the hadron tensor $\hat{H}^{\mu\nu}$ into nine invariant structure functions F_i [21].

where

$$\begin{aligned} \epsilon_\mu(\pm) &= \frac{1}{\sqrt{2}}(0;\pm 1,-i,0) \\ \epsilon_\mu(0) &= (0;0,0,1) \end{aligned} \tag{82}$$

are the polarization vectors for the gauge boson defined with respect to some coordinate axis in its rest frame (see below):

$$\begin{aligned}
H^{U+L} &= H^{00}+H^{++}+H^{--} && = H_{11}+H_{22}+H_{33} \\
H^{L} &= H^{00} && = H_{33} \\
H^{T} &= \frac{1}{2}(H^{+-}+H^{-+}) && = \frac{1}{2}(H_{22}-H_{11}) \\
H^{I} &= \frac{1}{4}\left(H^{+0}+H^{0+}-H^{-0}-H^{0-}\right) && = -\frac{1}{2\sqrt{2}}(H_{31}+H_{13}) \\
H^{P} &= H^{++}-H^{--} && = -i(H_{12}-H_{21}) \\
H^{A} &= \frac{1}{4}\left(H^{+0}+H^{0+}+H^{-0}+H^{0-}\right) && = -\frac{i}{2\sqrt{2}}(H_{23}-H_{32}) \\
H^{7} &= -\frac{i}{2}(H^{+-}-H^{-+}) && = -\frac{1}{2}(H_{21}+H_{12}) \\
H^{8} &= -\frac{i}{4}\left(H^{+0}-H^{0+}+H^{-0}-H^{0-}\right) && = -\frac{1}{2\sqrt{2}}(H_{23}+H_{32}) \\
H^{9} &= -\frac{i}{4}\left(H^{+0}-H^{0+}-H^{-0}+H^{0-}\right) && = -\frac{i}{2\sqrt{2}}(H_{31}-H_{13})
\end{aligned} \tag{83}$$

To calculate H^α it is convenient (and for later integration requisite) to project out the Cartesian components of the hardon tensor (right hand side of eq. (83)) with the help of covariant projection operators. The following projection operators $P_{\mu\nu}^{(\beta)}$ are useful for the calculation of the parity conserving[5] helicity cross sections $H^{U+L,L,T,I}$:

$$\begin{aligned}
P_{\mu\nu}^{U+L} &= -\hat{g}_{\mu\nu} \\
P_{\mu\nu}^{L_1} &= \hat{p}_{1\mu}\hat{p}_{1\nu} \\
P_{\mu\nu}^{L_2} &= \hat{p}_{2\mu}\hat{p}_{2\nu} \\
P_{\mu\nu}^{L_{12}} &= \hat{p}_{1\mu}\hat{p}_{2\nu}+\hat{p}_{1\nu}\hat{p}_{2\mu}
\end{aligned} \tag{84}$$

We have introduced the hatted tensors

$$\hat{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{Q^2}$$

$$\hat{p}_\mu = p_\mu - \frac{p.q}{Q^2}q_\mu$$

We define the following covariant projections on the hadonic matrixelements:

$$T_{ab}^{\beta}(\mathrm{B,V,T}) = N_\beta\, P_{\mu\nu}^{\beta}\hat{H}_{ab}^{\mu\nu}(\mathrm{B,V,T}) \qquad \beta\in\mathcal{M}' := \{U+L, L_1, L_2, L_{12}\} \tag{85}$$

[5] A complete set of projection operators is obtained by adding: $\hat{p}_{1\mu}\hat{p}_{2\nu}-\hat{p}_{1\nu}\hat{p}_{2\mu}$, $\epsilon(\mu,\nu,q,p_1)$, $\epsilon(\mu,\nu,q,p_2)$, $p_{1\mu}\epsilon(\nu,p_1,p_2,q)+(\mu\leftrightarrow\nu)$, $p_{2\mu}\epsilon(\nu,p_1,p_2,q)+(\mu\leftrightarrow\nu)$

where the normalization factor N_β is defined by:

$$N_{U+L}=1 \qquad N_{L_1}=\frac{1}{\hat{E}_1^2} \qquad N_{L_2}=\frac{1}{\hat{E}_2^2} \qquad N_{L_{12}}=\frac{1}{\hat{E}_1\hat{E}_2} \tag{86}$$

$$\hat{E}_1=\frac{Q^2-t}{2\sqrt{Q^2}} \qquad \hat{E}_2=\frac{Q^2-u}{2\sqrt{Q^2}}$$

Using eqs. (73-75) the partonic projection cross sections $\frac{s d\hat{\sigma}^\beta}{dt\, du}$ $(\beta \in \mathcal{M}')$ are written with the help of these projections. They are given in eqs. (40-42). In the following, we will show how the helicity cross sections $\frac{s d\hat{\sigma}^\alpha}{dt\, du}$ $(\alpha \in \{U+L, L, T, I\})$ are obtained from these covariant projection cross sections in the Collins-Soper frame. Using the parametrization of the partonic momenta in the Collins-Soper frame defined in eq. (20) one has $(\hat{E}_i = x_i E_i)$:

$$\begin{aligned} p_1^\mu &= \hat{E}_1(1\,, \sin\gamma_{cs}, 0, \cos\gamma_{cs}) \\ p_2^\mu &= \hat{E}_2(1\,, \sin\gamma_{cs}, 0, -\cos\gamma_{cs}) \end{aligned} \tag{87}$$

the covariant projections T^β_{ab} are written in the CS-frame as:

$$\begin{aligned} T^{U+L} &= H_{11}+H_{22}+H_{33} \\ T^{L_1} &= [\sin^2\gamma_{CS}\, H_{11}+\cos^2\gamma_{CS}\, H_{33}+\sin\gamma_{CS}\cos\gamma_{CS}\,(H_{13}+H_{31})] \\ T^{L_2} &= [\sin^2\gamma_{CS}\, H_{11}+\cos^2\gamma_{CS}\, H_{33}-\sin\gamma_{CS}\cos\gamma_{CS}\,(H_{13}+H_{31})] \\ T^{L_{12}} &= [2\sin^2\gamma_{CS}\, H_{11}-2\cos^2\gamma_{CS}\, H_{33}] \end{aligned} \tag{88}$$

The Cartesian components of $H^{\mu\nu}$ may be expressed in terms of the projections T^β by inverting eqs. (88). The helicity cross sections $d\sigma^{U+L,L,T,I}$ are therefore linearly related to these projection cross sections. One has in the Collins Soper frame:

$$\frac{s\, d\sigma^\alpha}{dt\, du} = (M)_{\alpha\beta}\,\frac{s\, d\sigma^\beta}{dt\, du} \tag{89}$$

with

$$\left(M^{CS}\right)_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \dfrac{1}{4\cos^2\gamma_{cs}} & \dfrac{1}{4\cos^2\gamma_{cs}} & \dfrac{-1}{4\cos^2\gamma_{cs}} \\ \dfrac{1}{2} & -\dfrac{(1+\cos^2\gamma_{cs})}{8\sin^2\gamma_{cs}\cos^2\gamma_{cs}} & -\dfrac{(1+\cos^2\gamma_{cs})}{8\sin^2\gamma_{cs}\cos^2\gamma_{cs}} & \dfrac{(1-3\cos^2\gamma_{cs})}{8\sin^2\gamma_{cs}\cos^2\gamma_{cs}} \\ 0 & \dfrac{1}{4\sqrt{2}\sin\gamma_{cs}\cos\gamma_{cs}} & \dfrac{-1}{4\sqrt{2}\sin\gamma_{cs}\cos\gamma_{cs}} & 0 \end{pmatrix} \tag{90}$$

D Phase space integrals

In this appendix we will present the phase space integrals for the $O(\alpha_s)$ and $O(\alpha_s^2)$ tree contributions to high q_T gauge boson production. We will work in $n = 4 - 2\varepsilon$ dimensions. The invariants are defined in eqs. (5) and (32). We introduce the following abbreviation:

$$d\mu(p) \equiv \frac{d^{n-1}p}{(2\pi)^{n-1} 2E_p} \tag{91}$$

D.1 The $O(\alpha_s)$-phase space

The phase space integral for the DY process

$$a\,(p_1) + b\,(p_2) \longrightarrow V\,(q) + c\,(k_1) \tag{92}$$
$$\longrightarrow l_1 + l_2 + c\,(k_1) \tag{93}$$

is defined by

$$d\mathrm{PS}^{(3)} = (2\pi)^n\, \delta^n(p_1 + p_2 - k_1 - l_1 - l_2)\; d\mu(k_1)\, d\mu(l_1)\, d\mu(l_2) \tag{94}$$

Using

$$1 = \int dQ^2 \int \frac{d^{n-1}q}{2E_q}\, \delta^n(q - l_1 - l_2) \tag{95}$$

we write $d\mathrm{PS}^{(3)}$ as a product of two 2-particle-phase-spaces:

$$d\mathrm{PS}^{(3)} = \int dQ^2 \left\{ d\mu(k_1) \frac{d^{n-1}q}{2E_q} \delta^n(p_1 + p_2 - k_1 - q) \right\} \times \left\{ d\mu(l_1)\, d\mu(l_2)\, (2\pi)^n\, \delta^n(q - l_1 - l_2) \right\} \tag{96}$$

$$:= \int dQ^2\, \{A_1\}\, \{B\} \tag{97}$$

A_1 is then given by:

$$A_1 = \frac{1}{16\pi^2 s} \left(\frac{ut}{4\pi s}\right)^{-\varepsilon} \frac{1}{\Gamma(1-\varepsilon)}\, dt\, du\, \delta(s + t + u - Q^2) \tag{98}$$

Since we are interested in the angular distributions of the leptons in its restframe, we parametrize the momenta as follows:

$$l_1 = \frac{\sqrt{Q^2}}{2} (1, \cos\theta, \sin\theta\cos\phi, \sin\theta\sin\phi, \dots)$$
$$l_2 = \frac{\sqrt{Q^2}}{2} (1, -\cos\theta, -\sin\theta\cos\phi, -\sin\theta\sin\phi, \dots)$$

Since the integration over the lepton angles is not divergent, we take the limit $\varepsilon \to 0$ for the calculation of B.

$$B\Big|_{\varepsilon=0} = \frac{1}{32\pi^2} d\cos\theta\, d\phi =: \frac{1}{32\pi^2} d\Omega^* \tag{99}$$

The 3-particle phase space is then given by

$$\begin{aligned} d\mathrm{PS}^{(3)} &= \int dQ^2 \{A_1\}\{B\}\Big|_{\varepsilon=0} &(100)\\ &= \frac{1}{2^9\pi^4 s}\left(\frac{ut}{4\pi s}\right)^{-\varepsilon}\frac{1}{\Gamma(1-\varepsilon)}\,\delta(s+t+u-Q^2)\,dQ^2\,dt\,du\,d\Omega^* \end{aligned}$$

D.2 The $O(\alpha_s^2)$-phase space

The 4-particle phase space integral is defined by:

$$d\mathrm{PS}^{(4)} = (2\pi)^n\,\delta^n(p_1+p_2-k_1-k_2-l_1-l_2)\,d\mu(k_1)\,d\mu(k_2)\,d\mu(l_1)\,d\mu(l_2) \qquad (101)$$

Using eq. (95) $d\mathrm{PS}^{(4)}$ can be written as:

$$\begin{aligned} d\mathrm{PS}^{(4)} &= \int dQ^2 \left\{d\mu(k_1)\,d\mu(k_2)\,\frac{d^{n-1}q}{2E_q}\,\delta^n(p_1+p_2-k_1-k_2-q)\right\} &(102)\\ &\qquad \left\{d\mu(l_1)\,d\mu(l_2)\,(2\pi)^n\,\delta^n(q-l_1-l_2)\right\} \\ &\equiv \int dQ^2\,\{A_2\}\{B\} &(103) \end{aligned}$$

A_2 is most easily written in the CMS of k_1 and k_2. We use the parametrization:

$$\begin{aligned} k_1 &= \frac{\sqrt{s_2}}{2}(1,\cos\theta_1,\sin\theta_1\cos\theta_2,\sin\theta_1\sin\theta_2,\dots) = \frac{\sqrt{s_2}}{2}(1,\hat{\boldsymbol{k}}) &(104)\\ k_2 &= \frac{\sqrt{s_2}}{2}(1,-\cos\theta_1,-\sin\theta_1\cos\theta_2,-\sin\theta_1\sin\theta_2,\dots) = \frac{\sqrt{s_2}}{2}(1,-\hat{\boldsymbol{k}}) &(105)\\ p_1 &= \frac{s_2-u}{2\sqrt{s_2}}(1,1,0,\dots) = \frac{s_2-u}{2\sqrt{s_2}}(1,\,\hat{\boldsymbol{p}}_1) &(106)\\ p_2 &= \frac{s_2-t}{2\sqrt{s_2}}(1,\cos\vartheta,0,\dots) = \frac{s_2-t}{2\sqrt{s_2}}(1,\,\hat{\boldsymbol{p}}_2) &(107) \end{aligned}$$

with

$$\cos\vartheta = 1 - \frac{2ss_2}{(s_2-u)(s_2-t)} \qquad (108)$$

After integration over irrelevant angles in (104) and (105) one abtains for A_2:

$$A_2 = \frac{(4\pi)^{2\varepsilon}}{2^9\pi^5 s\,\Gamma(1-2\varepsilon)}\int dt\,du\left(\frac{s}{s_2(ut-s_2Q^2)}\right)^{\varepsilon}\int_0^\pi\int_0^\pi d\Omega_{k_1k_2} \qquad (109)$$

where

$$d\Omega_{k_1k_2} = d\theta_2\,\sin^{-2\varepsilon}\theta_2\,d\theta_1\,\sin^{1-2\varepsilon}\theta_1 \qquad (110)$$

Taking again the limit $\varepsilon \to 0$ for the leptonic part of the phase space one obtains:

$$\begin{aligned} d\mathrm{PS}^{(4)} &= \int dQ^2\,\{A_2\}\{B\}\Big|_{\varepsilon=0} &(111)\\ &= \frac{1}{(4\pi)^7 s\,\Gamma(1-2\varepsilon)}\int dt\,du\,dQ^2\left(\frac{16\pi^2 s}{s_2(ut-s_2Q^2)}\right)^{\varepsilon}\int_0^\pi\int_0^\pi d\Omega_{k_1k_2}d\Omega^* \end{aligned}$$

where Ω^* characterizes the angles of the leptons in its restframe (see eq. (99)).

D.3 Angular Integrals

The angular integration of the two final state partons $d\Omega_{k_1 k_2}$ has to be done analytically as the integrals are singular. The necessary set of integrals are listed in this Appendix. Note that the reduction to this basic set of integrals is achieved only after very involved partial fractioning of the tree graph matrix elements.

Many of the necessary integrals can be found in [18]. Integrals of class III are done following methods introduced in [19]. A new class of integrals not available in the literature arise from the momentum projected integrands. They can be reduced to the known integrals with the recursion relations (113) and (114).

Class I

We use the Notation:

$$\text{Integrand} \longrightarrow \frac{(1-2\varepsilon)}{2\pi}\int_0^\pi \int_0^\pi d\Omega_{k_1 k_2}\ \text{Integrand}$$

$$1 \longrightarrow 1$$

$$\frac{1}{1+\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_1} \longrightarrow -\frac{(1-2\varepsilon)}{2\varepsilon}$$

$$\frac{\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_2}{1+\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_1} \longrightarrow \frac{1}{2\varepsilon}\cos\vartheta$$

$$\frac{(\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_2)^2}{1+\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_1} \longrightarrow \frac{1}{2(1-\varepsilon)}\left(1-\frac{\cos^2\vartheta}{\varepsilon}\right)$$

$$\frac{1}{(1+\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_1)^2} \longrightarrow -\frac{(1-2\varepsilon)}{2(1+\varepsilon)}$$

$$\frac{\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_2}{(1+\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_1)^2} \longrightarrow -\frac{(1-2\varepsilon)}{2\varepsilon(1+\varepsilon)}\cos\vartheta$$

$$\frac{(\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_2)^2}{(1+\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_1)^2} \longrightarrow -\frac{1}{2\varepsilon}+\frac{3}{2\varepsilon(1+\varepsilon)}\cos^2\vartheta$$

$$\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_1 \longrightarrow 0$$

$$(\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_1)^2 \longrightarrow \frac{1}{3-2\varepsilon}$$

$$(\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_1)(\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_2) \longrightarrow \frac{1}{3-2\varepsilon}\cos\vartheta$$

$$\frac{1}{(1+\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_1)(1+\hat{\boldsymbol{k}}\,\hat{\boldsymbol{p}}_2)} \longrightarrow -\frac{(1-2\varepsilon)}{2\varepsilon}\left(\sin^2\frac{\vartheta}{2}\right)^{-1-\varepsilon}\ {}_2F_1(-\varepsilon,-\varepsilon,1-\varepsilon;\cos^2\frac{\vartheta}{2})$$

$$= -\frac{(1-2\varepsilon)}{2}\left(\sin^2\frac{\vartheta}{2}\right)^{-1-\varepsilon}\left(\frac{1}{\varepsilon}+\varepsilon \mathrm{Li}_2\left(\cos^2\frac{\vartheta}{2}\right)+O(\varepsilon^2)\right)$$

$$\frac{1}{(1+\hat{k}\,\hat{p}_1)(1-\hat{k}\,\hat{p}_2)} \longrightarrow -\frac{(1-2\varepsilon)}{2\varepsilon}\left(\cos^2\frac{\vartheta}{2}\right)^{-1-\varepsilon}{}_2F_1(-\varepsilon,-\varepsilon,1-\varepsilon;\sin^2\frac{\vartheta}{2})$$

$$= \left(\cos^2\frac{\vartheta}{2}\right)^{-1}\left[-\frac{(1-2\varepsilon)}{2\varepsilon}+\frac{1}{2}\ln\left(\cos^2\frac{\vartheta}{2}\right)+O(\varepsilon)\right]$$

Li_2 is the dilogarithm function defined in eq. (117). The cosine of the polar angle ϑ is given by:

$$\cos\vartheta = 1-\frac{2ss_2}{(s_2-u)(s_2-t)} \tag{112}$$

Class II

The general structure of the integrals of this class is:

$$\widehat{I_n^{(k,l)}} = \frac{(1-2\varepsilon)}{2\pi}\int_0^\pi\int_0^\pi d\Omega_{k_1k_2}\,(1-\cos\theta_1)^{-k}(A+B\cos\theta_1+C\sin\theta_1\,\cos\theta_2)^{-l}$$

$$\begin{aligned}\widehat{I_n^{(1,1)}} \longrightarrow\ & \frac{(1-2\varepsilon)}{2(A+B)}\left\{-\frac{1}{\varepsilon}+\ln\left(\frac{(A+B)^2}{A^2-B^2-C^2}\right)\right.\\ & -\varepsilon\left[\ln^2\left(\frac{A-\sqrt{B^2+C^2}}{A+B}\right)-\frac{1}{2}\ln^2\left(\frac{A+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\right)\right.\\ & \left.\left.+2\,\mathrm{Li}_2\left(-\frac{B+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\right)-2\,\mathrm{Li}_2\left(\frac{B-\sqrt{B^2+C^2}}{A+B}\right)\right]+O(\varepsilon^2)\right\}\end{aligned}$$

$$\begin{aligned}\widehat{I_n^{(1,2)}} \longrightarrow\ & \frac{(1-2\varepsilon)}{2(A+B)^2}\left\{-\frac{1}{\varepsilon}+\ln\left(\frac{(A+B)^2}{A^2-B^2-C^2}\right)+\frac{2(B^2+C^2+AB)}{A^2-B^2-C^2}\right.\\ & -\varepsilon\left[\ln^2\left(\frac{A-\sqrt{B^2+C^2}}{A+B}\right)-\frac{1}{2}\ln^2\left(\frac{A+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\right)\right.\\ & -2\,\frac{(A+B)\sqrt{B^2+C^2}}{A^2-B^2-C^2}\ln\left(\frac{A+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\right)-2\ln\left(\frac{(A+B)^2}{A^2-B^2-C^2}\right)\\ & \left.\left.+2\,\mathrm{Li}_2\left(-\frac{B+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\right)-2\,\mathrm{Li}_2\left(\frac{B-\sqrt{B^2+C^2}}{A+B}\right)\right]+O(\varepsilon^2)\right\}\end{aligned}$$

$$\widehat{I_n^{(2,1)}} \longrightarrow \frac{(1-2\varepsilon)}{2(A+B)}\left\{\frac{B^2+AB+C^2}{(A+B)^2}\left[-\frac{1}{\varepsilon}+\ln\left(\frac{(A+B)^2}{A^2-B^2-C^2}\right)\right]\right.$$

$$-\frac{2C^2}{(A+B)^2}-1+O(\varepsilon)\Bigg\}$$

$$\widehat{I_n^{(2,2)}} \longrightarrow \frac{(1-2\varepsilon)}{2(A+B)^2}\Bigg\{\left[\frac{3C^2}{(A+B)^2}+\frac{2B}{(A+B)}\right]\left[-\frac{1}{\varepsilon}+\ln\left(\frac{(A+B)^2}{A^2-B^2-C^2}\right)\right]$$
$$-\frac{8C^2}{(A+B)^2}+\frac{2(B^2+C^2)}{A^2-B^2-C^2}-1+O(\varepsilon)\Bigg\}$$

Class III

The general structure of the integrals is given by:

$$\overline{I_n^{(i,j)}}=\frac{(1-2\varepsilon)}{2\pi}\int_0^\pi\int_0^\pi d\Omega_{k_1k_2}\,\frac{(-\cos\theta_1)^j}{(A+B\cos\theta_1+C\sin\theta_1\,\cos\theta_2)^i}$$

Although these integrals are finite some of them are needed up to to $O(\varepsilon)$. This is due to the "plus" prescription defined in eq. (33) which brings in $1/\varepsilon$ poles that generate finite pieces after mutiplication with the $O(\varepsilon)$ terms.

$$\overline{I_n^{(1,0)}} \longrightarrow \frac{(1-2\varepsilon)}{2\sqrt{B^2+C^2}}\Bigg\{\ln\left(\frac{A+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\right)$$
$$-\varepsilon\left[\mathrm{Li}_2\left(\frac{-2\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\right)-\mathrm{Li}_2\left(\frac{2\sqrt{B^2+C^2}}{A+\sqrt{B^2+C^2}}\right)\right]\Bigg\}$$

$$\overline{I_n^{(2,0)}} \longrightarrow \frac{(1-2\varepsilon)}{A^2-B^2-C^2}\left\{1+\varepsilon\frac{A}{\sqrt{B^2+C^2}}\ln\left(\frac{A+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\right)\right\}$$

Further integrals belonging to class III $\overline{I_n^{(1,j)}}$ and $\overline{I_n^{(2,j)}}$ with $j=1,2,3,4$ are calculated with the help of recursion relations:

$$\overline{I_n^{(1,j)}} \longrightarrow \frac{1}{j(B^2+C^2)}\Big[\frac{1}{2}\{(A-B)+(-1)^j(A+B)\}$$
$$+(2j-1)AB\,\overline{I_n^{(1,j-1)}}-(j-1)(A^2-C^2)\,\overline{I_n^{(1,j-2)}}\Big] \tag{113}$$

$$\overline{I_n^{(2,j)}} \longrightarrow \frac{-1}{j(B^2+C^2)}\Big[\frac{1}{2}\{1+(-1)^j\}+(2j-1)B\left(\overline{I_n^{(1,j-1)}}-A\,\overline{I_n^{(2,j-1)}}\right)$$
$$-(j-1)\left(2A\,\overline{I_n^{(1,j-2)}}-(A^2-C^2)\,\overline{I_n^{(2,j-2)}}\right)\Big] \tag{114}$$

E Born-cross sections

In this section we list the $O(\alpha_s)$ contributions corresponding to the diagrams in fig. 1.

E.1 Diagram: $|L_1 + L_2|^2$ for $(q\bar{q} + \bar{q}q) \to VG$

$$\begin{aligned}
A_{q\bar{q}}^{U+L}(s,t,u,Q^2) &= (1-\varepsilon)\left(\frac{u}{t}+\frac{t}{u}\right)+\frac{2Q^2(Q^2-u-t)}{ut}-2\varepsilon \\
A_{q\bar{q}}^{L_1}(s,t,u,Q^2) &= \frac{s}{2} \\
A_{q\bar{q}}^{L_2}(s,t,u,Q^2) &= \frac{s}{2} \\
A_{q\bar{q}}^{L_{12}}(s,t,u,Q^2) &= \frac{\varepsilon}{(1-\varepsilon)}\,s
\end{aligned}$$

E.2 Diagram: $|L_3 + L_4|^2$ for $qG \to Vq$

$$\begin{aligned}
A_{qG}^{U+L}(s,t,u,Q^2) &= -A_{q\bar{q}}^{U+L}(u,t,s,Q^2) \\
A_{qG}^{L_1}(s,t,u,Q^2) &= -\frac{u}{2} \\
A_{qG}^{L_2}(s,t,u,Q^2) &= -\frac{u}{(1-\varepsilon)} \\
A_{qG}^{L_{12}}(s,t,u,Q^2) &= \frac{u}{(1-\varepsilon)}
\end{aligned}$$

E.3 Diagram: $|L_3 + L_4|^2$ for $Gq \to Vq$

$$\begin{aligned}
A_{Gq}^{U+L}(s,t,u,Q^2) &= A_{qG}^{U+L}(s,u,t,Q^2) \\
A_{Gq}^{L_1}(s,t,u,Q^2) &= -\frac{t}{(1-\varepsilon)} \\
A_{Gq}^{L_2}(s,t,u,Q^2) &= -\frac{t}{2} \\
A_{Gq}^{L_{12}}(s,t,u,Q^2) &= \frac{t}{(1-\varepsilon)}
\end{aligned}$$

F Notation

In this appendix we list analytical formulae for the various functions $T^{\beta}_{a,b}$(B,V,S,H) defined in eq. (85) with $\beta \in \{U+L, L_1, L_2, L_{12}\}$. As corresponding formulae on the spin-averaged case $\beta = U+L$ have already been published in [1][2] there is no need to list them here. We have independently recalculated the spin-averaged case and are in complete agreement with [1][2]. All of the following functions are functions of s, t, u, Q^2 and $s_2 = s+t+u-Q^2$. For reason of compactness we introduce the following abbreviations:

$$d_t = \frac{1}{s_2 - t}\,,\ d_u = \frac{1}{s_2 - u}\,,\ d_{st} = \frac{1}{s+t-s_2}\,,\ d_{su} = \frac{1}{s+u-s_2}$$
$$d_s = \frac{1}{s+Q^2-s_2}\,,\ d_{tu} = \frac{1}{tu - s_2 Q^2}\,,\ \lambda = \sqrt{(u+t)^2 - 4 s_2 Q^2} \tag{115}$$

Similar we introduce a short hand notation for some transcendental functions:

$$f_s = \ln\left(\frac{s}{Q^2}\right)\,,\ f_t = \ln\left(\frac{-t}{Q^2}\right)\,,\ f_u = \ln\left(\frac{-u}{Q^2}\right)\,,\ f_{s_2} = \ln\left(\frac{s_2}{Q^2}\right)$$

$$f_{M^2} = \ln\left(\frac{M^2}{Q^2}\right)\,,\ f_{\mu^2} = \ln\left(\frac{\mu^2}{Q^2}\right)\,,\ f_A = \ln\left(\frac{A}{Q^2}\right)$$

$$f_{stu} = \ln\left(\frac{sQ^2}{(s_2-t)(s_2-u)}\right)\,,\ f_{tu} = \ln\left(\frac{tu - s_2Q^2}{(s_2-t)(s_2-u)}\right)$$

$$f_{\lambda t} = \ln\left(\frac{sQ^2(s_2-t)^2}{[s_2(2Q^2-u)-Q^2t]^2}\right)\,,\ f_{\lambda u} = \ln\left(\frac{sQ^2(s_2-u)^2}{[s_2(2Q^2-t)-Q^2u]^2}\right) \tag{116}$$

$$f_{st} = \ln\left(\frac{st^2}{Q^2(s_2-t)^2}\right)\,,\ f_{su} = \ln\left(\frac{su^2}{Q^2(s_2-u)^2}\right)\,,\ f_\lambda = \ln\left(\frac{s+Q^2-s_2+\lambda}{s+Q^2-s_2-\lambda}\right)$$

$$f_{1t} = \mathrm{Li}_2\left(\frac{Q^2}{Q^2-t}\right) + \frac{1}{2}\ln^2\left(\frac{Q^2}{Q^2-t}\right)\,,\ f_{2t} = \mathrm{Li}_2\left(\frac{Q^2}{s}\right) + \frac{1}{2}f_s^2 + f_s \ln\left(\frac{-t}{s-Q^2}\right)$$

$$f_{1u} = \mathrm{Li}_2\left(\frac{Q^2}{Q^2-u}\right) + \frac{1}{2}\ln^2\left(\frac{Q^2}{Q^2-u}\right)\,,\ f_{2u} = \mathrm{Li}_2\left(\frac{Q^2}{s}\right) + \frac{1}{2}f_s^2 + f_s \ln\left(\frac{-u}{s-Q^2}\right)$$

where the dilogarithm is defined by: :

$$\mathrm{Li}_2(x) = -\int_0^x \frac{dz}{z}\,\ln(1-z) = \sum_{n=1}^{\infty} \frac{x^n}{n^2} \qquad |x| \le 1 \tag{117}$$

A further class of integrals is defined by ($k = 1,2$):

$$H_k^{(i,j)} = \frac{1}{2\pi}\int_0^1\int_0^1 \sin\theta_1 d\theta_1 d\theta_2 \frac{(-\cos\theta_1)^j}{(A + B_k\cos\theta_1 + C_k \sin\theta_1\cos\theta_2)^i} \tag{118}$$

where

$$\boldsymbol{H}_k^{(1,0)} = \frac{1}{2\sqrt{B_k^2+C_k^2}} \ln\left(\frac{A+\sqrt{B_k^2+C_k^2}}{A-\sqrt{B_k^2+C_k^2}}\right) \tag{119}$$

$$\boldsymbol{H}_k^{(2,0)} = \frac{1}{A^2-B_k^2-C_k^2} \tag{120}$$

The other integrals $\boldsymbol{H}_k^{(i,j)}$ $(j = 1,2,3,4)$ can be reduced to the above two via the recursion relations:

$$\begin{aligned}\boldsymbol{H}_k^{(1,j)} = & \frac{1}{j(B_k^2+C_k^2)}\Big[\frac{1}{2}\{(A-B_k)+(-1)^j(A+B_k)\} \\ & +(2j-1)AB_k\,\boldsymbol{H}_k^{(1,j-1)} - (j-1)(A^2-C_k^2)\,\boldsymbol{H}_k^{(1,j-2)}\Big]\end{aligned} \tag{121}$$

$$\begin{aligned}\boldsymbol{H}_k^{(2,j)} = & \frac{-1}{j(B_k^2+C_k^2)}\Big[\frac{1}{2}\{1+(-1)^j\}+(2j-1)B_k\left(\boldsymbol{H}_k^{(1,j-1)} - A\,\boldsymbol{H}_k^{(2,j-1)}\right) \\ & -(j-1)\left(2A\,\boldsymbol{H}_k^{(1,j-2)} - (A^2-C_k^2)\,\boldsymbol{H}_k^{(2,j-2)}\right)\Big]\end{aligned} \tag{122}$$

The constants A, B_k and C_k $(k = 1,2)$ are given by

$$\begin{aligned} A &= \frac{2Q^2-u-t}{2} \\ B_1 &= \frac{1}{2}\left(s_2-t+(s_2-u)\cos\vartheta\right) \\ C_1 &= \frac{1}{2}(s_2-t)\sin\vartheta \\ B_2 &= \frac{1}{2}\left(s_2-u+(s_2-t)\cos\vartheta\right) \\ C_2 &= \frac{1}{2}(s_2-u)\sin\vartheta \end{aligned} \tag{123}$$

mit

$$\cos\vartheta = 1-2ss_2d_td_u \tag{124}$$

G $O(\alpha_s^2)$-virtual corrections

The one-loop contributions of fig. 7 and 8 can be obtained from the corresponding e^+e^--annihilation one-loop expressions given in [16,17] through crossing. As some of the arguments of the log and dilog functions turn negative after crossing there will be additional contributions proprtional to π. However, since we are only interested in the real parts of the crossed one-loop contributions all terms linear in $i\pi$ can be safely dropped.

The ultraviolet divergences appearing in the virtual diagrams were absorbed into the bare QCD coupling constant according to the $\overline{\mathrm{MS}}$ scheme. Thus we have added the counter terms:

$$\frac{s\,d\sigma_{ab}^{CT}}{dt\,du} = \frac{K_W^{ab}}{s}\frac{\alpha_s^2}{2\pi}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}(4\pi)^\varepsilon A_{ab}(s,t,u,Q^2) \tag{125}$$

$$\times\delta(s+t+u-Q^2)\left(\frac{2}{3}T_R-\frac{11}{6}N_C\right)\frac{1}{\varepsilon} \tag{126}$$

G.1 Diagrams $|\sum_{i=1}^{2}L_i+\sum_{i=1}^{11}V_i|^2$ for $(q\bar{q}+\bar{q}q)\to VG$

$$\begin{aligned}
B_{q\bar{q}}^{U+L}(s,t,u,Q^2) &= A_{q\bar{q}}^{U+L}(s,t,u,Q^2)\Big\{\frac{-2C_F-N_C}{\varepsilon^2}-\frac{1}{\varepsilon}\Big[3C_F-2C_Ff_s\\
&+\frac{11}{6}N_C+N_C(f_s-f_u-f_t)-\frac{2}{3}T_R\Big]-C_F(8+f_s^2)+\frac{\pi^2}{6}(4C_F-N_C)\\
&+\frac{1}{2}N_C(f_s^2-(f_t+f_u)^2)+N_C(f_{1t}+f_{1u})+\left(\frac{11}{6}N_C-\frac{2}{3}T_R\right)f_{\mu^2}\Big\}\\
&+C_F\left(\frac{s}{s+t}+\frac{s}{s+u}+\frac{s+t}{u}+\frac{s+u}{t}\right)\\
&+f_t\left(C_F\frac{4s^2+2st+4su+ut}{(s+u)^2}+N_C\frac{t}{s+u}\right)\\
&+f_u\left(C_F\frac{4s^2+2su+4st+ut}{(s+t)^2}+N_C\frac{u}{s+t}\right)\\
&+(2C_F-N_C)\Big[2f_s\left(\frac{s^2}{(u+t)^2}+\frac{2s}{u+t}\right)-\frac{Q^2}{ut}\left(\frac{u^2+t^2}{u+t}\right)\\
&+\frac{s^2+(s+u)^2}{ut}(f_{1t}-f_{2t})+\frac{s^2+(s+t)^2}{ut}(f_{1u}-f_{2u})\Big]
\end{aligned}$$

$$\begin{aligned}
B_{q\bar{q}}^{L_1}(s,t,u,Q^2) &= A_{q\bar{q}}^{L_1}(s,t,u,Q^2)\Big\{\frac{-2C_F-N_C}{\varepsilon^2}-\frac{1}{\varepsilon}\Big[3C_F-2C_Ff_s+\frac{11}{6}N_C\\
&+N_C(f_s-f_u-f_t)-\frac{2}{3}T_R\Big]-9C_F+N_C+\frac{\pi^2}{6}(4C_F-N_C)
\end{aligned}$$

$$- C_F(f_s^2 - 2f_t) + \frac{N_C}{2}(f_s^2 - (f_t + f_u)^2) + N_C\,(f_{1t} + f_{1u})$$

$$+ (2\,C_F - N_C)\left[f_s\left(\frac{u}{t+u} + \frac{su}{(t+u)^2}\right) + \frac{u}{t+u}\right] + \left(\frac{11}{6}N_C - \frac{2}{3}T_R\right) f_{\mu^2}$$

$$- (2\,C_F - N_C)\left[f_{2u} - f_{1u} - \frac{s}{u}(f_{2t} - f_{1t})\right]\Big\}$$

$$B_{q\bar{q}}^{L_2}(s,t,u,Q^2) = B_{q\bar{q}}^{L_1}(s,u,t,Q^2)$$

$$B_{q\bar{q}}^{L_{12}}(s,t,u,Q^2) \;=\; \frac{1}{\varepsilon} A_{q\bar{q}}^{L_{12}}(s,t,u,Q^2)$$

$$\times\Big\{\frac{-2\,C_F - N_C}{\varepsilon} - 3\,C_F - \frac{N_C}{2}$$

$$- \left(C_F - \frac{N_C}{2}\right)\frac{\pi^2}{6} - \frac{11}{6}N_C + \frac{2}{3}T_R$$

$$+ C_F\left(\frac{t}{2(s+t)} + \frac{u}{2(s+u)}\right) + \left(C_F - \frac{N_C}{2}\right)\left(3 - \frac{s}{u+t}\right) f_s$$

$$+ C_F\left[f_t\left(\frac{ut}{2(s+u)^2} - \frac{s}{s+u}\right) + f_u\left(\frac{ut}{2(s+t)^2} - \frac{s}{s+t}\right)\right]$$

$$+ \frac{N_C}{2}\left[f_t\left(\frac{t}{s+u} - 2\right) + f_u\left(\frac{u}{s+t} - 2\right)\right]$$

$$- \left(C_F - \frac{N_C}{2}\right)\left(\frac{s}{u}(f_{2t} - f_{1t}) + \frac{s}{t}(f_{2u} - f_{1u})\right)\Big\}$$

G.2 Diagrams $|\sum_{i=3}^{4} L_i + \sum_{i=1}^{11} W_i|^2$ for $qG \to Vq$

$$B_{qg}^{U+L}(s,t,u,Q^2) \;=\; A_{qg}^{U+L}(s,t,u,Q^2)\Big\{\frac{-2\,C_F - N_C}{\varepsilon^2} - \frac{1}{\varepsilon}\Big[3\,C_F - 2\,C_F f_u$$

$$+ \frac{11}{6}N_C + N_C(f_u - f_s - f_t) - \frac{2}{3}T_R\Big] - C_F(8 + f_u^2) - \frac{\pi^2}{3}(C_F - N_C)$$

$$+ \frac{1}{2}N_C(f_u^2 - f_s^2 - f_t^2) + N_C\,(f_{1t} - f_{2t}) + \left(\frac{11}{6}N_C - \frac{2}{3}T_R\right) f_{\mu^2}\Big\}$$

$$- \Big\{\, C_F\left(\frac{u}{t+u} + \frac{u}{s+u} + \frac{u+t}{s} + \frac{s+u}{t}\right)$$

$$+ f_t\left(C_F\frac{4u^2 + 2ut + 4su + st}{(s+u)^2} + N_C\frac{t}{s+u}\right)$$

$$+ f_s\left(C_F\frac{4u^2 + 2su + 4ut + st}{(u+t)^2} + N_C\frac{s}{u+t}\right)$$

$$+(2C_F - N_C)\left[2f_u\left(\frac{u^2}{(s+t)^2}+\frac{2u}{s+t}\right)-\frac{Q^2}{st}\left(\frac{s^2+t^2}{s+t}\right)\right.$$
$$\left.\left.+\frac{u^2+(s+u)^2}{st}\left(f_{1t}+f_{1u}-f_tf_u+\frac{\pi^2}{2}\right)+\frac{u^2+(u+t)^2}{st}(f_{1u}-f_{2u})\right]\right\}$$

$$B_{qg}^{L_1}(s,t,u,Q^2) = A_{qg}^{L_1}(s,t,u,Q^2)\left\{\frac{-2C_F - N_C}{\varepsilon^2}-\frac{1}{\varepsilon}\left[3C_F - 2C_F f_u\right.\right.$$
$$\left.+\frac{11}{6}N_C + N_C(f_u-f_s-f_t)-\frac{2}{3}T_R\right]-9C_F + N_C - \frac{\pi^2}{3}(C_F - N_C)$$
$$+(2C_F - N_C)\frac{s}{s+t}+\frac{N_C}{2}(f_u^2-f_s^2-f_t^2)-C_F(f_u^2-2f_t)$$
$$+(2C_F - N_C)f_u\left(\frac{s}{s+t}+\frac{su}{(s+t)^2}\right)+\left(\frac{11}{6}N_C - \frac{2}{3}T_R\right)f_{\mu^2}$$
$$\left.-(2C_F - N_C)\left[f_{2u}-f_{1u}-\frac{u}{s}\left(f_tf_u-\frac{\pi^2}{2}-f_{1t}-f_{1u}\right)\right]-N_C(f_{2t}-f_{1t})\right\}$$

$$B_{qg}^{L_2}(s,t,u,Q^2) = A_{qg}^{L_2}(s,t,u,Q^2)\left\{\frac{-2C_F - N_C}{\varepsilon^2}-\frac{1}{\varepsilon}\left[3C_F - 2C_F f_u\right.\right.$$
$$\left.+\frac{11}{6}N_C + N_C(f_s-f_u-f_t)-\frac{2}{3}T_R\right]-8C_F + \frac{\pi^2}{6}N_C$$
$$-C_F(f_u^2-2f_u)-N_C f_u+\frac{N_C}{2}(f_u^2-f_s^2-f_t^2)-N_C(f_{2t}-f_{1t})$$
$$+C_F\left(\frac{t}{2(u+t)}+\frac{s}{2(s+u)}\right)+\left(\frac{11}{6}N_C - \frac{2}{3}T_R\right)f_{\mu^2}$$
$$+\frac{C_F}{2}f_t\left(\frac{st}{(s+u)^2}+\frac{2s}{s+u}\right)+\frac{C_F}{2}f_s\left(\frac{st}{(u+t)^2}+\frac{2t}{u+t}\right)$$
$$-\frac{N_C}{2}\left(f_t\frac{t}{s+u}+f_s\frac{s}{u+t}\right)$$
$$\left.-\left(C_F - \frac{N_C}{2}\right)[f_tf_u-f_{1t}+f_{2u}-2f_{1u}]\right\}$$

$$B_{qg}^{L_{12}}(s,t,u,Q^2) = A_{qg}^{L_{12}}(s,t,u,Q^2)\left\{\frac{-2C_F - N_C}{\varepsilon^2}-\frac{1}{\varepsilon}\left[3C_F - 2C_F f_u\right.\right.$$
$$\left.+\frac{11}{6}N_C + N_C(f_u-f_s-f_t)-\frac{2}{3}T_R\right]-8C_F - \frac{\pi^2}{2}\left(C_F - \frac{5}{6}N_C\right)$$

$$
\begin{aligned}
& - C_F f_u^2 + C_F \left(\frac{t}{2(u+t)} + \frac{s}{2(s+u)}\right) - (2\,C_F - N_C)\,\frac{t-s}{2(s+t)} \\
& + \frac{N_C}{2}(f_u^2 - f_s^2 - f_t^2) - N_C\,(f_{2t} - f_{1t}) - \frac{N_C}{2}\left(f_t\,\frac{t}{s+u} + f_s\frac{s}{u+t}\right) \\
& + (2\,C_F - N_C)f_u\left(\frac{3s+t}{2(s+t)} - \frac{u(t-s)}{2(s+t)^2}\right) + \left(\frac{11}{6}\,N_C - \frac{2}{3}\,T_R\right) f_{\mu^2} \\
& + \frac{C_F}{2} f_t \left(\frac{st}{(s+u)^2} + \frac{2s}{s+u} + 2\right) + \frac{C_F}{2} f_s \left(\frac{st}{(u+t)^2} + \frac{2t}{u+t} - 2\right) \\
& - \left(C_F - \frac{N_C}{2}\right)\left[-\frac{u}{s}\left(f_u f_t - \frac{\pi^2}{2} - f_{1t} - f_{1u}\right) + \frac{2t+u}{t}(f_{2u} - f_{1u})\right] \Bigg\}
\end{aligned}
$$

G.3 Diagrams $|\sum_{i=3}^{4} L_i + \sum_{i=1}^{11} W_i|^2$ for $Gq \to Vq$

$$
\begin{aligned}
B_{Gq}^{U+L}(s,t,u,Q^2) &= B_{qG}^{U+L}(s,u,t,Q^2) \\
B_{Gq}^{L_1}(s,t,u,Q^2) &= B_{qG}^{L_2}(s,u,t,Q^2) \\
B_{Gq}^{L_2}(s,t,u,Q^2) &= B_{qG}^{L_1}(s,u,t,Q^2) \\
B_{Gq}^{L_{12}}(s,t,u,Q^2) &= B_{qG}^{L_{12}}(s,u,t,Q^2)
\end{aligned}
$$

H $O(\alpha_s^2)$-treegraph contributions

In this subsection we list the results of the integrated $O(\alpha_s^2)$ tree graph contributions listed in eqs. (28). The initial state singularities have been factored out according to the $\overline{\text{MS}}$-factorization scheme as described in sec. 6.

H.1 Diagrams $|\sum_{i=1}^{8} G_i|^2$ for $(q\bar{q}+\bar{q}q) \to VGG$

We split our results into contributions $C^{\beta}_{1,q\bar{q}}$ proportioal to $\delta(s_2)$ and remaining terms $C^{\beta}_{2,q\bar{q}}$

Note that the IR/M singularities in $C^{\beta}_{1,q\bar{q}}$ cancel against corresponding singularities in the virtual contributions given in section G.1. The factorization terms are given in sec. 6.1.

$$
\begin{aligned}
C^{\beta}_{1,q\bar{q}}(s,t,u,Q^2) &= A^{\beta}_{q\bar{q}}(s,t,u,Q^2)\Big\{\frac{2\,C_F + N_C}{\epsilon^2} + \frac{1}{\epsilon}\Big[3\,C_F - 2\,C_F f_s \\
&+ \frac{11}{6}N_C + N_C(f_s - f_u - f_t)\Big] + N_C\left[\frac{67}{18} - \frac{11}{6}f_A + f_A^2\right] + C_F(2f_t \\
&+ 2f_u - 4f_A - 3)f_{M^2} + \left(C_F - \frac{1}{2}N_C\right)\left[\frac{\pi^2}{3} + (2f_A + f_s - f_t - f_u)^2\right]\Big\}
\end{aligned}
$$

$A^{\beta}_{q\bar{q}}$ are the $O(\alpha_s)$-results given in sec. E.1.

$$
\begin{aligned}
C^{U+L}_{2,q\bar{q}}(s,t,u,Q^2) &= \left\langle \frac{1}{2}\left(\frac{u}{t} + \frac{t}{u} + \frac{2Q^2 s}{ut}\right)\left\{\left(\frac{f_{s_2}}{s_2}\right)_{A+}(8\,C_F - 2\,N_C)\right.\right. \\
&\left. + \frac{1}{(s_2)_{A+}}\left[-\frac{11}{6}N_C + N_C(f_{tu} - 2f_{stu}) + 4\,C_F(f_{stu} - f_{M^2})\right]\right\} \\
&+ C_F\left[s d_t^2 + 2d_t - \frac{s}{tu} + d_{tu}\left(s_2 + \frac{2Q^2(u-s_2)}{t}\right) - 2d_t Q^2\left(\frac{1}{u} - \frac{1}{t}\right)\right] \\
&+ N_C\left[-\frac{11}{6}\frac{s}{tu} + \frac{d_t^2 s^2}{u}\left(\frac{3s_2}{2t} - 2\right) + \frac{2d_t s}{u} + \frac{Q^2}{3t^2}\right] \\
&+ (2\,C_F - N_C)\left[\frac{d_t}{u}(Q^2-u)^2\left(d_u - \frac{1}{t}\right)(f_{stu} + f_{s_2} - f_{tu})\right. \\
&\left. + \frac{s - Q^2}{tu}(f_{stu} + f_{s_2})\right] + f_{tu}\left[C_F\, d_{tu}\left(\frac{4Q^2}{tu}(Q^2-t)^2\right.\right. \\
&\left.\left. + 2(Q^2+s) - s_2\right) + C_F\frac{s_2 - 2s}{tu} - N_C\frac{Q^2}{tu}\right] \\
&+ C_F(f_{M^2} - f_{s_2})\left[d_{tu}\left(4(u-Q^2) - s_2 - \frac{4Q^2}{tu}(u-Q^2)^2\right)\right.
\end{aligned}
$$

$$+ d_t^2(Q^2-u) - \frac{d_t}{t}(2Q^2-u) - \frac{2}{t} + \frac{Q^2}{t^2} + \frac{4Q^2}{tu}\Big]\Big\rangle + \langle u \leftrightarrow t\rangle$$

$$\begin{aligned}
C^{L_1}_{2,q\bar{q}}(s,t,u,Q^2) &= \frac{s}{2}\Bigg\{\left(\frac{f_{s_2}}{s_2}\right)_{A+}(8\,C_F - 2\,N_C) \\
&+ \frac{1}{(s_2)_{A+}}\left[-\frac{11}{6}N_C + N_C\,(f_{tu}-2f_{stu}) + 4\,C_F\,(f_{stu}-f_{M^2})\right] \\
&+ N_C\,\frac{1}{2u}\left[d_t(2(u-t)-s) + d_t^2 st - \frac{1}{3}\left(\frac{2s_2}{u}-7\right)\right] \\
&+ 2\left(C_F - \frac{1}{2}N_C\right)\left[s d_t d_u\,(f_{tu}-f_{stu}-f_{s_2}) - \frac{1}{u}(f_{stu}+f_{s_2})\right] \\
&- C_F\,(f_{M^2}-f_{s_2})\left[\frac{s_2}{u^2} - \frac{2}{u} - d_t + t d_t^2\right] \\
&+ C_F\,d_t\left(\frac{(s_2-u)}{s} + s_2\left(d_t - \frac{2}{u}\right)\right)\Bigg\}
\end{aligned}$$

$$C^{L_2}_{2,q\bar{q}}(s,t,u,Q^2) = C^{L_1}_{2,q\bar{q}}(s,u,t,Q^2)$$

$$\begin{aligned}
C^{L_{12}}_{2,q\bar{q}}(s,t,u,Q^2) &= s\Bigg\{C_F\left[s_2\left(\frac{1}{ut} - \frac{d_t}{t} - \frac{d_u}{u}\right) + \frac{1}{s}\right] \\
&+ N_C\left[\frac{u+t}{4ut} + s d_t d_u\left(\frac{5}{6} - \frac{s_2}{12ut}(7u+7t-4s_2)\right)\right] \\
&- \left(C_F - \frac{1}{2}N_C\right)\frac{1}{ut}\Big((2s+u+t)(f_{s_2}+f_{stu})\Big) \\
&+ C_F\,\frac{1}{sut}\left[2sQ^2 - (u+t)(2s_2-u-t) + s_2^2\right] f_{tu}\Bigg\}
\end{aligned}$$

H.2 Diagrams $|\sum_{i=1}^{8} F_i|^2$ for $(q\bar{q}+\bar{q}q) \to Vq\bar{q}$

The contributions of the $qq + \bar{q}\bar{q}$-initiated tree graph processes have to be further subdivided into the subclasses D_{xy} according to their $Vq\bar{q}$ coupling structure. The index doublet (xy) in D_{xy} with $x, y \in \{a,b,c,d\}$ takes the values $a \in \{1,2\}, b \in \{3,4\}, c \in \{5,6\}, d \in \{7,8\}$. The correspondence of the various D_{xy} terms with their graphical counterparts can be readily read-off from fig. 4 .
Note that there are factorization contributions only for the terms D_{cc} and D_{dd}. Note also that the contributions from the $q\bar{q}$- and $\bar{q}q$-initiated processes are identical to one another after integration.

H.2.1 Diagrams $|F_1 + F_2|^2$ for $(q\bar{q}+\bar{q}q) \to Vq\bar{q}$

We split our results into contributions $D^{\beta}_{1,aa}$ proportioal to $\delta(s_2)$ and the remaining terms $D^{\beta}_{2,aa}$. Note that the IR/M singularities in $D^{\beta}_{1,aa}$ cancel against corresponding singularities in the virtual contributions given in G.1.

$$D^{U+L}_{1,aa}(s,t,u,Q^2) = \frac{2}{3}\, T_R \left(-\frac{1}{\varepsilon} + f_A - \frac{5}{3}\right) A^{U+L}_{q\bar{q}}(s,t,u,Q^2)$$

$$D^{U+L}_{2,aa}(s,t,u,Q^2) = \frac{2}{3}\, T_R \left(\frac{1}{(s_2)_{A+}} A^{U+L}_{q\bar{q}}(s,t,u,Q^2) + \frac{2(s+Q^2)}{ut} - \frac{Q^2}{t^2} - \frac{Q^2}{u^2}\right)$$

$$D^{L_1}_{1,aa}(s,t,u,Q^2) = \frac{2}{3}\, T_R \left(-\frac{1}{\varepsilon} + f_A - \frac{5}{3}\right) A^{L_1}_{q\bar{q}}(s,t,u,Q^2)$$

$$D^{L_1}_{2,aa}(s,t,u,Q^2) = \frac{2}{3}\, T_R \left(\frac{1}{(s_2)_{A+}} A^{L_1}_{q\bar{q}}(s,t,u,Q^2) + \frac{(s_2 - 2u)}{u^2}\right)$$

$$D^{L_2}_{1,aa}(s,t,u,Q^2) = D^{L_1}_{1,aa}(s,u,t,Q^2)$$

$$D^{L_2}_{2,aa}(s,t,u,Q^2) = D^{L_1}_{2,aa}(s,u,t,Q^2)$$

$$D^{L_{12}}_{1,aa}(s,t,u,Q^2) = \frac{2}{3}\, T_R \frac{-1}{\varepsilon}\, A^{L_{12}}_{q\bar{q}}(s,t,u,Q^2)$$

$$D^{L_{12}}_{2,aa}(s,t,u,Q^2) = -\frac{2}{3}\, T_R\, \frac{s^2}{ut}$$

H.2.2 Diagrams $2(F^V_1 + F^V_2)^*(F^V_3 + F^V_4)$ for $(q\bar{q}+\bar{q}q) \to Vq\bar{q}$

Because of the occurrence of two separate fermion loops the contributions of diagrams $2(F_1+F_2)^*(F_3+F_4)$ differ for the vector-vector (VV) and axialvector-axialvector (AA)

coupling cases. In fact the VV-coupling contribution can be seen to vanish due to Furry's theorem.

H.2.3 Diagrams $2(F_1^A + F_2^A)^*(F_3^A + F_4^A)$

These interferences only contributes to Z production. The analytical results can be found in [20].

H.2.4 Diagrams $2(F_1 + F_2)^*(F_5 + F_6)$ for $(q\bar{q} + \bar{q}q) \to Vq\bar{q}$

$$
\begin{aligned}
D_{ac}^{U+L}(s,t,u,Q^2) &= \left(C_F - \frac{N_C}{2}\right)\frac{1}{ut} \\
&\times\Bigg\{ 2u - \frac{s^2+3uQ^2}{t} + \frac{s_2^2 s^2}{t} d_t^2 + \frac{(s+u)^2}{s_2} + \frac{s^2 t}{s_2} d_t \\
&+ \frac{2}{\lambda^2}(u^2+ut+st)\left(2Q^2 - t + \frac{u}{s_2}(s-Q^2)\right) \\
&+ (u+t)d_s\left[\frac{2s}{\lambda^2}\left(1+12\frac{sQ^2}{\lambda^2}\right)(ut - s_2Q^2) + \frac{(s_2-u)^2}{s_2} - \frac{4s^2u^2}{s_2\lambda^2}\right] \\
&+ \frac{1}{s_2\lambda}\Bigg[u^2(s_2+Q^2) - 2us_2Q^2 + \frac{s^2}{4}(5u+17t) \\
&+ 3stu + \frac{s}{\lambda^2}(u+t)\left(u(u^2-t^2) - s(3t^2 - 2s_2Q^2)\right) \\
&+ \frac{3s^2}{4\lambda^4}(u+t)(u^2-t^2)^2\Bigg] f_\lambda - \left[\frac{2uQ^2}{t} - \frac{u^2+2s(s+u)}{s_2}\right] f_{st} \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
D_{ac}^{L_1}(s,t,u,Q^2) &= \left(C_F - \frac{N_C}{2}\right)\frac{-1}{8u} \\
&\times\Bigg\langle \frac{1}{s_2}\Bigg\{ H_1^{(0,1)}\ \left[(u+t-2s_2)(2s+t-s_2)^2 + 2ts(4s+t-s_2)\right] \\
&- H_3^{(1,1)}\ 2(s_2-t)\left[(u+t-2s_2)(2s+t-s_2)+st\right] \\
&+ H_3^{(1,2)}\ (s_2-t)^2(u+t-2s_2) - 2(4s+t)t + 4s^2 f_{st}\Bigg\} \\
&+ 4\left(s+t-\frac{s_2}{2} - (2t-3s_2)s^2 d_t^2\right)\Bigg\rangle
\end{aligned}
$$

$$
D_{ac}^{L_2}(s,t,u,Q^2) = \left(C_F - \frac{N_C}{2}\right)\frac{-1}{8t}
$$

$$\times \left\langle \frac{1}{s_2} \left\{ \left(\boldsymbol{H}_2^{(1,1)} - \boldsymbol{H}_1^{(1,0)} \right) 2(4s_2 - 2t - u)(s_2 - t)s \right.\right.$$

$$- \boldsymbol{H}_1^{(1,0)} \left[4s^2(2s_2 - t) - (s_2 - t)^2(2s_2 - t - u)\right]$$

$$\left. + \boldsymbol{H}_2^{(1,2)} (u + t - 2s_2)(s_2 - t)^2 - 2t(2s - t) - 4st\, f_{st} \right\}$$

$$\left. + f_{st}(3t - 2s_2)\frac{4s}{t} + 2(8s + s_2) - \frac{12ss_2}{t} - 4t \right\rangle$$

$$D_{ac}^{L_{12}}(s,t,u,Q^2) = \left(C_F - \frac{N_C}{2} \right) \frac{-1}{8ut}$$

$$\times \left\langle \frac{1}{s_2} \left\{ - \boldsymbol{H}_1^{(1,0)} \left[2s \left(4ut^2 - 10us_2(t - s_2) + u^2(3t - 7s_2) \right.\right.\right.\right.$$

$$\left. + 3t(t^2 - 3ts_2 + 2s_2^2)\right) + 4s^2(2u + 3t)(t - 2s_2)$$

$$\left. + \left(u(2u + t - 3s_2) + t(t - s_2)\right) (u + t - 2s_2)(t - s_2)\right]$$

$$+ \boldsymbol{H}_2^{(1,1)}\, 2(s_2 - t) \left[\left(u(u + t - 2s_2) + t(t - s_2)\right) (u + t - 2s_2) \right.$$

$$\left. s(u + 3t - 6s_2)(u + t)\right] - \boldsymbol{H}_2^{(1,2)} (s_2 - t)^2(u + t)(u + t - 2s_2)$$

$$\left. + 2ut(2s + 2u + t) + 8st^2 + 2t^3 - 4ts^2 f_{st} \right\} + f_{st}(2s - u)4s$$

$$\left. - (4u + 3t)4s - 2u(4t - 3s_2 + 2u) + 4s^2(2 + s_2 d_t) - 2t(2t - s_2) \right\rangle$$

H.2.5 Diagrams $2(F_1 + F_2)^*(F_7 + F_8)$ for $(q\bar{q} + \bar{q}q) \to Vq\bar{q}$

$$D_{ad}^{U+L}(s,t,u,Q^2) = D_{ac}^{U+L}(s,u,t,Q^2)$$

$$D_{ad}^{L_1}(s,t,u,Q^2) = D_{ac}^{L_2}(s,u,t,Q^2)$$

$$D_{ad}^{L_{12}}(s,t,u,Q^2) = D_{ac}^{L_{12}}(s,u,t,Q^2)$$

$$D_{ad}^{L_2}(s,t,u,Q^2) = D_{ac}^{L_1}(s,u,t,Q^2)$$

H.2.6 Diagrams $|F_3 + F_4|^2$ for $(q\bar{q} + \bar{q}q) \to Vq\bar{q}$

$$D_{bb}^{U+L}(s,t,u,Q^2) = \left(\frac{1}{2}\right) \left\{ \frac{d_s}{2} \left(\frac{u}{s} - 1\right) - \frac{5}{4s} \right.$$

$$+\frac{1}{\lambda^2}\left[ud_s\left(-2s+\frac{3u}{2s}(t-u)+4u-2t\right)+\frac{u}{2s}(2s+2s_2+t-u)\right]$$
$$+\frac{3u^2(u-t)}{\lambda^4}\left[(2s-u-t)d_s-\frac{s+s_2}{s}\right]$$
$$+\frac{f_\lambda}{\lambda}\left[d_s\left(\frac{2u^2}{s}+\frac{5}{2}u+\frac{3}{2}s\right)+\frac{1}{s}\left(\frac{3}{4}s+u-\frac{s_2}{2}\right)\right]$$
$$+\frac{f_\lambda}{\lambda^3}\left[u^2d_s\left(3u-t-\frac{u^2}{s}+\frac{t^2}{s}-2s\right)\right.$$
$$\left.+\frac{u}{s}\left(2s_2t-ut-2s_2^2+4us_2-3u^2+2ss_2-us\right)\right]$$
$$\left.+\frac{f_\lambda}{\lambda^5}\left[3(u^2-t^2)u^2d_s(s-Q^2)+\frac{3u^2Q^2}{s}(u-t)(u+t-2s_2)\right]\right\}+\left\{(u\leftrightarrow t)\right\}$$

$$D_{bb}^{L_1}(s,t,u,Q^2) = \left(\frac{1}{2}\right)\frac{-(s_2-u)}{4s^2}\left\{H_1^{(1,0)}\,d_s\right.$$
$$\times\left[4s^3+2s^2(3u-2s_2+t)+2s(s_2-u)(s_2-2u)-(s_2-u)^3\right]$$
$$-H_1^{(2,0)}\,\frac{1}{2}(2s+u-s_2)\left[(u-s_2)^2+2s(s+u-s_2)\right]$$
$$-H_1^{(1,1)}\,2s(s+u-s_2)-H_1^{(1,4)}\,(s_2-u)^3d_s$$
$$+H_1^{(1,2)}\,2(s_2-u)d_s\left[2s^2+s(3u-3s_2+t)+(s_2-u)^2\right]$$
$$-H_1^{(2,4)}\,\frac{(s_2-u)^3}{2}+H_1^{(2,1)}\,\left(2s^3-2s(s_2-u)^2+(s_2-u)^3\right)$$
$$\left.-H_1^{(2,2)}\,s(s_2-u)(3s-2(s_2-u))+H_1^{(2,3)}\,(s_2-u)^2(2s+u-s_2)\right\}$$

$$D_{bb}^{L_2}(s,t,u,Q^2) = D_{bb}^{L_1}(s,u,t,Q^2)$$

$$D_{bb}^{L_{12}}(s,t,u,Q^2) = \left(\frac{1}{2}\right)\frac{-1}{4s^2}$$
$$\times\left\{H_1^{(1,0)}\,2d_s\left[-4s^3(u-2s_2)+2s^2\left(3s_2\left(3u-2s_2+t\right)-2u(u+t)\right)\right.\right.$$
$$\left.+s(s_2-u)\left(u(u-8s_2+4t)+2s_2\left(3s_2-2t\right)+t^2\right)-(s_2-t)(s_2-u)^3\right]$$
$$-H_1^{(2,0)}\,\left[2s(s+u-s_2)+(s_2-u)^2\right]\left[2ss_2-(s_2-t)(s_2-u)\right]$$

$$+\boldsymbol{H}_1^{(1,1)}\,2(s_2-u)\left[s(2s+7u-6s_2+t)+2(u-s_2)^2\right]$$

$$-\boldsymbol{H}_1^{(1,2)}\,2(s_2-u)^2 d_s\left[4s^2+2s(5u-5s_2+2t)-(s_2-u)(3u-4s_2+t)\right]$$

$$-\boldsymbol{H}_1^{(2,1)}\,(s_2-u)\left[4s^3+2s^2(3u-6s_2+t)-4s(s_2-u)(u-3s_2+t)\right.$$

$$\left.+(s_2-u)^2(u-4s_2+3t)\right]+\boldsymbol{H}_1^{(1,4)}\,2(s_2-u)^4 d_s$$

$$+\boldsymbol{H}_1^{(2,2)}\,(s_2-u)^2\left[6s^2+2s(4u-6s_2+t)-3(s_2-u)(u-2s_2+t)\right]$$

$$\left.-\boldsymbol{H}_1^{(2,3)}\,(s_2-u)^3(4s+3u-4s_2+t)+\boldsymbol{H}_1^{(2,4)}\,(s_2-u)^4+4s(u-s_2)\right\}$$

H.2.7 Diagrams $2(F_3+F_4)^*(F_5+F_6)$ for $(q\bar{q}+\bar{q}q)\to Vq\bar{q}$

$$D_{bc}^{U+L}(s,t,u,Q^2) = \left(C_F-\frac{N_C}{2}\right)$$

$$\times\left\{\frac{48ss_2Q^2}{\lambda^4 t}(ut-s_2Q^2)d_s+\frac{2}{\lambda^2}\left(\frac{-2}{t}d_s\left[Q^2(s_2-u)^2+3s(t-2Q^2)^2\right]\right.\right.$$

$$+\frac{1}{st}\left(s_2(s^2+Q^4)+7sQ^2(s+Q^2)-s^3-Q^6\right)+\frac{1}{s}(Q^4+s_2^2-2s_2u)$$

$$\left.+2t-5s-8Q^2\right)+\frac{2}{st}\left(u+2t+2s_2+Q^2-\frac{2s_2Q^2}{t}+s_2(Q^2-u)d_t\right.$$

$$\left.+\left((s_2+t)^2-4Q^2(s+t)\right)d_s\right)+\frac{(s+Q^2)^2-s_2^2}{st}d_s\,f_{st}$$

$$+\frac{f_\lambda}{\lambda}\left[\frac{-24ss_2Q^2}{\lambda^4 t}(ut-s_2Q^2)+\frac{2}{\lambda^2}\left(\frac{s-Q^2}{s}(u-s_2)(s+s_2-Q^2)\right.\right.$$

$$\left.+\frac{u^2}{t}(Q^2-s)-u(3s+3s_2-Q^2)-\frac{s_2}{t}(t^2-2s_2t-6sQ^2+2Q^4)\right)$$

$$+\frac{1}{st}\left(2td_s\left[2Q^2t+s_2(4s-s_2-t)\right]+(s_2-u)^2\right.$$

$$\left.\left.\left.+3(s_2-t)^2+2s^2+2st-4t^2\right)\right]+\frac{(s_2-t)^2+s_2^2}{st}d_s\,(f_{st}+f_{\lambda t})\right\}$$

$$D_{bc}^{L_1}(s,t,u,Q^2) = \left(C_F-\frac{N_C}{2}\right)\frac{-1}{4s}\left\{-\boldsymbol{H}_1^{(1,0)}\,d_s\left[4s^2(2Q^2-3t)\right.\right.$$

$$+2s\left((u-t)^2-6(s_2-t)^2-t(s_2-t)\right)$$

$$\left.+\left(2u^2-11us_2+7ut+13s_2^2-15s_2t+4t^2\right)(s_2-t)\right]$$

$$+ \boldsymbol{H}_2^{(1,1)} (s_2 - t)(4s + 3(s_2 - t)) - \boldsymbol{H}_2^{(1,2)} (s_2 - t)^2(2s + s_2 - t)d_s$$

$$+ \left(\boldsymbol{H}_1^{(2,2)} - \boldsymbol{H}_1^{(2,0)} - \boldsymbol{H}_1^{(2,1)} + \boldsymbol{H}_1^{(2,3)}\right)\frac{(u - s_2)^3}{2} + 4s(s_2 d_t + 1)$$

$$+ f_{st}\, 2sd_s(2s + u - s_2) + f_{\lambda t}\, 2(Q^2 - s_2)(t - s_2)d_s + 2(u + s_2 - 2t)\Big\}$$

$$D_{bc}^{L_2}(s,t,u,Q^2) = \left(C_F - \frac{N_C}{2}\right)\frac{-1}{8st}$$

$$\times\Big\{ \boldsymbol{H}_1^{(1,0)}\, 2\Big[td_s\left(4s^2s_2 + 2s(u - 2s_2 + 2t)(s_2 - t) + (u - s_2)(s_2 - t)^2\right)$$

$$- 2s^2(2s_2 - 3t) + s(u + 2s_2 + 3t)(s_2 - t) + (3u - 6s_2 + 2t)(s_2 - t)^2\Big]$$

$$- \boldsymbol{H}_1^{(2,0)}\, t\left[8s^3 + 12s^2(s_2 - t) - 6s(s_2 - t)^2 + (s_2 - t)^3\right]$$

$$+ \boldsymbol{H}_2^{(1,1)}\, 2(s_2 - t)\left[(s_2 - t)(6s + 2u + 3t - 4s_2) + s(t - u)\right]$$

$$+ \boldsymbol{H}_2^{(1,2)}\, 2(s_2 - t)^2 d_s\left[2s(u + t - 2s_2) + (t - s_2 + u)(3t - 4s_2) + u^2\right]$$

$$+ \boldsymbol{H}_2^{(2,1)}\, t(s_2 - t)\left[12s^2 - (s_2 - t)^2\right] - \boldsymbol{H}_2^{(2,2)}\, t(s_2 - t)^2(6s + t - s_2)$$

$$+ 4\frac{s_2 - t}{Q^2 t}\Big[s^2(t - 4s_2) + s\left(4(t - s_2)^2 - 2s_2(2u - t) + 3ut\right)$$

$$- (s_2 - t)(u + t - s_2)t\Big] + \boldsymbol{H}_2^{(2,3)}\, t(s_2 - t)^3$$

$$+ 4s(s_2 - t)d_s\left[f_{st}(2Q^2 - u) + \frac{f_{\lambda t}}{s}\left((s_2 - t)(Q^2 - s_2) + ss_2\right)\right]\Big\}$$

$$D_{bc}^{L_{12}}(s,t,u,Q^2) = \left(C_F - \frac{N_C}{2}\right)\frac{-1}{4st}\Big\{$$

$$\boldsymbol{H}_1^{(1,0)}\Big[2td_s\Big(2s^2(2s + u + s_2 - 2t) + s(u - 6s_2 + t)(s_2 - t)$$

$$+ \left(2(u + t)^2 - u^2 + s_2(7s_2 - 6u - 8t)\right)(s_2 - t)\Big) + (s_2 - t)(4s_2 t - t^2 + u^2)$$

$$+ 2s^2(4s_2 - 9t) - (s_2 - t - s)\left(7(s_2 - t)u - 10s_2^2 + 19s_2 t - 5t^2\right)\Big]$$

$$+ \boldsymbol{H}_1^{(2,0)}\, t\left[4s^2(2s + u - 3s_2) + 2s(s_2 - t)(3s_2 - 2u) - (s_2 - u)(s_2 - t)^2\right]$$

$$+ \boldsymbol{H}_2^{(1,1)}\, (s_2 - t)\Big[s(u + 13t - 6s_2) + (u + t)^2 + 10(t - s_2)^2 + 4ut$$

$$- 2s_2(3u + s_2)\Big] - \boldsymbol{H}_2^{(1,2)}\, (s_2 - t)^2 d_s\Big[s(2u + 6t - 4s_2) + 4s_2(s_2 - u)$$

$$- 7t(2s_2 - t) + u(u + 6t)\Big] + \boldsymbol{H}_2^{(2,2)}\, t(s_2 - t)^2(6s + u + 2t - 3s_2)$$

$$
\begin{aligned}
& - H_2^{(2,1)}\, t(s_2 - t)\left[2s(6s + 2u + 3t - 6s_2) - (s_2 - t)(2u + t - 3s_2)\right] \\
& - H_2^{(2,3)}\, t(s_2 - t)^3 + 2sd_s f_{st}\left(2ss_2 + us_2 - 3s_2 t + 3t^2\right) \\
& + 2d_s f_{\lambda t}\left[(2s_2 - t)st - (s_2 - t)\left(s_2 u - 2tu + s_2 t - 2(s_2 - t)^2\right)\right] \\
& + \frac{2}{Q^2}\left[s^2(3t - 5s_2) + s\left(t(t + s_2) + 8(s_2 - t)^2 - 6u(s_2 - t)\right)\right. \\
& \left. - (s_2 - t)(u + t - s_2)(u + 4t - 3s_2)\right]\Bigg\}
\end{aligned}
$$

H.2.8 Diagrams $2(F_3 + F_4)^*(F_7 + F_8)$ for $(q\bar{q} + \bar{q}q) \to Vq\bar{q}$

$$
\begin{aligned}
D_{bd}^{U+L}(s,t,u,Q^2) &= D_{bc}^{U+L}(s,u,t,Q^2) \\
D_{bd}^{L_1}(s,t,u,Q^2) &= D_{bc}^{L_2}(s,u,t,Q^2) \\
D_{bd}^{L_{12}}(s,t,u,Q^2) &= D_{bc}^{L_{12}}(s,u,t,Q^2) \\
D_{bd}^{L_2}(s,t,u,Q^2) &= D_{bc}^{L_1}(s,u,t,Q^2)
\end{aligned}
$$

H.2.9 Diagrams $|F_5 + F_6|^2$ for $(q\bar{q} + \bar{q}q) \to Vq\bar{q}$

The facorization terms for these diagrams are listed in eq. (61).

$$
\begin{aligned}
D_{cc}^{U+L}(s,t,u,Q^2) = & \left(\frac{1}{2}\right)\Bigg\{\left[2\frac{s_2 - u}{t^2}\left(2\frac{s_2}{t} - 1\right)^2 - 4\frac{s_2 u}{t^3} - 2\frac{s}{t^2}\left(2\frac{s_2^2}{t^2} + 1\right)\right. \\
& \left. - \frac{1}{s}\left(2\frac{s_2^2}{t^2} - 2\frac{s_2}{t} + 1\right)\left(2\left(\frac{s_2 - u}{t}\right)^2 - 2\frac{s_2 - u}{t} + 1\right)\right] \\
& \times (3 - f_{st} - f_{s_2} + f_{M^2}) + \left(sd_t^2 + 2\frac{st + s_2 q^2}{t^2} d_t\right)(f_{s_2} - f_{M^2}) \\
& + \left(s\frac{u^2 - t^2}{t\lambda^3} + \frac{2u + 3s}{t\lambda}\right) f_\lambda + (u - t)\frac{s + s_2 - q^2}{t\lambda^2} + 4\frac{s_2}{st}\left(\frac{s_2}{t} - 1\right) \\
& + 6\frac{s_2 - u}{st}\left(\frac{s_2 - u}{t} - 1\right) + \frac{3}{s} + \frac{9u + 7s - 5s_2}{t^2} + \frac{u + t}{t} d_t - sd_t^2\Bigg\}
\end{aligned}
$$

$$
D_{cc}^{L_1}(s,t,u,Q^2) = \left(\frac{1}{2}\right)\frac{-1}{8}\Bigg\{ H_1^{(1,0)}\, 2(12s + 4u + 3t - 7s_2) - H_2^{(2,2)}\,(s_2 - t)^2
$$

$$- \boldsymbol{H}_1^{(2,0)} \left[16s^2 + 2s(3t - 7s_2 + 4u) - (s_2 - t)(t - 3s_2 + 2u)\right]$$

$$- \boldsymbol{H}_2^{(1,1)} \, 6(s_2 - t) + \boldsymbol{H}_2^{(2,1)} \, 2(s_2 - t)(Q^2 + 2s - s_2)$$

$$- \frac{8}{Q^2 t^2} \left[3s_2 (Q^2 - t)^2 + t(4s_2 - 2t - 2u - s)(Q^2 - t)\right]$$

$$+ \frac{4}{t^2} d_t^2 s(s_2 - 2t) \left(2s_2 (s_2 - t) + t^2\right) - \frac{8}{t^2} \left((s_2 - u)(s_2 - t) + t^2\right)$$

$$+ \frac{4}{t^2} (f_{s_2} - f_{M^2}) d_t^2 (Q^2 s_2 - ut) \left((s_2 - t)^2 + s_2^2\right)$$

$$+ f_{st} \frac{8}{t^2} \left(s_2 Q^2 + t(s - s_2)\right) \Bigg\}$$

$$D_{cc}^{L_2}(s,t,u,Q^2) = \left(\frac{1}{2}\right) \frac{-1}{4t} \Bigg\{ \boldsymbol{H}_1^{(1,0)} \left[2s(4t - 4s_2 + u) + (s_2 - t)(2t - 2s_2 + u)\right]$$

$$- \boldsymbol{H}_1^{(2,0)} \frac{t}{2} (8s + t - s_2)(2s - t + s_2) - \boldsymbol{H}_2^{(1,1)} (s_2 - t)(u + 2t - 2s_2)$$

$$+ \boldsymbol{H}_2^{(2,1)} \, 3st(s_2 - t) - \boldsymbol{H}_2^{(2,2)} \frac{t}{2} (s_2 - t)^2 - f_{st} \frac{2}{t} \left((s_2 - t)(s - t) + ss_2\right)$$

$$+ \frac{2}{Q^2 t^3} \Big[- s^2 s_2 \left(16 s_2 (s_2 - t) + 5t^2\right)$$

$$+ s \left(32 s_2^3 (s_2 - 2t - u) + t^3 (t - 4s_2 + 2u) + t s_2 (37 t s_2 + 48 s_2 u - 21 tu)\right)$$

$$- (s_2 - t) \Big(16 u s_2 (u s_2 - ut - t^2 + 3 t s_2 - 2 s_2^2) + t^3 (s_2 - 3t - u)$$

$$+ 16 s_2^2 (s_2 - t)^2 + 2u^2 t^2\Big) \Big]$$

$$+ \frac{4}{t^3} (f_{s_2} - f_{M^2} + f_{st} + 1)(Q^2 s_2 - ut) \left[(s_2 - t)^2 + s_2^2\right] \Bigg\}$$

$$D_{cc}^{L_{12}}(s,t,u,Q^2) = \left(\frac{1}{2}\right) \frac{-1}{4t}$$

$$\times \Bigg\{ - \boldsymbol{H}_1^{(1,0)} \left[2s(u + 10t - 6s_2) + u(5t - 7s_2) + (s_2 - t)(6s_2 - t) + 2u^2\right]$$

$$+ \boldsymbol{H}_1^{(2,0)} \, t \left[16s^2 + 2s(2u - 3s_2) + (s_2 - u)(s_2 - t)\right] + \boldsymbol{H}_2^{(2,2)} \, t(s_2 - t)^2$$

$$- \boldsymbol{H}_2^{(2,1)} \, t(s_2 - t)(Q^2 + 5s - s_2) + \boldsymbol{H}_2^{(1,1)} (s_2 - t)(u + 5t - 2s_2)$$

$$+ \frac{2}{Q^2 t^2} \Big[- 4s^2 s_2 (3s_2 - 2t) - 12 s_2^3 (s_2 - 2t) - 3 s_2 t^2 (3s_2 + 2t) + 5t^4$$

$$+ u \left[8 s_2^2 (3s_2 - 5t) + t^2 (11 s_2 + 3t)\right] - 2u^2 \left[2 s_2 (3s_2 - 4t) + t^2\right]$$

$$- s\left[2u\left(12(s_2 - t)s_2 + t^2\right) - 4s_2^2(6s_2 - 8t) - t^2(5s_2 - t)\right]\Bigg]$$

$$+ f_{st}\, 2d_t\left((3t - 5s_2)s + (t - s_2)u - t^2 + s_2^2\right)$$

$$+ (f_{s_2} - f_{M^2} + f_{st})\frac{4d_t}{t^2}(Q^2 s_2 - ut)\left((s_2 - t)^2 + s_2^2\right)\Bigg\}$$

H.2.10 Diagrams $2(F_5^V + F_6^V)^*(F_7^V + F_8^V)$ for $(q\bar{q} + \bar{q}q) \to Vq\bar{q}$

The result for the interference of these diagrams differ for vector-vector (relevant for γ^* production) and axial-axial (relevant for Z production) couplings. They do not contribute to W production. The analytical results can be found in [20].

H.2.11 Diagrams $2(F_5^A + F_6^A)^*(F_7^A + F_8^A)$ for $(q\bar{q} + \bar{q}q) \to Vq\bar{q}$

The analytical results can be found in [20].

H.2.12 Diagrams $|F_7 + F_8|^2$ for $(q\bar{q} + \bar{q}q) \to Vq\bar{q}$

The structure of the factorization terms is closely related to the factorization contributions for the diagrams in sec. H.2.9.

$$\begin{aligned}
D_{dd}^{U+L}(s,t,u,Q^2) &= D_{cc}^{U+L}(s,u,t,Q^2) \\
D_{dd}^{L_1}(s,t,u,Q^2) &= D_{cc}^{L_2}(s,u,t,Q^2) \\
D_{dd}^{L_{12}}(s,t,u,Q^2) &= D_{cc}^{L_{12}}(s,u,t,Q^2) \\
D_{dd}^{L_2}(s,t,u,Q^2) &= D_{cc}^{L_1}(s,u,t,Q^2)
\end{aligned}$$

H.3 Diagrams $|\sum_{i=1}^{8} H_i|^2$ for $qq \to Vqq$ and $\bar{q}\bar{q} \to V\bar{q}\bar{q}$

Due to the different Vqq coupling structures the contributions of the $qq \to Vqq$ and $\bar{q}\bar{q} \to V\bar{q}\bar{q}$ (see fig. 5) have to be again subdivided into different subclasses. There is a simplification in as much as most of the resulting contributions can be obtained from the results in sec. H.2 via $u \leftrightarrow t$ exchange after phase space integration. Note that the following results hold both for qq and $\bar{q}\bar{q}$-initiated processes.

H.3.1 Diagrams $|\sum_{i=1}^{4} H_i|^2$ and $|\sum_{i=5}^{8} H_i|^2$ for $qq \to Vqq$

mit $\beta \in \{U+L, L_1, L_{12}, L_2\}$ gilt:

$$E_{aa}^{\beta}(s,t,u,Q^2) = E_{cc}^{\beta}(s,u,t,Q^2) = D_{cc}^{\beta}(s,u,t,Q^2)$$

$$E^{\beta}_{bb}(s,t,u,Q^2) = E^{\beta}_{dd}(s,u,t,Q^2) = D^{\beta}_{dd}(s,u,t,Q^2)$$

In analogy to the results in sec. H.2.10 the (VV)-contributions differ from the (AA) contributions.

$$E^{\beta}_{ab\,VV}(s,t,u,Q^2) = E^{\beta}_{cd\,VV}(s,u,t,Q^2) = -D^{\beta}_{cd\,VV}(s,u,t,Q^2)$$

$$E^{\beta}_{ab\,AA}(s,t,u,Q^2) = E^{\beta}_{cd\,AA}(s,u,t,Q^2) = D^{\beta}_{cd\,AA}(s,u,t,Q^2)$$

H.3.2 Diagrams $2(H_1+H_2)^*(H_5+H_6)$ for $qq \to Vqq$

$$E^{U+L}_{ac}(s,t,u,Q^2) = \left(C_F - \frac{N_C}{2}\right)\frac{-4}{t} \times \left\{-\frac{Q^2}{t} f_{st} - \frac{1}{2}\left((s_2-Q^2)^2+s^2\right) d_t\, d_s\, (f_{\lambda t}+f_{st})\right\}$$

$$E^{L_1}_{ac}(s,t,u,Q^2) = \left(C_F - \frac{N_C}{2}\right) s^2 d_s\, d_t \left\{2f_\lambda \frac{s_2-t}{\lambda} - f_{st} - f_{\lambda t}\right\}$$

$$E^{L_2}_{ac}(s,t,u,Q^2) = \left(C_F - \frac{N_C}{2}\right) s \times \left\{2sd_s\frac{f_\lambda}{\lambda} - \left(\frac{2s_2-t}{t^2} + sd_s\, d_t\right) f_{st} - \left(\frac{1}{t} + sd_s\, d_t\right) f_{\lambda t}\right\}$$

$$E^{L_{12}}_{ac}(s,t,u,Q^2) = \left(C_F - \frac{N_C}{2}\right)\frac{-sd_s}{t} \times \left\{4st\frac{f_\lambda}{\lambda} + f_{st}\,(Q^2+3s-s_2) - f_{\lambda t}\,(Q^2-s-s_2)\right\}$$

H.3.3 Diagrams $2(H_1+H_2)^*(H_7+H_8)$ for $qq \to Vqq$

$$E^{U+L}_{ad}(s,t,u,Q^2) = \left(C_F - \frac{N_C}{2}\right)\frac{2}{sut}\left\{2s_2Q^2\frac{u^2+t^2}{ut} - (u+t)^2 + \left(2s_2(Q^2-s)-(u+t)^2\right)\left(f_{stu}+f_{s_2} - \frac{1}{2}(f_{su}+f_{st})\right)\right\}$$

$$E^{L_1}_{ad}(s,t,u,Q^2) = \left(C_F - \frac{N_C}{2}\right)\frac{-1}{2u}\left\{(u-s_2)(2f_{stu}+2f_{s_2}-f_{su}-f_{st}) \frac{2}{uQ^2}\left[2s_2^2(Q^2+u) - us_2(2s+3t+3u) + u^2(Q^2+s_2)\right]\right\}$$

$$E^{L_2}_{ad}(s,t,u,Q^2) = E^{L_1}_{ad}(s,u,t,Q^2)$$

$$E^{L_{12}}_{ad}(s,t,u,Q^2) = \left(C_F - \frac{N_C}{2}\right)\frac{-1}{2ut}\left\{ 4(u+t)s_2 - 4ut\left(1+\frac{s_2}{Q^2}\right)\right.$$
$$\left. -(s_2(Q^2-s-s_2) - 2ut)(f_{st} + f_{su} - 2f_{stu} - 2f_{s_2})\right\}$$

H.3.4 Diagrams $2(H_3+H_4)^*(H_5+H_6)$ for $qq \to Vqq$

$$E^{\beta}_{bc}(s,t,u,Q^2) = E^{\beta}_{ad}(s,u,t,Q^2)$$

mit $\beta \in \{U+L, L_1, L_2, L_{12}\}$

H.3.5 Diagrams $2(H_3+H_4)^*(H_5+H_6)$ for $qq \to Vqq$

$$E^{\beta}_{bd}(s,t,u,Q^2) = E^{\beta}_{ac}(s,u,t,Q^2)$$

mit $\beta \in \{U+L, L_1, L_2, L_{12}\}$

H.4 Diagrams $|\sum_{i=1}^{8} I_i|^2$ for $qG \to VqG$

We split our results into contributions $C^{\beta}_{1,qG}$ proportional to $\delta(s_2)$ and remaining terms $C^{\beta}_{2,qG}$. Note that the IR/M in $C^{\beta}_{1,qG}$ cancel against corresponding singularities in the virtual contributions given in sec. G.2.

The factorization terms for this class of diagrams are listed in sec. 6.2. They carry the flavour factor $T_R = n_f/2$, which is needed to cancel the corresponding singularities in B^{β}_{qg} given in sec. G.2.

$$C^{\beta}_{1,qg}(s,t,u,Q^2) = A^{\beta}_{qg}(s,t,u,Q^2)\left\{\frac{2C_F + N_C}{\epsilon^2} + \frac{1}{\epsilon}\left[3C_F - 2C_F f_u\right.\right.$$
$$\left. + \frac{11}{6}N_C + N_C(f_u - f_s - f_t) - \frac{2}{3}T_R\right] + C_F\left[\frac{7}{2} + 2f_{M^2}(f_u - f_A) + f_A^2\right.$$
$$\left. - \frac{3}{2}(f_{M^2} + f_A)\right] + N_C\left[\frac{\pi^2}{6} + \frac{1}{2}(f_s - f_t - f_u)^2\right.$$
$$\left.\left. + 2f_t(f_{M^2} - f_A) + 2f_A(f_s - f_u - f_{M^2} + f_A)\right] - \left(\frac{11}{6}N_C - \frac{2}{3}T_R\right)f_{M^2}\right\}$$

Due to the length of $C_{2,qG}$-contributions we further divide them into terms proportional to C_F and N_C .

$$C^{\beta}_{2,qg}(s,t,u,Q^2) = A^{\beta}_{qg}N_C + B^{\beta}_{qg}C_F$$

The results for $\mathcal{A}_{qg}^{U+L}$ and $\mathcal{B}_{qg}^{U+L}$ have already been published in [1] (eq. (A.3)) [2] (eq. (A.12)). We do not list them here.

$$\mathcal{A}_{qg}^{L_1} = \frac{-1}{32s^2}\Big\{ 2s(s_2-u)^3\Big[\boldsymbol{H}_1^{(2,1)} - \boldsymbol{H}_1^{(2,3)}$$
$$+\boldsymbol{H}_1^{(2,2)}(2s+u-s_2)d_u - \boldsymbol{H}_1^{(1,3)}\frac{2+sd_{st}}{2s}\Big]$$
$$-2s(s_2-t)^3\Big[\boldsymbol{H}_2^{(2,3)} - \boldsymbol{H}_2^{(2,2)}(2Q^2+t-3s_2)d_t + \boldsymbol{H}_2^{(1,3)}\frac{2+sd_{st}}{2s}$$
$$-\boldsymbol{H}_2^{(2,1)}\left(8s^2+(s_2-t)(4Q^2-t-3s_2)\right)d_t^2\Big]$$
$$+\boldsymbol{H}_1^{(2,0)}\,2s\Big[(u+t-2s_2)\left((s_2-u-2s)(u-t)+(s_2-t)^2\right)$$
$$-8s^2(2u+t+4s-3s_2)\Big]\Big\}$$
$$-\frac{1}{32s^2}\frac{d_{st}}{(s_2)_{A+}}\Big\{-\boldsymbol{H}_2^{(1,1)}(s_2-t)\Big[2s_2(s_2-t)^2(4u+3t-7s_2)$$
$$4s^3(2u+t+s_2)+s\left(4u^2(t-2s_2)+4ut(2t-7s_2)+2s_2^2(10u+17t)\right.$$
$$\left.+4t^2(t-6s_2)-11s_2^3+s_2t^2\right)+4s^2\left(5s_2^2+ut-(u+t)(8s_2-2t-u)\right)\Big]$$
$$+\boldsymbol{H}_2^{(1,2)}(s_2-t)^2\Big[2s^2(u+t)+st(2u+2t-s_2)-7ss_2^2$$
$$-2s_2(s_2-t)(2u+3t-5s_2)\Big]$$
$$+\boldsymbol{H}_1^{(1,1)}(s_2-u)\Big[4s^3(t-2s_2)+4s^2\left(t^2-s_2(2u+3t-4s_2)\right)$$
$$+ss_2(s_2-u)(u+4t-5s_2)-2s_2(s_2-t)(s_2-u)^2\Big]$$
$$+\boldsymbol{H}_1^{(1,2)}(s_2-u)^2\Big[-2s^2(u+t-3s_2)-ss_2(11s_2-10t-9u)$$
$$-2st(u+t)+2s_2(s_2-t)(s_2-u)\Big]$$
$$+\boldsymbol{H}_1^{(1,0)}\Big[8s^4(u+8s_2)+4s^3\left(u(5t-6s_2+2u)+t^2+3ts_2-7s_2^2\right)$$
$$+2s^2\left(u(9t^2-42ts_2+38s_2^2)+u^2(7t-15s_2+u)+3(t^3-15s_2^3)-4ts_2(10t\right.$$
$$\left.-21s_2)\right)+s(u+t-2s_2)\left(ut(2u+4t-21s_2)-us_2(5u-19s_2)+t^2(2t\right.$$
$$\left.-23s_2)+s_2^2(47t-25s_2)+2s_2\frac{s_2-t}{s}\left(u(u-t-s_2)-t(t-3s_2)-s_2^2\right)\right)\Big]\Big\}$$
$$-\Big\{-\frac{1}{8s}\frac{1}{(s_2)_{A+}}(2su+(u+t)^2)$$
$$-\frac{d_t^3 d_{st}}{24Q^2s^2t^2}\Big[12s^5\left(t^3(t-3s_2)+4s_2^2(4t^2-3ts_2+s_2^2)\right)$$

$$+12s^4\Big(15t^3s_2(3s_2-u)+3t^4(t-3s_2)+u(2t^4+37t^2s_2^2-26ts_2^3+8s_2^4)$$

$$+4s_2^4(11t-3s_2)-71t^2s_2^3\Big)-3s^3(s_2-t)\Big(16s_2^2u(16t^2-12s_2t+4s_2^2-s_2u)$$

$$+16s_2(10s_2^3t-2t^4-3s_2^4)+4ut^2(ut-11us_2+3t^2)+14t^2(t^3-17s_2^3)$$

$$+5ts_2(8u^2s_2+29t^2s_2-20ut^2)-t^5\Big)$$

$$+s^2(s_2-t)^2\Big(48s_2^3(3ts_2-(s_2-u)^2)+9t^3(3u^2+6ut-10ts_2)$$

$$+21ut^2s_2(17s_2-7u)+4t^2s_2^2(43t-53s_2)+15uts_2(8us_2-13t^2)+t^4s_2$$

$$+33t(t^4-8us_2^3)\Big)+2t^2(s_2-t)^4(Q^2-s)(u+t-2s_2)(u-2t+s_2)$$

$$-st^2(s_2-t)^3\Big(31u(t^2+s_2^2)+5s_2^2(7t-2s_2)+3u^3+14t^3$$

$$-20u^2(s_2-t)-13ts_2(3t+5u)\Big)\Big]$$

$$+\frac{u}{(s_2)_{A+}}\left(f_{stu}-f_{M^2}+\frac{f_{st}-f_{su}}{4}+\frac{u}{2s}f_{tu}\right)+2u\left(\frac{f_{s_2}}{s_2}\right)_{A+}+\frac{f_{su}}{4}$$

$$-(f_{s_2}-f_{M^2})\Big[-\frac{s}{t^2}(t^2-s_2t+s_2^2)^2d_t^3+\frac{d_t^3}{t^2}(t^4(3s_2-t)+s_2^4(s_2-3t))$$

$$-u\frac{s_2}{t^2}+s_2d_t^2(5s_2-u)\Big]-\frac{f_{stu}+f_{s_2}}{2}(1+(u-t)d_t)$$

$$+\frac{f_{st}}{4t^2}d_t\left(2s(2s_2^2-t^2)+u(2s_2-t)^2-(s_2-t)(4s_2^2+t^2)-s_2t^2\right)$$

$$+f_{\lambda t}\frac{Q^2+s-s_2}{4s}(Q^2-u)d_t-\frac{f_{tu}}{2}\left((s_2-u)d_t+\frac{2u-s_2}{s}\right)\Big\}$$

$$\mathcal{B}_{qg}^{L_1} = \frac{-1}{16s}\Big\{H_1^{(2,0)}\Big[8s^3-4s^2(u-s_2)-2s(u-s_2)^2-(u-s_2)^3\Big]$$

$$+H_1^{(2,1)}(2s+u-s_2)(2s-u+s_2)(u-s_2)+H_1^{(2,3)}(u-s_2)^3$$

$$+H_1^{(2,2)}(2s+u-s_2)(u-s_2)^2+H_2^{(1,2)}2d_{st}(Q^2+3s-s_2)(s_2-t)^2$$

$$-H_2^{(1,1)}4(s_2-t)d_{st}\Big[8s^2+s(5u-14s_2+8t)+(u-2s_2+t)(u-3s_2+2t)\Big]$$

$$-H_1^{(1,2)}2d_{st}(3s+u-s_2)(s_2-u)^2+H_1^{(1,1)}4s(2Q^2-t)(s_2-u)d_{st}$$

$$+H_1^{(1,0)}2d_{st}\Big[4s^2(3s+u-12s_2+7t)$$

$$-s\left(16(2s_2-t)u-7u^2-63s_2^2+78s_2t-24t^2\right)+u^2(u+6t)$$

$$-9s_2(u^2+3s_2^2)+s_2t(53s_2-34t-38u)+28us_2^2+13ut^2+7t^3\Big]$$

$$+ \boldsymbol{H}_1^{(1,0)} \; \frac{8su(u+t)}{(s_2)_{A+}} - \frac{8s}{u^2}\left(f_{s_2} - f_{M^2}\right)\left[4u^2 - 3s_2u + s_2^2 + su^2(s_2^2+t^2)d_t^3\right]$$

$$- \frac{16su}{(s_2)_{A+}} f_{M^2} + 16 f_{tu}\left(\frac{-u^2}{(s_2)_{A+}} + 2u - s_2\right) + 16su\left(\frac{f_{s_2}}{s_2}\right)_{A+}$$

$$-8sf_{su}\left[\frac{-u}{(s_2)_{A+}} + \frac{3u(u-s_2)+s_2^2}{u^2}\right]$$

$$+8f_{\lambda t}(Q^2+s-s_2) - \frac{12su}{(s_2)_{A+}} + 12s - 8s^2 d_t(s_2d_t+1)$$

$$-16s(s_2-u)\left(d_t + \frac{1}{2u}\right) + 8s\frac{s_2-u}{Q^2u^2}\left(su + s_2(Q^2-u)\right)$$

$$+8s^2(t^2+s_2^2)d_t^3 + 4d_{st}\left[4s^3s_2d_t^3(s_2+2t) + 4s^2d_t^2(ut-2s_2^2-2t^2)\right.$$

$$\left.-2sd_t\left(u(Q^2-s) - t^2 - 4(s_2-t)^2\right) + (u-2s_2+t)(u+2s_2-3t)\right]\Bigg\}$$

$$\mathcal{A}_{qg}^{L_2} = \frac{-1}{16st(s_2)_{A+}}\Bigg\{ - \boldsymbol{H}_1^{(1,1)}\;(s_2-u)\left[2s\left(u(u-t+s_2) - 2(t-s_2)^2 + t^2\right)\right.$$

$$\left.-2t(u+t)^2 + 8s_2^2(s_2-u) + s_2(2u^2+13ut+11t^2-14ts_2)\right]$$

$$+\boldsymbol{H}_1^{(1,0)}\;\left[8s^3s_2 - 2s\left((t-4s_2-u)u + s_2(s_2-7t)\right)(2s-u)\right.$$

$$-2s\left(t^2(t+7s_2) - 15s_2^2(u+2t) + 22s_2^3 - 2u^3 + ut^2\right)$$

$$\left.-(u+t-2s_2)\left((u+t)^2t + us_2(2s_2-u-6t) - 3t^2s_2\right)\right]$$

$$-\boldsymbol{H}_1^{(1,2)}\;(s_2-u)^2\left[(s_2-t)^2 - (s_2-t)u + s_2^2\right]$$

$$-\boldsymbol{H}_2^{(1,1)}\;(s_2-t)\left[4s^2s_2 + 2s(3us_2+t^2+4ts_2)\right.$$

$$\left.-s_2\left(8s_2(s_2-u) + (2u+t)(u+t) - 10ts_2\right)\right]$$

$$+\boldsymbol{H}_2^{(1,2)}\;(s_2-t)^2\left[(2s_2-u)(s_2-t) + t^2\right]$$

$$-2\left[2su(u-t) - t(u+t)^2\right]\Bigg\} - \frac{1}{8}\Bigg\{ - \boldsymbol{H}_2^{(2,2)}\;(s_2-t)^2$$

$$-\boldsymbol{H}_1^{(2,0)}\;(8s+t-s_2)(2s-t+s_2) + \boldsymbol{H}_2^{(2,1)}\;6s(s_2-t)$$

$$-\frac{d_td_u}{Q^2st^4}\left[4s^3s_2\left(t^3(t-4s_2+3u) + 12s_2^2(s_2^2-2ts_2-s_2u+2tu)\right.\right.$$

$$\left.+17t^2s_2(s_2-u)\right) + 2(s_2-u)s^2\left(3t^3u(t-11s_2) + 2t^2s_2^2(53u-77s_2)\right.$$

$$\left.+48s_2^4(u-s_2+3t) - 6t^4(2s_2-t) + 8ts_2^2(8t^2-15us_2)\right)$$

$$+ s(s_2 - t)\left(48 s_2^3\left(8uts_2 - 7ut(u+t) + (s_2-u)^3 - 3ts_2(s_2-t)\right)\right.$$

$$+ 8u^2 ts_2(31 s_2 t + 12us_2 - 7ut) + 5u^2t^3(u - 9s_2) + 4t^4(4us_2 - u^2 - 3s_2^2)$$

$$+ t^5(11s_2 - 7u) + 4t^3 s_2^2(21u - 11s_2)\Big)\Big] - \frac{Q^2 - s}{Q^2 st}\left(u^2 + 6ut - 2us_2 + 3t^2\right)$$

$$+ 16\left(f_{s_2} + f_{st} - f_{M^2}\right)\frac{d_t}{t^4}\left[(2t^2 - ts_2 + s_2^2)(s_2 - t)^2 u\right.$$

$$\left. + (t^2 - ts_2 + s_2^2)^2 s - 5t^2 s_2^2(s_2 - t) + 3ts_2(s_2^3 - t^3) + t^5 - s_2^5\right]$$

$$+ 2f_{st}\frac{d_t}{t^2}\left[t^3 - 3t^2(3s_2 + s) + 4s_2^2(2t - s)\right] + 4f_{s_2}\left(sd_t - \frac{2u+t}{t}\right)$$

$$+ \frac{8u}{(s_2)_{A+}}\left(\frac{f_{st}}{2} - \frac{f_{su}}{4} - \frac{f_{tu}}{2} + 2f_{stu} - 2f_{M^2}\right) - 4f_{tu}(Q^2 - u)d_t$$

$$+ 4f_{stu}\left(sd_t - \frac{2u+t}{t}\right) + f_{su}\frac{2u}{t} + 32u\left(\frac{f_{s_2}}{s_2}\right)_{A+} + \frac{2f_{\lambda t}}{t}\left(ss_2 d_t + Q^2 - s_2\right)\Big\}$$

$$\mathcal{B}_{qg}^{L_2} = \frac{-1}{8st(s_2)_{A+}}\Big\{ \boldsymbol{H}_1^{(1,2)}(s_2 - u)^2\left[s_2(4Q^2 - 2s - 3t) - u(u+t)\right]$$

$$- \boldsymbol{H}_1^{(1,1)}\, 2(s_2 - u)\left[2s(ss_2 - u^2) + 4ss_2(2u + t - 2s_2)\right.$$

$$\left. - ut(s + 2u + t - 6s_2) + ts_2(t - 5s_2) + (2s_2 - u)^3\right]$$

$$- \boldsymbol{H}_1^{(1,0)}\, u\left[u\left(2s(2s + 2u + t) + u^2 + 3ut + 2t^2\right) - 4st^2\right]$$

$$- 2u\left(6st - u(2s + 2t + u)\right) + 4stu\left(2f_{tu} + f_{su} - 4f_{M^2}\right)\Big\}$$

$$+ \frac{-1}{16s}\Big\{ \boldsymbol{H}_1^{(2,0)}\left(4s^2 + (u - s_2)^2\right)(2Q^2 - u - s_2) - \boldsymbol{H}_1^{(2,3)}(s_2 - u)^3$$

$$- \boldsymbol{H}_1^{(2,1)}(s_2 - u)\left[4s^2 - 4s(s_2 - u) - (s_2 - u)(3u + 4t - 7s_2)\right]$$

$$- \boldsymbol{H}_2^{(1,1)}\, 4(s_2 - t)d_{st}\left[2s^2(2t - 3s_2) + s(Q^2 - s - s_2)(3t - 4s_2)\right.$$

$$\left. - (Q^2 - s - s_2)^2(s_2 - t)\right] + \boldsymbol{H}_1^{(2,2)}(s_2 - u)^2(2Q^2 + u - 3s_2)$$

$$+ \boldsymbol{H}_1^{(1,1)}\, 4sd_{st}\frac{(s_2 - u)}{t}\left[u(2Q^2 - t - u) + 2s(s_2 - t)\right]$$

$$+ \boldsymbol{H}_1^{(1,0)}\, 2s\left[6(u - 2s_2) - 4s + \frac{1}{s}\left(2t^2 + 4t(2u - 3s_2)\right.\right.$$

$$\left. + 5(3u^2 + 5s_2^2 - 6us_2) - us_2\right) + \frac{2}{t}\left(2s(5u - s_2) + 2us_2 + 11(u - s_2)^2\right)$$

$$\left. + \frac{1}{st}\left(4u(2u^2 + 9s_2^2) - 5s_2(4s_2^2 + 5u^2)\right)\right]$$

$$- \boldsymbol{H}_1^{(1,0)}\ 4s\frac{d_{st}}{t}(4Q^2 - 3u - t)\left(2s(s_2 - t) + u(2Q^2 - u - t)\right)$$

$$+ \frac{4}{t^2}\left[-s^2\left(3s_2 + (9u - s_2)td_u\right) + 2d_u\, ts\left(u(u - 5s_2 + 4t) + 2s_2(2s_2 - t)\right)\right.$$

$$\left.- t\left(t^2 + 6u(t + u) + s_2(10s_2 - 7t - 13u)\right)\right]\Bigg\}$$

$$- \Bigg\{ - \frac{1}{t}d_{st}d_u\left((2u - s_2)(s_2 - t)s - s^2 u\right) + \frac{1}{2t}d_{st}(u - t)u + s(s_2^2 + t^2)\frac{d_t}{2t^2}$$

$$- s_2 d_u - (f_{s_2} - f_{M^2})\left(\frac{s}{2t^2}(s_2^2 + t^2)d_t + s_2 d_u\right) + f_{tu}\left(\frac{su}{t}d_{st} - 1\right)$$

$$- f_{su}\left(u\frac{s}{2t}d_{st} + \frac{1}{2}\right) + f_{\lambda t}\left(\frac{u}{2t}(s_2 - t)d_{st} + \frac{2s_2 - t}{2t}\right) + 2u\left(\frac{f_{s_2}}{s_2}\right)_{A+}\Bigg\}$$

$$\mathcal{A}_{qg}^{L_{12}} = \frac{-1}{8s}\Bigg\{$$

$$\boldsymbol{H}_1^{(2,0)}\ \left[32s^3 - 8s^2(s_2 - u) + (u + t - 2s_2)\left(2ss_2 - (s_2 - t)(s_2 - u)\right)\right]$$

$$+ \boldsymbol{H}_1^{(1,3)}\ d_{st}\frac{(s_2 - u)^3}{2s}(3s - 2s_2 + 2t) - \boldsymbol{H}_1^{(2,2)}\ (s_2 - u)^2(2Q^2 - t - s_2)$$

$$- \boldsymbol{H}_1^{(2,1)}\ (s_2 - u)\left[2ss_2 + (s_2 - u)(2Q^2 - u - s_2)\right] + \boldsymbol{H}_1^{(2,3)}\ (s_2 - u)^3$$

$$+ \boldsymbol{H}_2^{(1,3)}\ \frac{(s_2 - t)^3}{2s}d_{st}(3s - 2s_2 + 2t) - \boldsymbol{H}_2^{(2,2)}\ (s_2 - t)^2(2Q^2 - u - s_2)$$

$$- \boldsymbol{H}_2^{(2,1)}\ (s_2 - t)\left[2s(4s + s_2) + (s_2 - t)(2Q^2 - t - s_2)\right] + \boldsymbol{H}_2^{(2,3)}\ (s_2 - t)^3\Bigg\}$$

$$- \frac{d_{st}}{32s^2 t(s_2)_{A+}}\Bigg\{\ \boldsymbol{H}_1^{(1,1)}\ 2s(s_2 - u)\left[2s^3\left(u(u + s_2 - t) + 2t(2s_2 - t)\right)\right.$$

$$-s^2\left(2us_2(u + s_2 - 10t) + 6t^2(u + t) + s_2 t(16s_2 - 17t)\right)$$

$$+st(2t^2(4s_2 - t - 2u) - 20s_2 t(s_2 - u) + s_2^2(17s_2 - 26u) + u^2(9s_2 - 2t))$$

$$\left.+2s_2 t(s_2 - u)(s_2 - t)(u - 3s_2 + 2t)\right]$$

$$- \boldsymbol{H}_1^{(1,0)}\ \left[8s^4\left(u^2 + s_2(7Q^2 + 10t - 5s)\right)\right.$$

$$- 4s^3\left(u^2(u - 4s_2 - t) + 4s_2(5s_2^2 - 2s_2 t - 2t^2) + us_2(6s_2 - 11t)\right)$$

$$+ 2s^2\left(u^2(20s_2^2 - 5s_2 t - 2t^2 - 3s_2 u - 2tu) - 14us_2(6s_2^2 - 8s_2 t + 3t^2)\right.$$

$$\left.+ 29s_2 t^2(7s_2 - 2t) + 50s_2^3(2s_2 - 5t) + s_2^2(8s_2^2 + 3t^2)\right)$$

$$+ ss_2(u - 2s_2 + t)\left(6s_2^2(8s_2 - 19t) + 10s_2 t(7u + 10t) - 4u(7t^2 + 11s_2^2)\right)$$

$$- 3t(3u^2 + 11t^2) + 10u^2 s_2\Big) - 4(s_2 - t)^2 s_2 t(s_2 - u)(u - 2s_2 + t)\Big]$$

$$+ \boldsymbol{H}_2^{(1,1)}\; 2(s_2 - t)\Big[2s^3\left(2t(u + 3s_2 + t) + s_2(3u + 2s)\right)$$

$$- s^2\Big(2us_2(2u - 5s_2 + 10t) - 2ut(u + 4t) + 4s_2^2(5s_2 - 6t) + t^2(17s_2 - 6t)\Big)$$

$$+ s\Big(9us_2t(4s_2 - u) + 2ut^2(u + 2t) + 16s_2^3(s_2 - u) + 2us_2(2us_2 - 11t^2)$$

$$- 2t^3(8s_2 - t) + s_2t(34s_2t - 35s_2^2)\Big) + 2s_2t(s_2 - t)^2(2u - 3s_2 + t)\Big]$$

$$+ \boldsymbol{H}_1^{(1,2)}\; (s_2 - u)^2\Big[-2s^2(s_2 - 2t)(u - 2s_2 + t)$$

$$+ s\Big(2s_2^2(u + 9t) - s_2t(7t + 25u) - 4(s_2 - t)^3 + 4ut^2\Big)$$

$$+ 4s_2t(s_2 - t)(2u - 3s_2 + t)]$$

$$+ \boldsymbol{H}_2^{(1,2)}\; (s_2 - t)^2\Big[2s^2\left(s_2(u - 2s_2 - t) - 2t(u + t)\right) - s\Big(4t^2(u + t)$$

$$+ 2s_2^2(u - 2s_2) - s_2t(7u + 6s_2 + t)\Big) + 4s_2t(s_2 - t)(2t - 3s_2 + u)\Big]\Bigg\}$$

$$+ \frac{1}{48st^3} d_{st} d_t^2 \Big\{12s^3\Big(2t^3(t + 5s_2) + s_2^3(43t - 24s_2) - 19t^2s_2^2\Big)$$

$$-12s^2(s_2 - t)\Big(3t^3(3t + 4s_2 - 2u) - 3t^2s_2(13s_2 - 7u) + 43ts_2^2(2s_2 - u)$$

$$-24s_2^3(2s_2 - u)\Big) + 2s(s_2 - t)^2\Big(15t^3(5t - u) - 8t^2s_2(7t + 12s_2)$$

$$+3t^2u(36s_2 + u) + 2s_2^2(s_2 - u)(129t - 72s_2)\Big)$$

$$+t^2(s_2 - t)^2(u + t - 2s_2)\left(6s_2(4s_2 - u) + t(49t - 68s_2 + u)\right)\Big\}$$

$$+ \frac{d_t}{12Q^2s^2t^3}\Big\{3s_2s^4(9t - 16s_2)(2t - 3s_2)$$

$$+3s^3\Big(6s_2^3(33t - 16(s_2 - u)) + 3t^3(5s_2 - 3u) + 2tus_2(39t - 79s_2) - 121t^2s_2^2\Big)$$

$$+s^2\Big(3t^4(12t + 5u) - t^3s_2(79t + 89s_2) + 3t^3u(44s_2 - 9u) + 15t^2s_2^2(27s_2 - 39u)$$

$$+144s_2^3(s_2 - u)^2 + 9ts_2u^2(20t - 33s_2) - 3ts_2^3(139s_2 - 238u)\Big)$$

$$-st^3(s_2 - t)\Big(14s_2(s_2 - t) + 4u(2t - 5s_2) + t^2 + 7u^2\Big)$$

$$-t^3(t - 2s_2 + u)^2(s_2 - t)(t + u - s_2)\Big\}$$

$$- \Bigg\{ \frac{u}{(s_2)_{A+}}\Bigg(2f_{M^2} - 2f_{stu} - \frac{f_{st}}{2} - \frac{u}{2s}f_{tu} + \frac{u + 2t}{4t}f_{su} + \frac{u}{4t}\Bigg)$$

$$+ 2\left(f_{s_2} - f_{M^2}\right)\frac{d_t^2}{t^3}\Big[(s_2 - t)\Big((s_2 - t)^3u - (s_2 - t)^2(t^2 + s_2^2) - ts_2^2(t - u)\Big)$$

$$+ s(t^2 - ts_2 + s_2^2)^2\Big] - \frac{f_{s_2} + f_{stu}}{2t}(-utd_t + s - 2t + s_2)$$

$$- 4u\left(\frac{f_{s_2}}{s_2}\right)_{A+} - \frac{f_{tu}}{2s}d_t u(Q^2 - u) - \frac{f_{su}}{4t}(Q^2 - s)$$

$$+ \frac{f_{st}}{4t^3}d_t\Big(4(s_2 - t)(2ss_2^2 + st^2 - 5s_2t^2) + 8s_2^2(s_2 - 2t)(u - s_2)$$

$$+ ut^2(20s_2 - 11t) - 2t^3(4t - s)\Big) + \frac{f_{\lambda t}}{4st}\Big[s^2(3s_2 - t)d_t + u(3t - 4s_2)$$

$$+ s(-6s_2 + 3t + 2u + us_2d_t) + u^2 + d_t\left(4s_2^3 - 2t(t^2 - 4s_2t + 5s_2^2)\right)\Big]\Big\}$$

$$\mathcal{B}_{qg}^{L_{12}} = \frac{-d_{st}}{8st(s_2)_{A+}}\Big\{ \boldsymbol{H}_1^{(1,0)}\ \Big[4s^3\left((u + s_2)^2 + 2s_2(2s_2 - 3t)\right)$$

$$+ 2s^2s_2\left(s_2u + 12tu + 8u^2 - 31s_2^2 + 43s_2t - 14t^2\right) - 2s^2u(4t^2 + 3tu + 2u^2)$$

$$- s\Big(5s_2^2u(10s_2 - 11t) + 8t^3u + (7s_2^2 - 18s_2t + 10t^2)u^2$$

$$- u^3(8s_2 - 5t - u) - 6s_2^2(13s_2^2 + 21t^2) + 4s_2t(44s_2^2 + 7t^2)\Big)$$

$$+ (s_2 - t)\Big(11s_2^2(4us_2 - 7ut + 8s_2t) + 14ts_2(u^2 + t^2) - 4s_2^2(9s_2^2 + 16t^2)$$

$$+ 2us_2(15t^2 - u^2) + u^2(u^2 + ut - 13s_2^2)\Big)\ \Big]$$

$$+ \boldsymbol{H}_1^{(1,1)}\ 2(s_2 - u)\Big[2s^3s_2 - s^2\left(3s_2(2s_2 - u) + u(2u + t) - 5s_2t\right)$$

$$+ s\left(s_2(u - s_2)^2 - u(s_2 + t)^2 + s_2^3 - 2u^2t + s_2t(4s_2 - t)\right)$$

$$+ (s_2 - u)(u - 3s_2 + 2t)ts_2\Big] - \boldsymbol{H}_1^{(1,2)}\ (s_2 - u)^2\big[2ss_2(s - 3s_2)$$

$$- su(u - 4s_2 + t) + (us_2 - ut - 2s_2^2)(u - 2s_2 + t)\big]\ \Big\}$$

$$- \frac{1}{8st}\Big\{\ \boldsymbol{H}_1^{(2,0)}\ t\left[-8s^3 + (s_2 - t)(4s^2 + (s_2 - u)^2) - 2ss_2(s_2 - u)\right]$$

$$+ \boldsymbol{H}_1^{(2,1)}\ t(s_2 - u)\left[4s^2 - 2s(2s_2 - u) - (s_2 - u)(u - 3s_2 + 2t)\right]$$

$$- \boldsymbol{H}_1^{(2,2)}\ t(s_2 - u)^2(2Q^2 - t - s_2) - \boldsymbol{H}_2^{(1,2)}\ d_{st}2t(s_2 - t)^2(Q^2 + s - s_2)$$

$$- \boldsymbol{H}_2^{(1,1)}\ 2d_{st}(s_2 - t)\Big[6s^2(s_2 - 2t) + 2s\left(s_2(2u - 4s_2 + 11t) - t(3u + 5t)\right)$$

$$+ (u - 2s_2 + t)\left(s_2(u - 2s_2 + 7t) - 2t(u + 2t)\right)\Big] + \boldsymbol{H}_1^{(2,3)}\ t(s_2 - u)^3\ \Big\}$$

$$- \Big\{\ \frac{1}{(s_2)_{A+}}\left(2uf_{M^2} + \frac{u^2}{s}f_{tu} - u\left(1 + \frac{u}{2t}\right)f_{su} - \frac{u}{4st}(2su - 6st - u^2)\right)$$

$$
- 2u \left(\frac{f_{s_2}}{s_2} \right)_{A+} + (f_{s_2} - f_{M^2}) \frac{2u - s_2}{u} + f_{su} \left(2 - \frac{s_2}{u} - (ss_2 - u(s-u)) \frac{d_{st}}{2t} \right)
$$

$$
+ f_{tu} \frac{d_{st}}{st} \left[s^2(Q^2 + t) + s \left((s_2 - u)^2 + t(Q^2 - s - s_2) \right) + ut(s_2 - t) \right]
$$

$$
+ f_{\lambda t} \frac{d_{st}}{2st} \left[u^2(s_2 - t) + us_2(3s - 4s_2 + 7t) - ut(2s + 3t) + (3s_2 - t)s^2 \right.
$$

$$
\left. - 3(2s_2 - t)(s_2 - t)s + 2s_2^2(2s_2 - 5t) + 2t^2(4s_2 - t) \right]
$$

$$
+ \frac{d_{st}}{4st} \left[-2s^3 d_t^2 \left(5s_2(s_2 - t) + 2t^2 \right) - 4s^2 d_t \left(us_2 + ut - 2s_2^2 + t^2 \right) \right.
$$

$$
- s \left(6u(u - t) + 4t(2s_2 - t) - 7us_2 \right) + (s_2 - t) \left(s_2(14t - 2s_2 - 3u) \right.
$$

$$
\left. \left. - 6t(u + t) + 4u^2 \right) \right] - \frac{u + t}{2Q^2} + 2 + \frac{s}{t} d_t^2 (s_2^2 + t^2) \Big\}
$$

H.5 Diagrams $|\sum_{i=1}^{8} I_i|^2$ for $Gq \to VqG$

$$
\begin{aligned}
C_{i,Gq}^{U+L}(s, t, u, Q^2) &= C_{i,qG}^{U+L}(s, u, t, Q^2) \\
C_{i,Gq}^{L_1}(s, t, u, Q^2) &= C_{i,qG}^{L_2}(s, u, t, Q^2) \\
C_{i,Gq}^{L_2}(s, t, u, Q^2) &= C_{i,qG}^{L_1}(s, u, t, Q^2) \\
C_{i,Gq}^{L_{12}}(s, t, u, Q^2) &= C_{i,qG}^{L_{12}}(s, u, t, Q^2)
\end{aligned}
$$

for $i = 1, 2$

H.6 Diagrams $|\sum_{i=1}^{8} J_i|^2$ for $GG \to Vq\bar{q}$

The factorization terms for this class of diagrams are listed in sec. 6.3. Due to the length of the C_{GG}-contributions we further divide them into terms proportional to C_F and N_C.

$$C^{\beta}_{GG}(s,t,u,Q^2) = \mathcal{A}^{\beta}_{GG}\, N_C + \mathcal{B}^{\beta}_{GG}\, C_F$$

The results for $\mathcal{A}^{U+L}_{GG}$ and $\mathcal{B}^{U+L}_{GG}$ have already been published in [1] (eq. A.4) [2] (eq. A.13). We do not list them here.

$$\begin{aligned}
\mathcal{A}^{L_1}_{GG} = & \frac{-d_{st}}{8s^2}\Big\{ \boldsymbol{H}_1^{(1,1)}\, s(s_2-u)\left[s(14Q^2-u+t-10s_2)-u^2+t^2\right.\\
& \left.-2(s_2-u)(6t-5s_2)\right] + \boldsymbol{H}_2^{(1,1)}\, s(s_2-t)\left[s(10s+3u+21t-8s_2)-(u-t)^2\right.\\
& \left.-12t(s_2-t)-2s_2(s_2-u)\right]\\
& -\boldsymbol{H}_1^{(1,2)}\,(s_2-u)^2 d_s\left[s^2(14s+21u+25t-36s_2)-4(s_2-t)(s_2-u)^2\right.\\
& \left.+s\left(4u^2+11t^2-3u(10s_2-7t)+2s_2(13s_2-16t)\right)\right]\\
& -\boldsymbol{H}_2^{(1,2)}\, s(s_2-t)^2 d_s\left[2s_2(s_2-4s-u-4t)+t(5t+11s+3u)+3s(2s+u)\right]\\
& +\boldsymbol{H}_1^{(1,0)}\, d_s\left[+4s^3\left(s_2(7s_2+2t)+u(2u+12s_2-19t)-30(s_2-t)^2\right)\right.\\
& -8s^4(4s+3u+13t-11s_2)+2s^2\left(14t^2(8s_2-3u)-5u^2(2s_2+t)+4u^3\right.\\
& \left.-4us_2(4s_2-19t)+22s_2^2(s_2-5t)-31t^3\right)-s\left(4s_2ut(21s_2-25t)\right.\\
& -4s_2u^2(6s_2+4t-5u)+tu^2(19t-7u)+6s_2^3(s_2-12t)+24s_2t^2(5s_2-3t)\\
& \left.\left.+t^3(31u+14t)-3u^4\right)-2(s_2-u)^4(s_2-t)\right]\Big\}\\
& -\frac{1}{8s^2}\Big\{ \boldsymbol{H}_1^{(2,0)}\,(s_2-u)(2s+u-s_2)\left[2s(s+u-s_2)+(s_2-u)^2\right]\\
& +\boldsymbol{H}_1^{(1,4)}\, 2(s_2-u)^4 d_s - \boldsymbol{H}_1^{(2,1)}\, 2(s_2-u)\left[2s^3-(2s+u-s_2)(s_2-u)^2\right]\\
& +\boldsymbol{H}_1^{(2,2)}\, 2s(s_2-u)^2(3s+2u-2s_2)-\boldsymbol{H}_1^{(2,3)}\, 2(s_2-u)^3(2s+u-s_2)\\
& +\boldsymbol{H}_1^{(2,4)}\,(s_2-u)^4+2d_{st}d_t^2 s\left[-(s_2-t)^2\left(4s_2(u+t-s_2)+u(u+t)-6t^2\right)\right.\\
& \left.-2s(s_2-t)\left(4s_2(2s_2-u)-5s_2t+9t^2\right)+4s^2\left(s_2(3s_2-2t)+3t^2\right)\right]\Big\}\\
& -\Big\{ f_{tu}\left(s_2d_t-\frac{s_2-u}{s}\right)-(f_{s_2}+f_{stu})\,td_t-f_{\lambda u}\frac{Q^2-t}{2s}\\
& +(f_{\lambda u}+f_{su})\frac{2Q^2-t}{2}d_s+(f_{\lambda t}+f_{st})\,d_sd_t\frac{t}{2}(2s+u-s_2)-f_{\lambda t}\frac{t}{s}\Big\}
\end{aligned}$$

$$
\begin{aligned}
\mathcal{B}_{GG}^{L_1} \;=\; & \frac{-d_{su}d_{st}}{4s}\Big\{ \boldsymbol{H}_1^{(1,1)}\,(s_2-u)\Big[2s^2(u-t) \\
& + s\left(3u(u-2s_2)+4t(s_2-2u)+5s_2^2-2t^2\right)+2(s_2-u)(s_2-t)(u-2s_2+t)\Big] \\
& + \boldsymbol{H}_1^{(1,2)}\,d_s s(s_2-u)^2\Big[2s(2s+u+3t-4s_2)+4(s_2-t)^2+(s_2+3u)(t-u)\Big] \\
& - \boldsymbol{H}_1^{(1,3)}\,s(s_2-u)^3-\boldsymbol{H}_2^{(1,1)}\,(s_2-t)\Big[2s^2(6s+7u-12s_2+5t) \\
& - s\left(6u(2s_2-u)+2t(5s_2-8u)-7s_2^2+3t^2\right)-2(s_2-u)(s_2-t)(u-2s_2+t)\Big] \\
& + \boldsymbol{H}_2^{(1,2)}\,s(s_2-t)^2d_s\left[2s(6s+7u-12s_2+5t)+3u(2u-5s_2)+t(7u-t)\right. \\
& \left.+3s_2(4s_2-3t)\right]-\boldsymbol{H}_2^{(1,3)}\,s(s_2-t)^3 \\
& + \boldsymbol{H}_1^{(1,0)}\,d_s\Big[8s^4(2s+5u-11s_2+6t)-4s^3\left(u(41s_2-35t-11u)-38s_2^2\right. \\
& \left.+3t(17s_2-4t)\right)+4s^2\left(7s_2^2(6u+8t)-5u^2(5s_2-u)-s_2t(31t+82u)\right. \\
& \left.+27t^2u+2t(13u^2+2t^2)-22s_2^3\right)-s\left(4s_2^3(2s_2+u+7t)+13s_2t^2(7u-3s_2)\right. \\
& \left.-17tu(t^2+6s_2^2+u^2)-19u^2(s_2^2+2t^2)+9s_2(u^3+t^3)+t^4-u^4+83s_2tu^2\right) \\
& -2(s_2-t)(s_2-u)(u-2s_2+t)^3\Big]\Big\} \\
& -\Big\{\frac{\boldsymbol{H}_1^{(2,0)}}{2}\Big[2s^2+(s_2-u)(2s-s_2+u)\Big]-\boldsymbol{H}_1^{(2,1)}\,s(s_2-u) \\
& + \boldsymbol{H}_1^{(2,2)}\,\frac{(s_2-u)^2}{2}-(f_{s_2}-f_{M^2})\left[\frac{u^2+s_2^2}{u^2}+2(s_2^2+t^2)d_t^2\right] \\
& + 2f_{s_2}t(s_2-t)d_t^2-2f_{tu}d_{su}d_{st}d_t\Big[ss_2(Q^2-s_2)+(s_2^2-t^2)(s_2-u)\Big] \\
& + 2f_{stu}td_t+f_{su}d_s\frac{d_{st}}{u^2}\Big[ss_2^2(4s_2-3t-2s-u)+su^2(u-2s_2) \\
& -(u^2-s_2^2+s_2t)(u+t-2s_2)s_2\Big]-f_{\lambda u}d_sd_{su}\Big[(Q^2-s_2)(s_2-u)+ss_2\Big] \\
& -(f_{\lambda t}+f_{st})\,t\Big[-sd_t(d_s-d_{st})+s^2d_{st}d_sd_t-2d_{st}\Big] \\
& - f_{st}\left[2td_{su}+(s-s_2)d_{st}-t(s+2t-2u)d_{su}d_{st}\right] \\
& - d_{su}\frac{d_{st}}{3s}\Big[12s^2(Q^2-s_2)+s^2s_2+s\left(2u(u-4s_2+11t)+5s_2^2-t(14s_2+t)\right) \\
& +3(2s_2-t-u)(s_2-u)(s_2-t)\Big]+\frac{s_2-u}{u^2Q^2}\left(su+ss_2-s_2^2+s_2t\right)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{GG}^{L_{12}} \;=\; & \frac{-d_{st}d_{su}}{16s^2}\Big\{ \boldsymbol{H}_1^{(1,0)}\ \Big[16s^5 - 52s^4(3s_2 - t - u) \\
& + 2s^3\Big(3s_2(57s_2 - 49t) + 28(u^2 + t^2) - u(157s_2 - 77t)\Big) \\
& - 2s^2\Big(-5ut(12t + 13u) - 11(t^3 + u^3) - 33s_2u(7s_2 - 3u) \\
& + 4s_2t(21t + 64u) + s_2^2(145s_2 - 206t)\Big) + s\Big(u^4 + t^4 - 4u^3(9s_2 - 8t) \\
& + 8u^2s_2(21s_2 - 29t) - 10s_2^3(23u + 19t) + 7ut^2(10u - 26s_2) \\
& + 2s_2t^2(59s_2 - 13t) + 2s_2^2(49s_2^2 + 193ut) + 22ut^3\Big) - 10(s_2 - u)^3(s_2 - t)^2\Big] \\
& - \boldsymbol{H}_1^{(1,1)}\ 2(s_2 - u)\Big[40s^4 + s^3(109u - 148s_2 + 65t) + s^2\big(u(73u - 257s_2) \\
& + 4(34ut + 45s_2^2) + t(29t - 161s_2)\big) + s\Big(11t(7u^2 - 3s_2t) + 34s_2^2(3t + 5u) \\
& + 3(t^3 + 2u^3 - 58uts_2) - 97u^2s_2 + 26ut^2 - 80s_2^3\Big) + 8(s_2 - u)^3(s_2 - t)\Big] \\
& - \boldsymbol{H}_2^{(1,1)}\ 2s(s_2 - t)\Big[s^2(24s + 41u - 84s_2 + 53t) + s\big(u(21u - 97s_2) \\
& + 4(18ut + 23s_2^2) - t(113s_2 - 25t)\big) - 25s_2(u^2 + t^2) \\
& + 3u^2(u + 6t) + 6us_2(9s_2 - 13t) + t^2(29u - 2t) + 2s_2^2(29t - 16s_2)\Big] \\
& + 4s^2d_ud_t\Big[8s^2(s_2(u + t + s_2) - 3ut) - s\Big(3u(13t^2 + 17s_2^2) \\
& - 2s_2(2s_2^2 + 61ut) - u^2(31s_2 - 47t) + s_2t(43s_2 - 23t)\Big)\Big] \\
& + 2s\Big[-2s\Big(u(23u - 73s_2) + 64ut + 28s_2^2 - 3t(19s_2 - 5t)\Big) - u^2(u + 47t) \\
& - 8s_2^2(10u - 4s_2 + 8t) - t^2(31u - 34s_2 + t) + 2us_2(25u + 54t)\Big]\Big\} \\
& - \frac{1}{16s^2}\Big\{ \boldsymbol{H}_1^{(2,3)}\ 4(s_2 - u)^3(5s + 3u - 4s_2 + t) - \boldsymbol{H}_1^{(2,4)}\ 4(s_2 - u)^4 \\
& + \boldsymbol{H}_2^{(1,2)}\ s(s_2 - t)^2\Big[4d_s(6s - 4s_2 + 5t + 3u) + d_{su}d_{st}\Big(5(u^2 - t^2) \\
& - 2s(12s_2 - 5t - 7u - 6s) - 6u(3s_2 - 2t) + 6s_2(2s_2 - t))\Big] \\
& - \boldsymbol{H}_1^{(1,4)}\ 8(s_2 - u)^4d_s - \boldsymbol{H}_1^{(2,1)}\ 4(s_2 - u)\Big[-2s^2(4s + 5u - 10s_2 + 2t) \\
& + s(s_2 - u)(5u - 15s_2 + 6t) - (s_2 - u)^2(u - 4s_2 + 3t)\Big] \\
& - \boldsymbol{H}_1^{(2,2)}\ 4(s_2 - u)^2\left[s(10s + 10u - 15s_2) + 3t) - 3(s_2 - u)(u - 2s_2 + t)\right] \\
& + \boldsymbol{H}_1^{(1,2)}\ (s_2 - u)^2\Big[d_s\Big(- 36s(s - 2s_2) - 6st - 2su - 2u(6u - 31s_2 + 19t) \\
& - 60s_2^2 + 58s_2t - 10t^2\Big) + d_{su}d_{st}\Big(2s^2(29s + 46u - 81s_2 + 35t)
\end{aligned}
$$

$$+ s\left(31u^2 + 10s_2(15s_2 - 13t - 17u) + 104ut + 15t^2\right)$$

$$+ 2(18u - 23s_2 + 5t)(s_2 - u)(s_2 - t)\Big)\Big]$$

$$+ \boldsymbol{H}_1^{(2,0)}\ 4\left[2ss_2 - (s_2 - u)(s_2 - t)\right]\left[s(t-u) + (2s + u - s_2)^2\right]$$

$$+ \boldsymbol{H}_1^{(1,0)}\ 2d_s\Big[4s^3\left(4(3s + 8u + 6t) - 57s_2\right) + 2s^2\Big(31(t^2 + 5s_2^2)$$

$$- 2s_2(82t + 97u) + u(77t + 47u)\Big) - s\Big(13u(13us_2 - 6t^2)$$

$$+ 3s_2t(47t + 86u) - 25t(9s_2^2 + t^2) + 4(32s_2^3 - 21u^2t) - 33u^3 - 251s_2^2u\Big)$$

$$+ (s_2 - u)^2(s_2 - t)(u - 6s_2 + 5t)\Big]\ \Big\}$$

$$- \frac{1}{4s}\Big\{\ \boldsymbol{H}_2^{(2,3)}\ (s_2 - t)^3 - \boldsymbol{H}_2^{(2,2)}\ (s_2 - t)^2(2Q^2 - u - s_2)$$

$$- \boldsymbol{H}_2^{(2,1)}\ (s_2 - t)\left[2ss_2 + (s_2 - t)(2s + 2u - 3s_2 + t)\right]$$

$$- 4s\left(f_{s_2} + f_{stu} - f_{tu}\right)\left(2utd_td_u + s_2(d_t + d_u)\right) - 4f_{tu}(2s + u + t)$$

$$+ 2f_{\lambda u}d_sd_u\Big[2s^2(2s_2 - u) - s\left(-4s_2(4u + t) + 3(2u^2 + 3s_2^2 + ut)\right)$$

$$+ (4u - 3s_2 + t)(s_2 - u)(u - 2s_2 + t)\Big]$$

$$+ 2f_{\lambda t}d_sd_t\Big[2s^2(2s_2 - t) - s\left(-4s_2(4t + u) + 3(2t^2 + 3s_2^2 + tu)\right)$$

$$+ (4t - 3s_2 + u)(s_2 - t)(t - 2s_2 + u)\Big]$$

$$+ 2f_{su}sd_sd_u\left(3u^2 + s_2(2s - u + t - s_2)\right)$$

$$+ 2sf_{st}d_sd_t\left(3t^2 + s_2(2s - t + u - s_2)\right)\ \Big\}$$

$$\mathcal{B}_{GG}^{L_{12}} \ = \ \Big\langle\ \frac{-d_{su}d_{st}}{2s}\Big\{\ \boldsymbol{H}_1^{(1,0)}\ sd_s\Big[-4s^3(3u - 5s_2)$$

$$-2s^2\left(19u(u - 4s_2) + 27ut + 34s_2^2\right) - 2u^4 - 28s_2^4$$

$$- 2s\left((142s_2^2 + 61tu)u - 77s_2u^2 + 13u^3 - s_2(101ut + 38s_2^2)\right)$$

$$+ (144s_2^3 + 191s_2ut)u - 33t^2u^2 - 117s_2^2u^2 - 155s_2^2ut + 33t^3(s_2 - u)\Big]$$

$$+ \boldsymbol{H}_1^{(1,3)}\ s(s_2 - u)^3 + \boldsymbol{H}_1^{(1,1)}\ (s_2 - u)\Big[2s^2(2s + u + 3t - 5s_2)$$

$$- s(3u^2 + 4us_2 - 7s_2^2 - 2t(7u - 9s_2 + 4t)) - (u - t)(u - 2s_2 + t)^2\Big]$$

$$- \boldsymbol{H}_1^{(1,2)}\ (s_2 - u)^2d_s\Big[2s^2(4s + 3u + 5t - 8s_2) + (t - u)(u - 2s_2 + t)^2$$

$$- s\left(4u^2 + us_2 - 5ut - 8s_2^2 + 15s_2t - 7t^2\right)\Big]\Big\}$$

$$- \left\{ \frac{-\boldsymbol{H}_1^{(2,0)}}{2}\left[4s(s-s_2) + s(u+t) + (s_2-t)(s_2-u)\right]\right.$$

$$- \boldsymbol{H}_1^{(2,2)}\frac{(s_2-u)^2}{2} + \frac{\boldsymbol{H}_1^{(2,1)}}{2}(Q^2+3s-s_2)(s_2-u)$$

$$+ \frac{1}{3Q^2}d_{su}d_{st}\Big[9s^3 + (30u - 29s_2)s^2 + s\left(19u^2 - 86us_2 + 37ut + 34s_2^2\right)$$

$$+ 62s_2^2u + 51t^2u - 59s_2ut - 41s_2t^2 + 7u^3 - 14s_2^3\Big]$$

$$- f_{tu}d_{su}d_{st}\left[2\left((s_2-t)^2 + 2ut\right) - 2sd_u\left(2ut + s_2(s_2-t)\right) + s^2d_ud_t(ut+s_2^2)\right]$$

$$+ (f_{s_2} - f_{M^2})\frac{1}{ut}\left[s_2\left(u^2d_u + t^2d_t\right) + d_ud_t\left(2u^2t^2 - s_2^3(u+t)\right)\right]$$

$$+ (f_{stu} + f_{s_2})\left[2utd_td_u + s_2(d_t+d_u)\right] + f_{su}d_sd_{st}\frac{d_u}{u}\Big[2s^2s_2(u-2s_2)$$

$$+ s\left(u^2(u-3s_2) - us_2(5s_2-3t) + 2s_2^2(4s_2-3t)\right)$$

$$- (u+t-2s_2)\left(u(u^2+us_2-ts_2+s_2^2) + 2s_2^2(t-s_2)\right)\Big]$$

$$+ f_{\lambda t}d_sd_{st}d_t\Big[2s^2(t-2s_2) + s\left(3t(2t+u) - 4s_2(4t+u) + 9s_2^2\right)$$

$$+ (u+4t-3s_2)(u+t-2s_2)(t-s_2)\Big]\Big\}\Big\rangle + \Big\langle u \leftrightarrow t\Big\rangle$$

Conclusions

We have calculated the angular decay distribution of the lepton pair decaying from W's produced in hadron hadron collisions at large transverse momentum. The general structure of the angular distribution is given by nine helicity cross sections. It is shown that one can measure six of these helicity cross sections without the complete reconstruction of the decay kinematics due to the unobservability of the neutrino. We show that large deviations from the $(1 + \cos\theta)^2$ distribution valid at low q_T are expected already at moderate transverse W momenta. We present the $O(\alpha_s^2)$ corrections to four parity conserving helicity cross sections. It appears that the NLO corrections to the angular coefficients A_0 and A_2 are not large (less that 8% for the sum of all contributions at $q_T = 200$ GeV) when they are normalized to the NLO rate. It is clear, that the $O(\alpha_s^2)$ results are more reliable than the $O(\alpha_s)$ results as they depend less on the renormalization and factorization scales. We have checked that the theoretical uncertainties due to the choice of the scales are negleglible for the predictions of the cross section ratios A_i.

We found that the $O(\alpha_s^2)$ contributions to the T−odd angular coefficients $A_5 - A_7$ are less than 2% even at large q_T.

The angular coefficient A_3 reaches about 0.2 at $q_T = 100$ GeV. This coefficient receives only contributions from the qG initiated process at LO and may therefore be used to extract the gluon structure function.

Acknowledgement:

I thank J. Körner and G. Schuler for many very helpful discussions. I would like to thank the DESY Theory Group and the FERMILAB Theory Group for hospitality.

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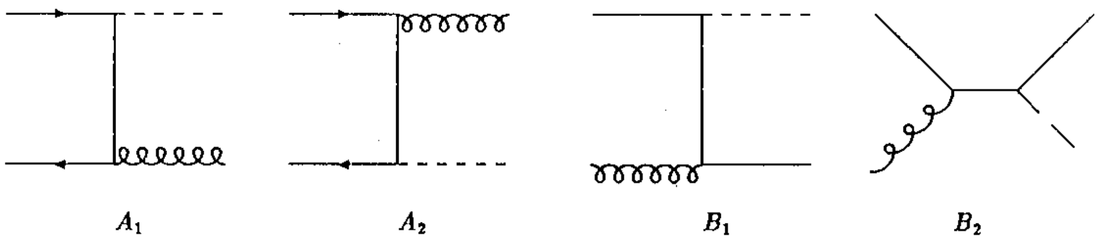

Fig. 1: Leading diagrams for $q\bar{q} \to GV$ and $qG \to qV$

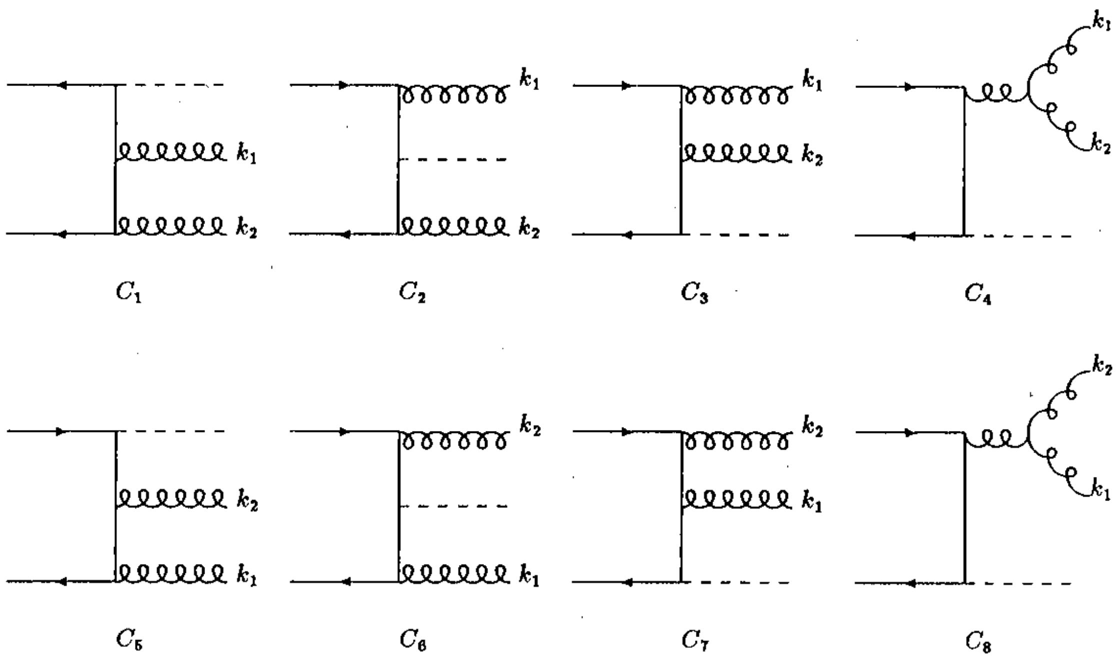

Fig. 2: Diagrams for $q\bar{q} \to GGV$

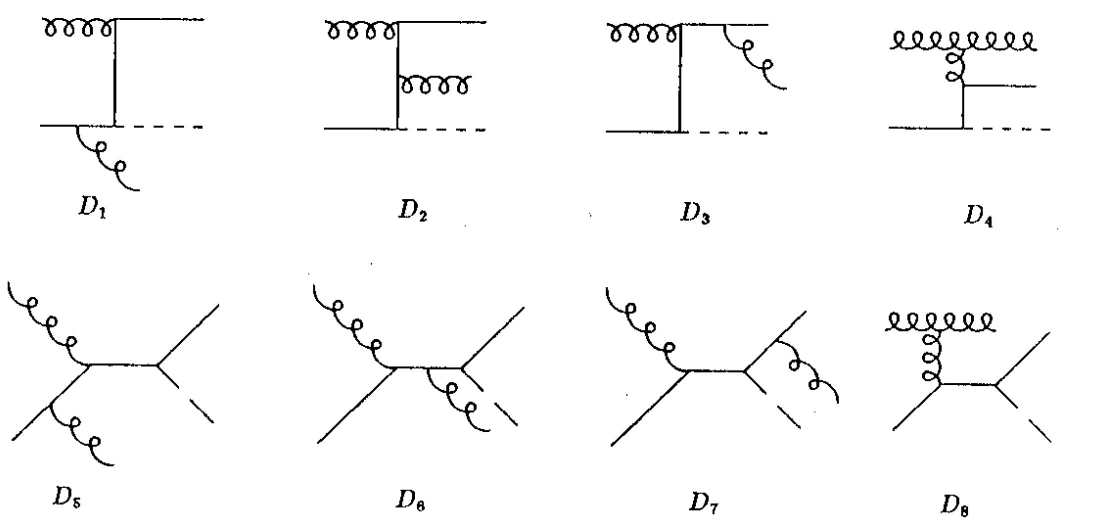

Fig. 3: Diagrams for $qG \to qGV$

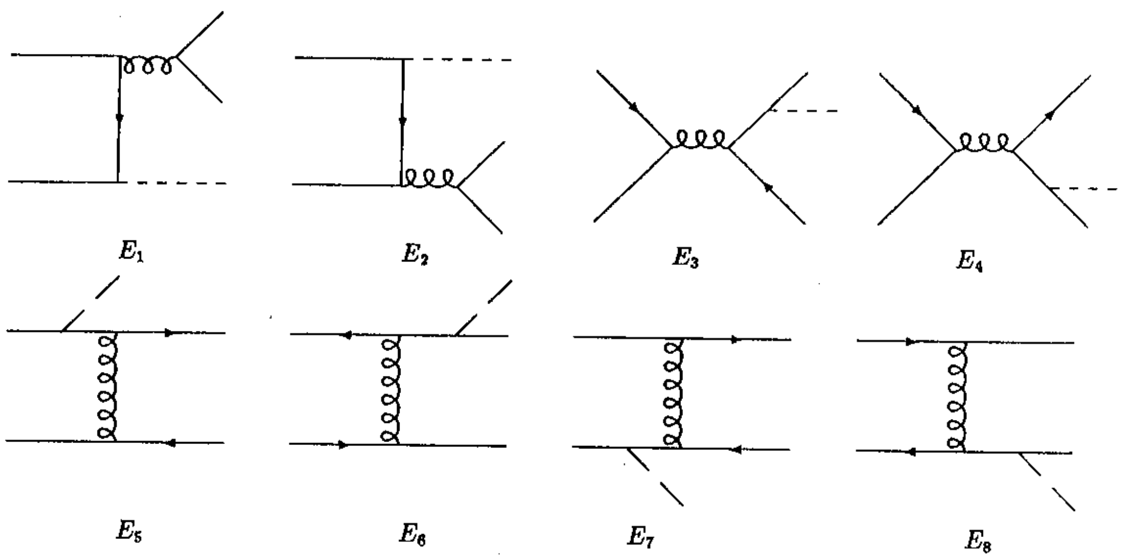

Fig. 4: Diagrams for $q\bar{q} \to q\bar{q}V$

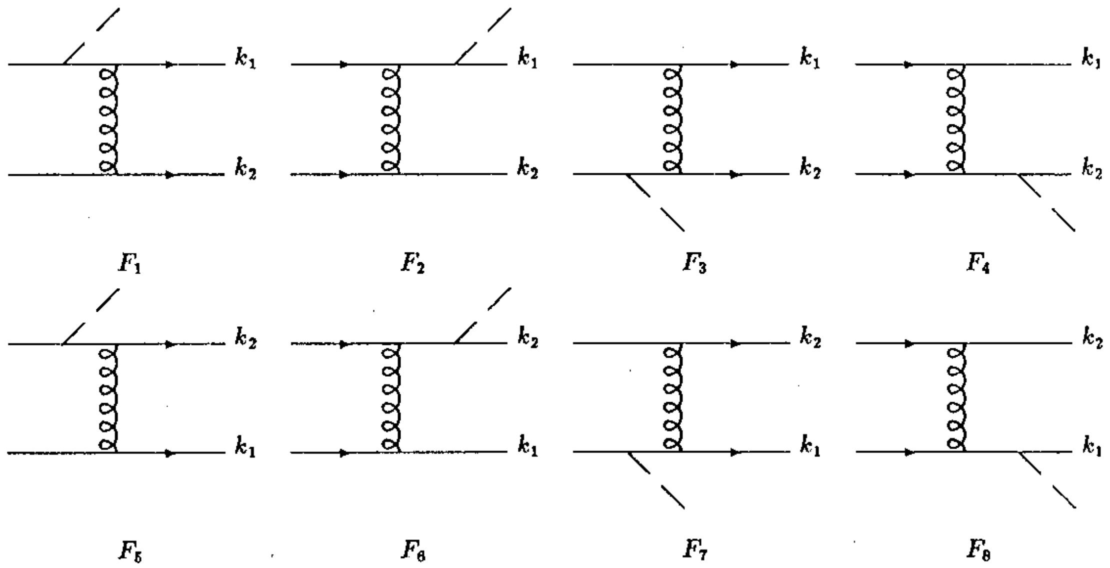

Fig. 5: Diagrams for $qq \to qqV$ and $\bar{q}\bar{q} \to \bar{q}\bar{q}V$

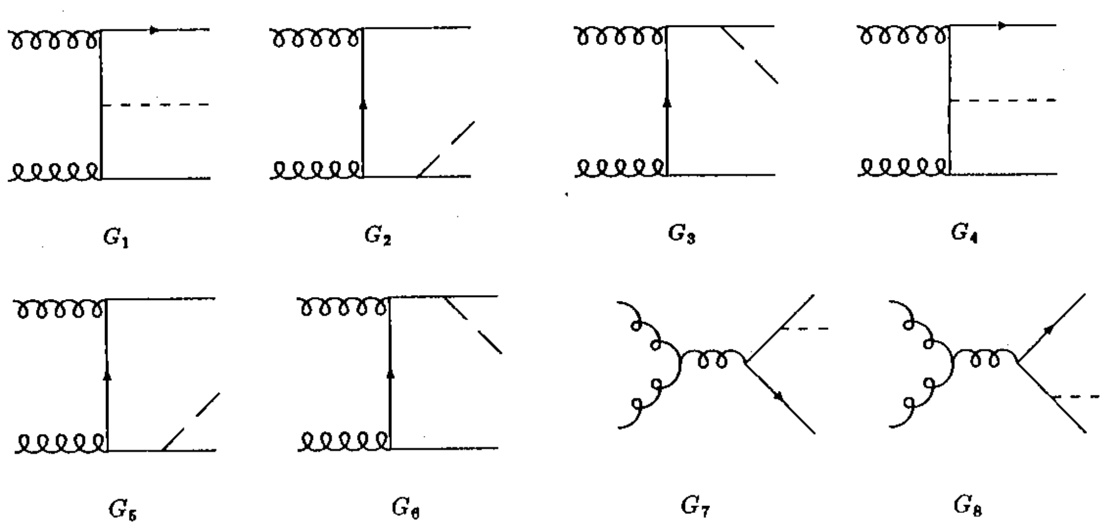

Fig. 6: Diagrams for $GG \to q\bar{q}V$

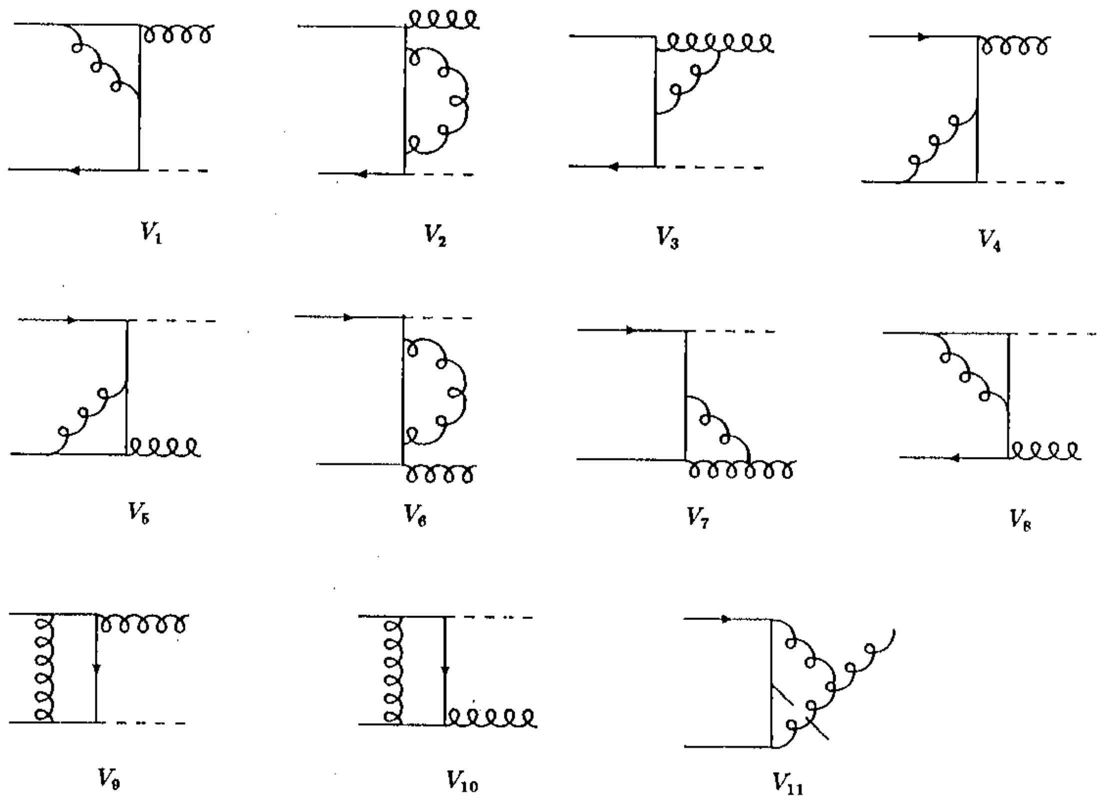

Fig. 7: Next-to-leading diagrams for $q\bar{q} \to GV$

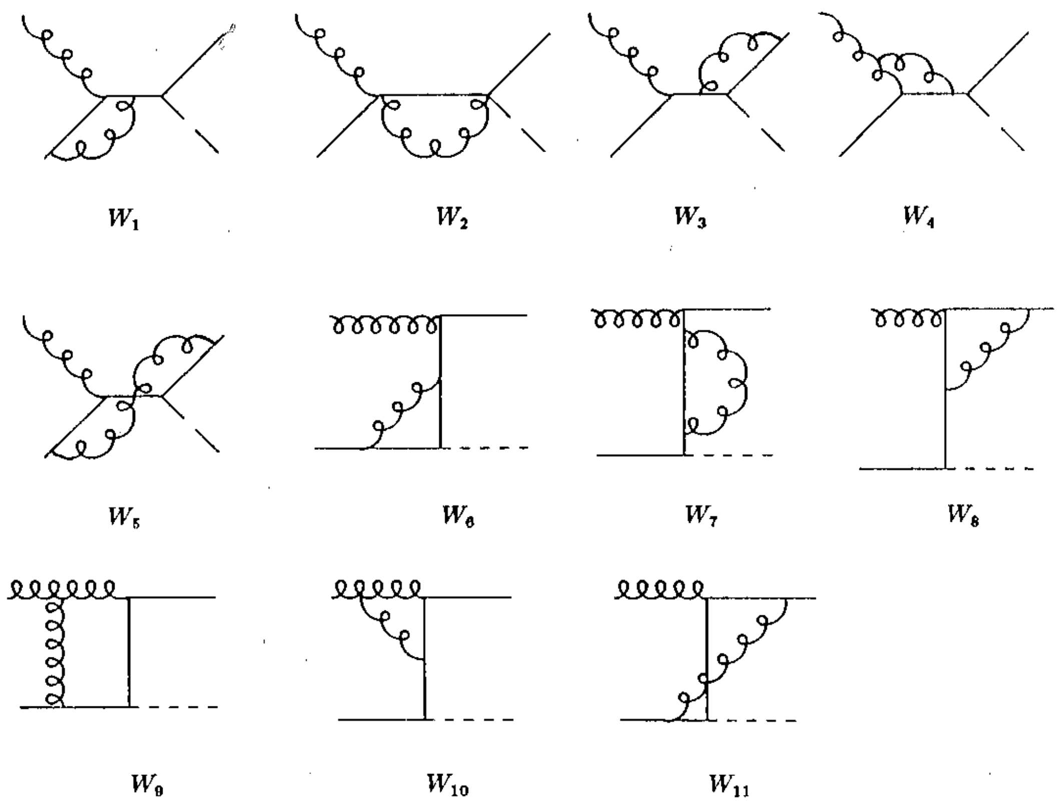

Fig. 8: Next-to-leading diagrams for $qG \to qV$

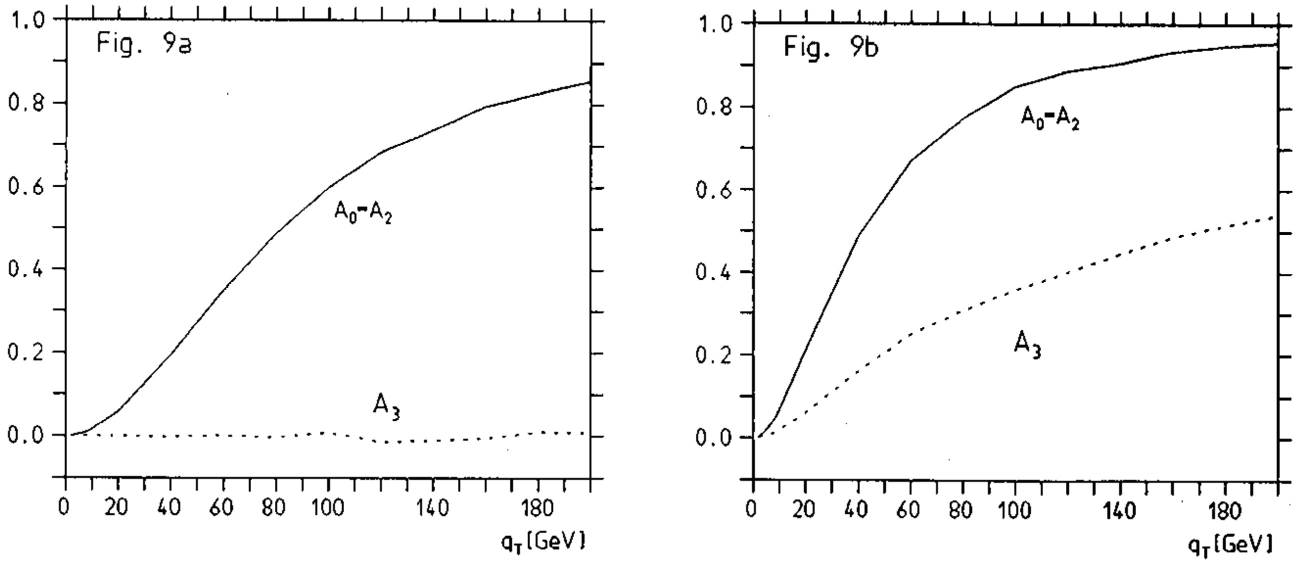

Figure 9:
Angular coefficients A_0, A_2 and A_3 as a function of q_T for $p\bar{p} \to W^+ + X$ at $\sqrt{s} = 1.8\,\text{TeV}$. Shown are the $O(\alpha_s)$ contributions from the $q\bar{q}$ initiated subprocess (fig. 9a: $[q\bar{q} \to G\,W^+]$) and the qG initiated subprocess (fig. 9b: $[qG \to q\,W^+]$).

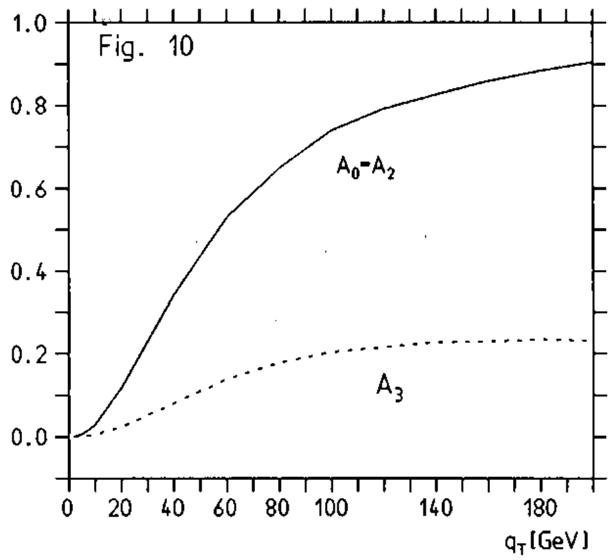

Figure 10:
Angular coefficients A_0, A_2 and A_3 as a function of q_T for $p\bar{p} \to W^+ + X$ at $\sqrt{s} = 1.8\,\text{TeV}$. Shown are the $O(\alpha_s)$ contributions from the $q\bar{q}$ and qG initiated subprocesses.

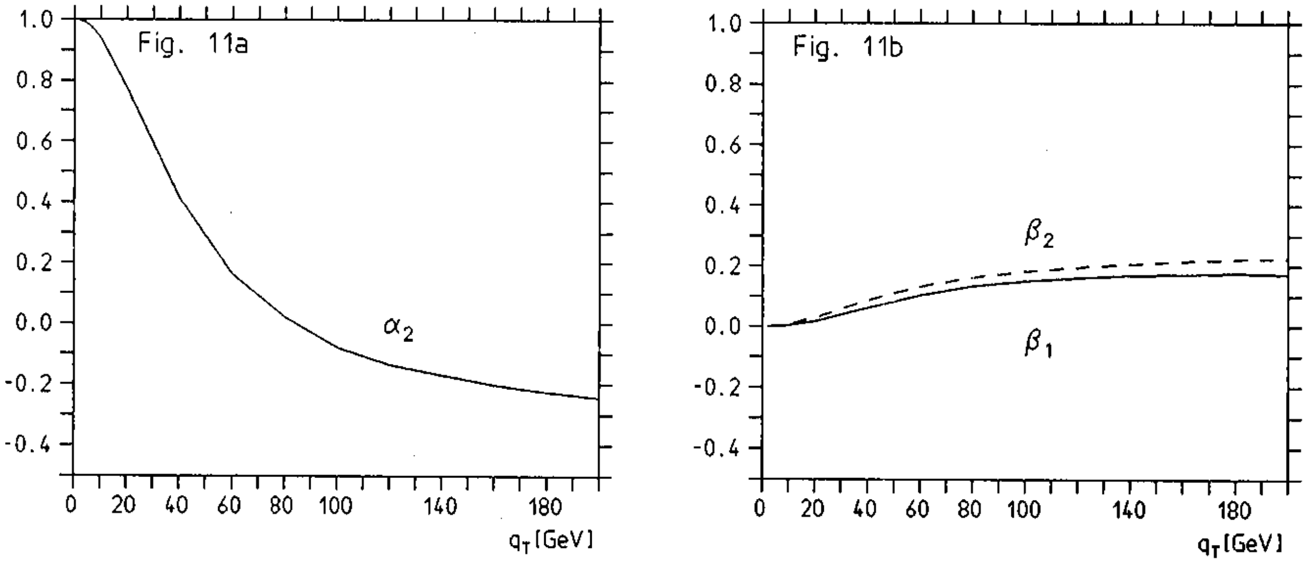

Figure 11:

Angular coefficients α_2 (fig. 11a) and β_1, β_2 (fig. 11b) as a function of q_T for $p\bar{p} \to W^+ + X$ at $\sqrt{s} = 1.8\,\text{TeV}$. Shown are the O($\alpha_s$) contributions from the $q\bar{q}$ and qG initiated subprocesses.

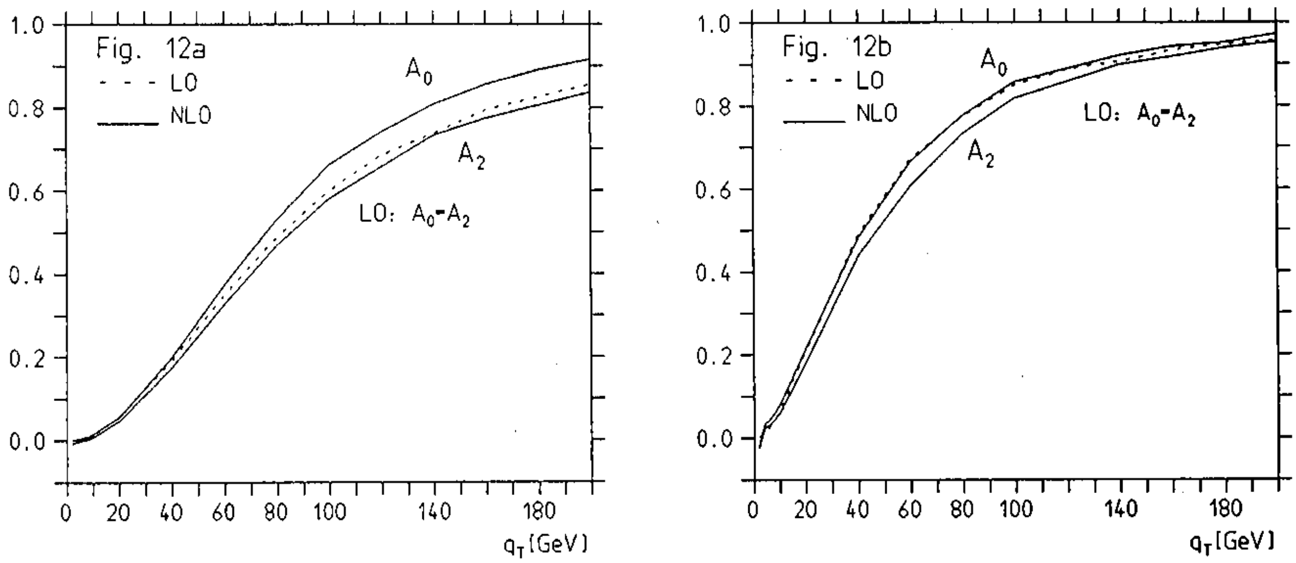

Figure 12:

Angular coefficients A_0 and A_2 as a function of q_T for $p\bar{p} \to W^+ + X$ at $\sqrt{s} = 1.8\,\text{TeV}$. Shown are the contributions from the $q\bar{q}$ initiated subprocesses (fig. 12a: $[q\bar{q} \to G\ (G)W^+] + [q\bar{q} \to q\bar{q}W^+]$) and the qG initiated subprocesses (fig. 12b: $[qG \to q\,(G)W^+]$) up to $O(\alpha_s^2)$. Dashed lines are the corresponding $O(\alpha_s)$ results.

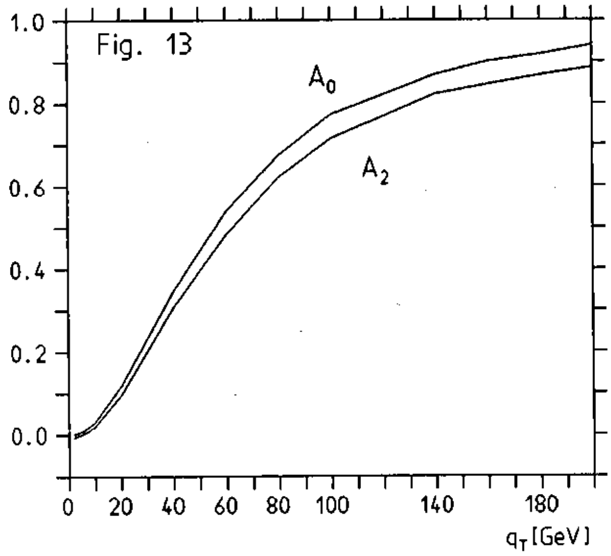

Figure 13:

Angular coefficients A_0 and A_2 as a function of q_T for $p\bar{p} \to W^+ + X$ at $\sqrt{s} = 1.8\,\text{TeV}$ at NLO including all parton processes of eqs. (23,28,29) at

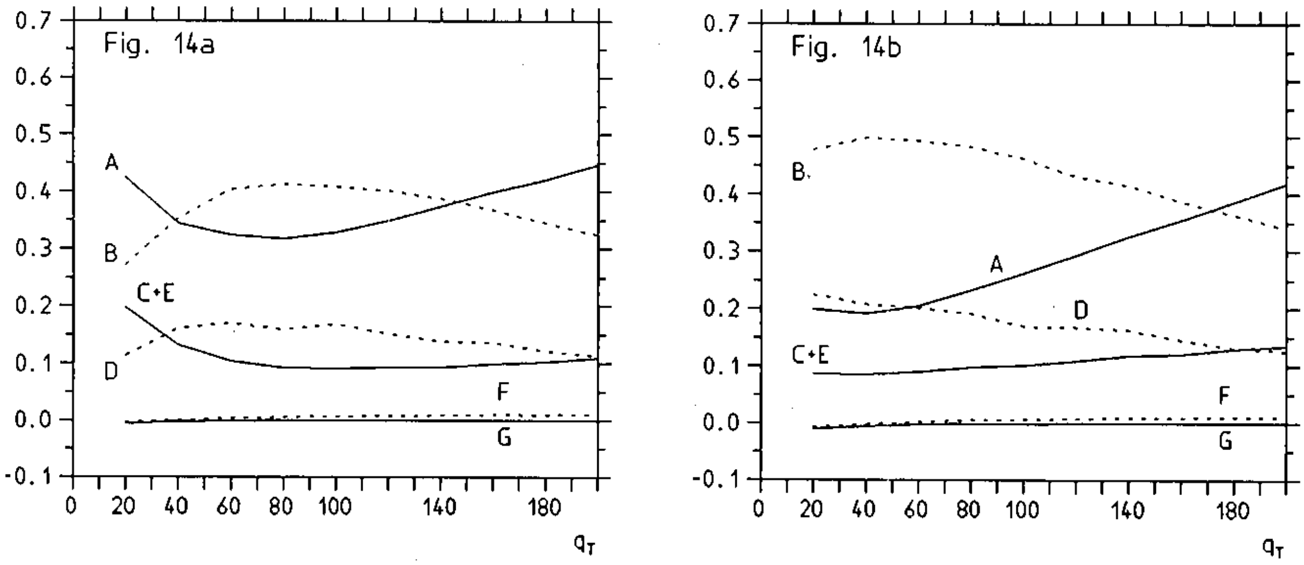

Figure 14:

Relative contributions to $d\sigma^{U+L}$ (fig. 14a) and $d\sigma^{L}$ (fig. 14b) for W^+ production at $\sqrt{S} = 1.8$TeV and $\mu^2 = (m_W^2 + q_T^2)/2$.

At LO:

(A) $[q\bar{q} \to GW^+]$ and (B) $[qG \to qW^+] + [\bar{q}G \to \bar{q}W^+]$.

At NLO:

(C+E) $[q\bar{q} \to GW^+ + GGW^+] + [q\bar{q} \to q\bar{q}W^+]$

(D) $[qG \to qW^+ + qGW^+] + [\bar{q}G \to \bar{q}W^+ + \bar{q}GW^+]$

(F) $[qq \to qqW^+] + [\bar{q}\bar{q} \to \bar{q}\bar{q}W^+]$

(G) $[GG \to q\bar{q}W^+]$.

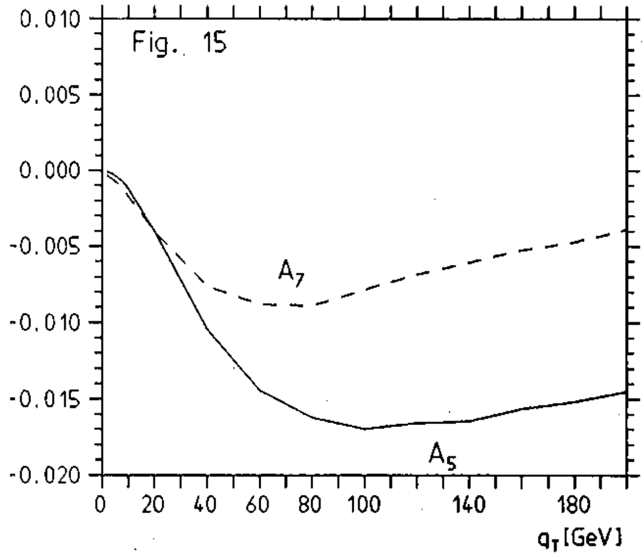

Figure 15:
T-odd angular distributions A_5 and A_7 as a function of q_T

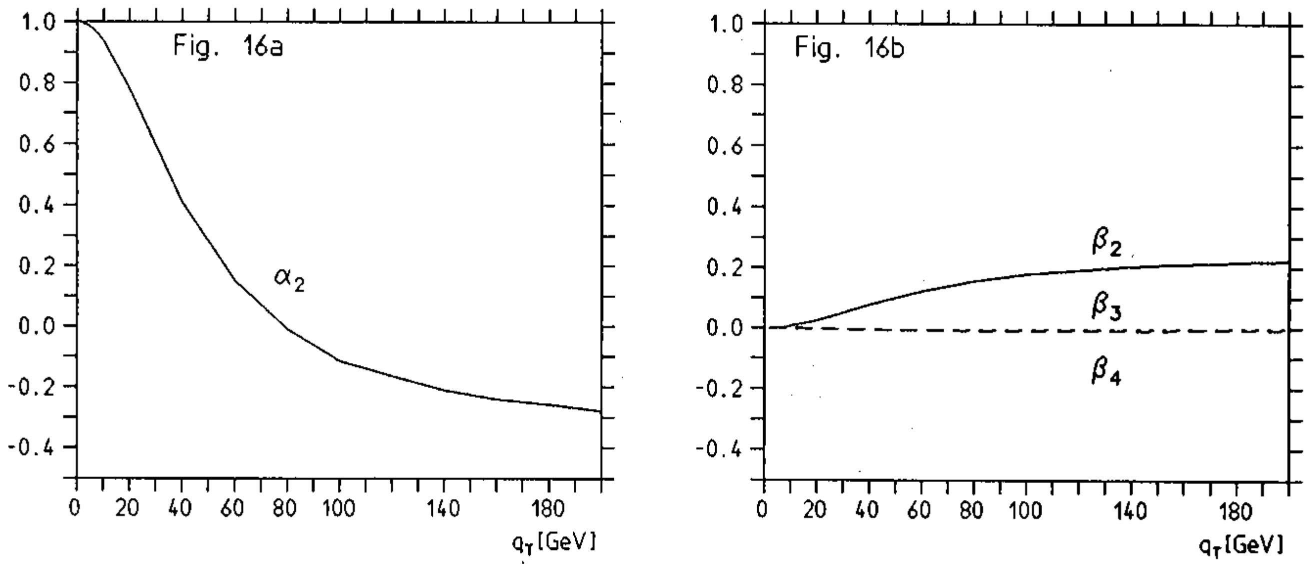

Figure 16:
Angular coefficients $\alpha_1, \beta_2, \beta_3$ and β_4 at $O(\alpha_s^2)$.