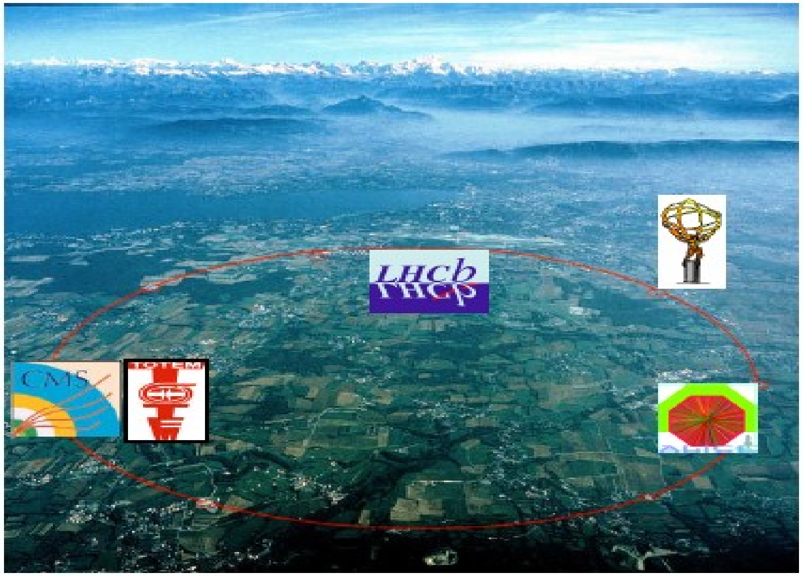
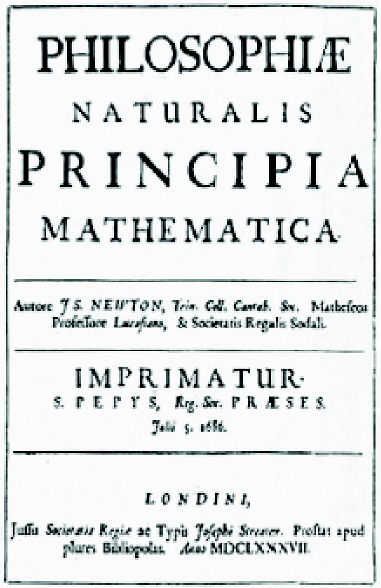
Physics Program

of the experiments at

Large Hadron Collider



Standard Model Higgs boson: addendum


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

Autore J. S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

Digression on the origin of Mass

- Gallilean and Newtonian concept of mass :

Inertial mass (F=ma) Gravitational mass (P=mg)

Single concept of mass

Conserved intrinsic property of matter where the total mass of a system is the sum of its constituents

- Einstein : Does the mass of a system depend of its energy content?

Mass = rest energy of a system or $m_0=E/c^2$

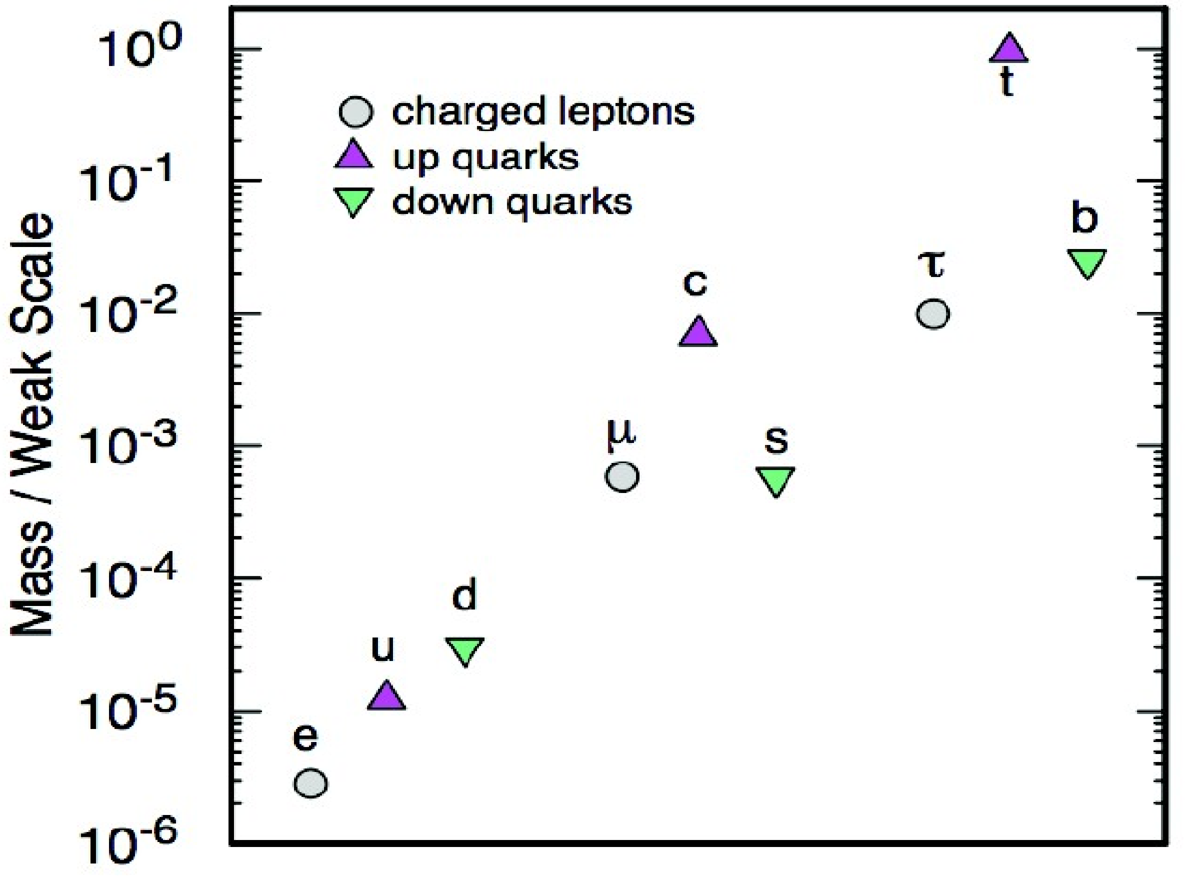
- Atomic level : binding energy ~O(10eV) which is ~10^{-8} of the mass
- Nuclear level : binding energy ~2% of the mass
- Nucleus parton level : binding energy ~98% of the mass

Most of the (luminous) mass in the universe comes from QCD confinement energy

- The insight of the Higgs mechanism :

New element in trying to understand the origin of mass of gauge bosons and fermions

The flavour hierarchy



Historical roots of the Standard Model

1864-1958 - Abelian theory of quantum electrodynamics

1933-1960 - Fermi model of weak interactions

1954 - Yang-Mills theories for gauge interactions...

1957-59 – Schwinger, Bludman and Glashow introduce W bosons for the weak charged currents…

…birth of the idea of unified picture for the electromagnetic and weak interaction in …

$$SU(2)_L \times U(1)_Y$$

Caution, not unified in the sense of unified forces, only unique framework

… but local gauge symmetry forbids gauge bosons and fermion masses.

Spontaneour Symmetry Breaking (SSB)

Global Symmetry

The Goldstone theorem is where it all began…

Massless scalars occur in a theory with SSB (or more accurately where the continuous symmetry is not apparent in the ground state).

Originates from the work of Landau (1937)

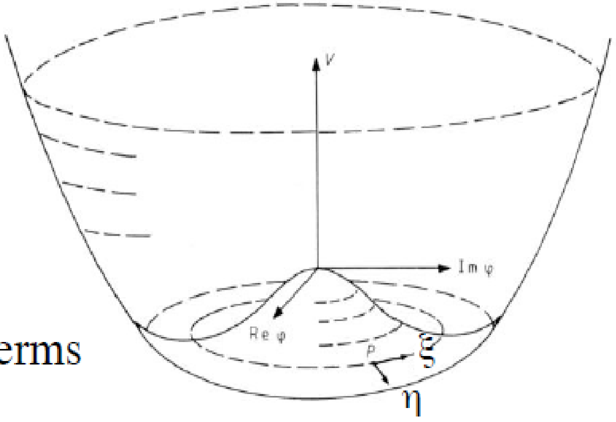
From a simple (complex) scalar theory with a U(1) symmetry

$$\varphi = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \qquad L = \partial_\nu \varphi^* \partial^\nu \varphi - V(\varphi) \qquad V(\varphi) = \mu^2 \varphi^* \varphi + \lambda(\varphi^* \varphi)^2$$

The Lagrangian is invariant under : $\varphi \to e^{i\alpha}\varphi$

$$v = -\frac{\mu^2}{\lambda}$$

Shape of the potential if $\mu^2<0$ and $\lambda>0$ necessary for SSB and be bounded from below.



Change frame to local minimum frame :

$$\varphi = \frac{v + \eta + i\xi}{\sqrt{2}}$$ No loss in generality.

$$L = \underbrace{\frac{1}{2}\partial_\nu \xi \partial^\nu \xi}_{\text{Massless scalar}} + \frac{1}{2}\partial_\nu \eta \partial^\nu \eta + \underbrace{\mu^2 \eta^2}_{\text{Massive scalar}} + \text{interaction terms}$$

Massless scalar Massive scalar

Spontaneour Symmetry Breaking (SSB)

All the players… in the same PRL issue

Local Symmetry

VOLUME 13, NUMBER 9 PHYSICAL REVIEW LETTERS 31 AUGUST 1964

BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

F. Englert and R. Brout
Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium
(Received 26 June 1964)

2 pages

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

1 page

GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES*

G. S. Guralnik,† C. R. Hagen,‡ and T. W. B. Kibble
Department of Physics, Imperial College, London, England
(Received 12 October 1964)

2 pages

1964 –The Higgs mechanism : How gauge bosons can acquire a mass.

SSB extended to Local Symmetry

Let the aforementioned continuous symmetry U(1) be local :

$$\varphi \to e^{i\alpha(x)}\varphi$$

α(x) now depends on the space-time x.

The Lagrangian can now be written : $L = (D_\nu \varphi)^* D^\nu \varphi - V(\varphi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

In terms of the covariant derivative : $D_\nu = \partial_\nu - ieA_\nu$

The gauge invariant field strength tensor : $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

And the Higgs potential : $V(\varphi) = \mu^2 \varphi^* \varphi + \lambda(\varphi^* \varphi)^2$

Here the gauge field transforms as : $A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha$

Again translate to local minimum frame : $\varphi = \frac{v + \eta + i\xi}{\sqrt{2}}$

$$L = \frac{1}{2}\partial_\nu \xi \partial^\nu \xi + \frac{1}{2}\partial_\nu \eta \partial^\nu \eta + \mu^2 \eta^2 - v^2 \lambda \eta^2 + \frac{1}{2}\underbrace{e^2 v^2 A_\mu A^\mu} - evA_\mu \partial^\mu \xi - F^{\mu\nu} F_{\mu\nu} + \text{ITs}$$

Mass term for the gauge field! But…

SSB extended to Local Symmetry

What about the field content?

A massless Goldstone boson ξ, a massive scalar η and a massive gauge boson!

Number of d.o.f. : 1 1 3

Number of initial d.o.f. : 4 Oooops… Problem!

But wait! Halzen & Martin p. 326 The term $evA_\mu \partial^\mu \xi$ is unphysical

The Lagrangian should be re-written using a more appropriate expression of the translated scalar field choosing a particular gauge where h(x) is real :

$$\varphi = (v + h(x))e^{i\frac{\theta(x)}{v}}$$

Gauge fixed to absorb θ

Then the gauge transformations are : $\varphi \to e^{-i\frac{\theta(x)}{v}}\varphi \qquad A_\mu \to A_\mu + \frac{1}{ev}\partial_\mu \theta$

$$L = \frac{1}{2}\partial_\nu h \partial^\nu h - \lambda v^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4$$ Massive scalar : The Higgs boson

$$+(1/2)e^2 v^2 A_\mu A^\mu - F^{\mu\nu}F_{\mu\nu}$$ Massive gauge boson

$$+(1/2)e^2 A_\mu A^\mu h^2 + ve^2 A_\mu A^\mu h$$ Gauge-Higgs interaction

The Goldstone boson does not appear anymore in the Lagrangian

1968 - The turning point:

2 pages

A MODEL OF LEPTONS*

Steven Weinberg†
Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 17 October 1967)

Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interactions. What could be more natural than to unite[1] these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences by imagining that the symmetries relating the weak and electromagnetic interactions are exact symmetries of the Lagrangian but are broken by the vacuum. However, this raises the specter of unwanted massless Goldstone bosons.[2] This note will describe a model in which the symmetry between the electromagnetic and weak interactions is spontaneously broken, but in which the Goldstone bosons are avoided by introducing the photon and the intermediate-boson fields as gauge fields.[3] The model may be renormalizable.

We will restrict our attention to symmetry groups that connect the observed electron-type leptons only with each other, i.e., not with muon-type leptons or other unobserved leptons or hadrons. The symmetries then act on a left-handed doublet

$$L = [\tfrac{1}{2}(1+\gamma_5)]\binom{\nu_e}{e} \qquad (1)$$

and on a right-hand...

$$R = [\tfrac{1}{2}(1-\gamma_5)]e. \qquad (2)$$

The conclusions of the paper…

Is this model renormalizable? We usually do not expect non-Abelian gauge theories to be renormalizable if the vector-meson mass is not zero, but our Z_μ and W_μ mesons get their mass from the spontaneous breaking of the symmetry, not from a mass term put in at the beginning. Indeed, the model Lagrangian we start from is probably renormalizable

Therefore, we shall construct our Lagrangian out of L and R, plus gauge fields $\vec{A}_\mu$ and B_μ cou...

Of course our model has too many arbitrary features for these predictions to be taken very seriously

whose ... and Y and give the electron its mass. The only renormalizable Lagrangian which is invariant under $\vec{T}$ and Y gauge transformations is

Milestone PRL 1967

The Weinberg Salam model (classical)

Before applying the Higgs mechanism to the $SU(2)_L$ x U(1) gauge symmetry

Why $SU(2)_L$ x U(1) ? And not $SU(2)_L$ only?

In order to describe the weak and electromagnetic interactions with a unique gauge group, Q the photon should be among the three generators of $SU(2)_L$

… then the electric charges of the multiplets must add up to 0. Which is not the case for the simple electron-neutrino doublet.

The ways out are$_{\text{Cheng and Li p.341}}$:

(i) Add an additional U(1) thus introducing an additional gauge boson

(ii) Add new fermions to form a triplet with charges adding up to 0

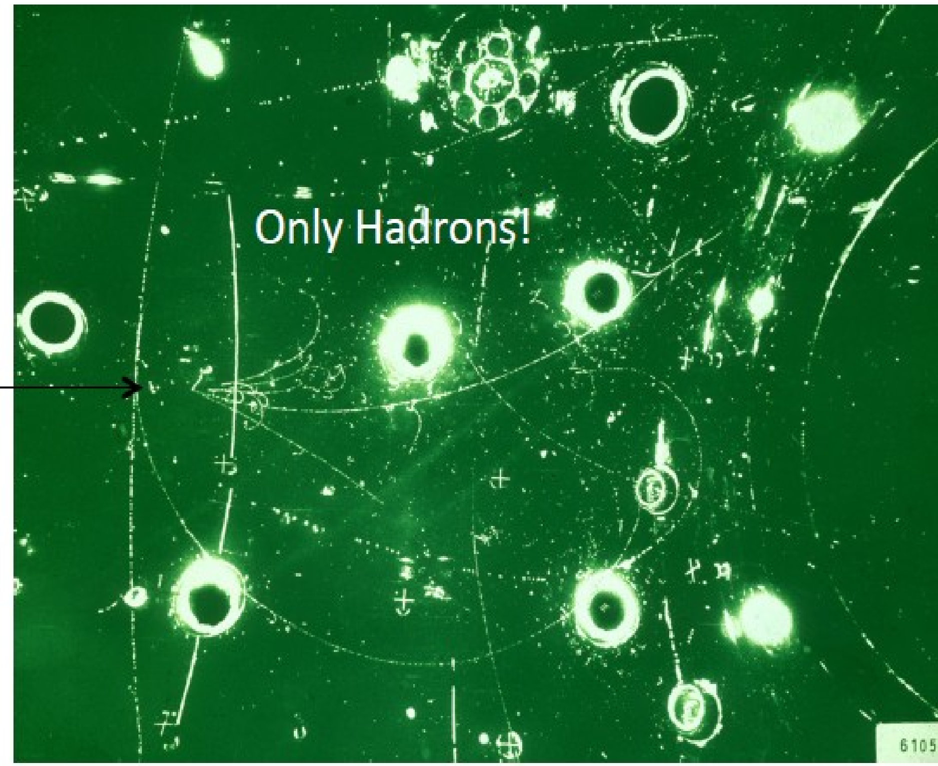
Georgi and Glashow followed (ii) in 1972 but their model was ruled out later in 1973 by a major discovery…

The Neutral Currents

1973: neutral current discovery (Gargamelle experiment, CERN)

Evidence for neutral current events $\nu + N \rightarrow \nu + X$ in ν-nucleon deep inelastic scattering

ν_μ



1973-1982: $\sin^2\theta_W$
Measurements in deep inelastic neutrino scattering experiments (NC vs CC rates of νN events)

Assuming a third weak gauge boson the initial number of gauge boson d.o.f. is 8, to give mass to three gauge bosons at least one doublet of scalar fields is necessary (4 d.o.f.) :

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}$$

Setting aside the gauge kinematic terms the Lagrangian can be written :

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi) \qquad \begin{cases} D_\mu = \partial_\mu - ig\vec{W}_\mu.\vec{\sigma} - ig'\frac{Y}{2}B_\mu \\ V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \end{cases}$$

The next step is to develop the Lagrangian near : $<\phi>= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

Choosing the specific real direction of charge 0 of the doublet is not fortuitous :

$$\phi = e^{-i\vec{\sigma}.\vec{\xi}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ H + v \end{pmatrix}$$

In particular for a non charged vacuum

Again choosing the gauge that will absorb the Goldstone bosons ξ...

Then developing the covariant derivative for the Higgs field :

Just replacing the Pauli matrices :

$$D_\mu \varphi = \partial_\mu \varphi - \frac{i}{2}\begin{pmatrix} gW_\mu^3 + g'B_\mu & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & -gW_\mu^3 + g'B_\mu \end{pmatrix}\varphi$$

Then using : $W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}}$

$$D_\mu \varphi = \partial_\mu \varphi - \frac{i}{2}\begin{pmatrix} gW_\mu^3 + g'B_\mu & \sqrt{2}gW_\mu^+ \\ \sqrt{2}gW_\mu^- & -gW_\mu^3 + g'B_\mu \end{pmatrix}\varphi = \begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \sqrt{2}gvW_\mu^+ + \sqrt{2}ghW_\mu^+ \\ -gvW_\mu^3 + g'vB_\mu - ghW_\mu^3 + g'hB_\mu \end{pmatrix}$$

For the mass terms only :

$$(D_\mu \varphi)^+ D^\mu \varphi = \partial_\mu h \partial^\mu h + \frac{1}{4}g^2 v^2 W_\mu^+ W^{-\mu} + \frac{1}{8}(W_\mu^3 \quad B_\mu)\begin{pmatrix} g^2 v^2 & -gg'v^2 \\ -gg'v^2 & g'^2 v^2 \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}$$

Explicit mixing of W^3 and B.

Finaly the full Lagrangian will then be written :

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2}\lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4}H^4 \\ &+ \frac{1}{2}\left[\frac{g'^2 v^2}{4}B_\mu B^\mu - \frac{g g' v^2}{2}W^3_\mu B^\mu + \frac{g^2 v^2}{4}\vec{W}_\mu.\vec{W}^\mu\right] \\ &+ \frac{1}{v}\left[\frac{g'^2 v^2}{4}B_\mu B^\mu H - \frac{g g' v^2}{2}W^3_\mu B^\mu H + \frac{g^2 v^2}{4}\vec{W}_\mu.\vec{W}^\mu H\right] \\ &+ \frac{1}{2v^2}\left[\frac{g'^2 v^2}{4}B_\mu B^\mu H^2 - \frac{g g' v^2}{2}W^3_\mu B^\mu H^2 + \frac{g^2 v^2}{4}\vec{W}_\mu.\vec{W}^\mu H^2\right]\end{aligned}$$

Massive scalar : The Higgs boson

Massive gauge bosons

Gauge-Higgs interaction

In order to derive the mass eigenstates :

Diagonalize the mass matrix $\frac{1}{4}\begin{pmatrix} g^2v^2 & -gg'v^2 \\ -gg'v^2 & g'^2v^2 \end{pmatrix} = \mathcal{M}^{-1}\begin{pmatrix} m_Z^2 & 0 \\ 0 & 0 \end{pmatrix}\mathcal{M}$

Where

$$\mathcal{M} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \qquad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

The Weinberg angle was actually first introduced by Glashow (1960)

The first very important consequences of this mechanism :

1.- Two massive charged vector bosons :

$$m_W^2 = \frac{g^2 v^2}{4}$$

Corresponding to the observed charged currents

Thus v = 246 GeV

Given the known W mass and g coupling

2.- One massless vector boson : $m_\gamma = 0$

The photon correponding to the unbroken $U(1)_{EM}$

3.- One massive neutral vector boson Z :

$$m_Z^2 = (g^2 + g'^2) v^2 / 4$$

4.- One massive scalar particle : The Higgs boson

Whose mass is an unknown parameter of the theory as the quartic coupling λ

$$m_H^2 = \frac{4\lambda(v) m_W^2}{g^2}$$

Which of these consequences are actually predictions ?

1.- The theory was chosen in order to describe the weak interactions mediated by charged currents.

2.- The masslessness of the photon is a consequence of the choice of developing the Higgs field in the neutral and real part of the doublet.

3 & 4.- The appearance of massive Z and Higgs bosons are actually predictions of the model.

One additional very important prediction which was not explicitly stated in Weinberg's fundamental paper… although it was implicitly clear :

There is a relation between the ratio of the masses and that of the couplings of gauge bosons :

$$\frac{M_W}{M_Z} = \frac{g^2}{g^2 + g'^2} = \cos^2\theta_W$$

or

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$$

The sector of Fermions

Taking a closer look at the neutral current interaction part of the Lagrangian :

$$L_L = -\frac{1}{2}\overline{\psi}_L\gamma_\mu \begin{pmatrix} gW_3^\mu + g'Y_L B^\mu & 0 \\ 0 & -gW_3^\mu + g'Y_L B^\mu \end{pmatrix}\psi_L \quad L_R = -\frac{1}{2}\overline{\psi}_R\gamma_\mu \begin{pmatrix} g'Y_R B^\mu & 0 \\ 0 & 0 \end{pmatrix}\psi_R$$

In the lepton sector :

$$\begin{aligned} -2L_{NC}^{leptons} &= \overline{\nu}_L\gamma_\mu\left[(c_W g - s_W g'Y_L)Z^\mu + (s_W g + c_W g'Y_L)A^\mu\right]\nu_L \\ &+ \overline{e}_L\left[(-c_W g - s_W g'Y_L)Z^\mu + (-s_W g + c_W g'Y_L)A^\mu\right]e_L \\ &+ \overline{e}_R\gamma_\mu\left[-s_W g'Y_R Z^\mu + c_W g'Y_R A^\mu\right]e_R \end{aligned}$$

1.- Eliminate neutrino coupling to the photon : $g\sin\theta_W = -g'Y_L\cos\theta_W$

2.- Same coupling e_R and e_L to the photon : $g'Y_R = 2g'Y_L$

3.- Link to the EM coupling constant *e* : $g\sin\theta_W = e$

Y the hypercharge is chosen to verify the Gell-Mann Nishijima formula : $Q = I_3 + \frac{Y}{2}$

The Minimal Standard Model

	Field	I_3	Y	Q	$SU(2)_L x U(1)_Y$	$SU(3)_C$
Leptons	(ν_L, e_L)	(1/2,-1/2)	-1	(0,-1)	(2,-1)	1
	e_R	0	-2	-1	(1,-2)	1
Quarks	(u_L, d_L)	(1/2,-1/2)	-1	(2/3,-1/3)	(2,1/3)	3
	u_R	0	4/3	2/3	(1,4/3)	$\bar{3}$
	d_R	0	-2/3	-1/3	(1,-2/3)	$\bar{3}$
IVB	B	0	0	-	(1,0)	1
	W	(1,0,-1)	0	-	(3,0)	1
	g	0	0	-	(1,0)	8
Higgs	H	(1/2,-1/2)	1	-	(2,1)	1

The sector of fermions

Another important consequence of the Weinberg Salam Model…

A specific $SU(2)_L xU(1)_Y$ problem : $m\overline{\psi}\psi$ manifestly not gauge invariant

$$m\overline{\psi}\psi = m\overline{\psi}(\frac{1}{2}(1-\gamma^5)+\frac{1}{2}(1+\gamma^5))\psi = m(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L)$$

- neither under $SU(2)_L$ doublet and singlet terms together
- nor under $U(1)_Y$ do not have the same hypercharge

Fermion mass terms are forbidden

Not the case when using Yukawa couplings to the Higgs doublet

Then after SSB one recovers :

$$\frac{\lambda_\psi v}{\sqrt{2}}\overline{\psi}\psi + \frac{\lambda_\psi}{\sqrt{2}}H\overline{\psi}\psi$$

Which is invariant under $U(1)_{EM}$

Very important : The Higgs mechanism DOES NOT predict fermion masses

…Yet the coupling of the Higgs to fermions is proportional to their masses

The coupling to the Higgs fields is the following :

$$\lambda_d (\overline{u}_L, \overline{d}_L) \begin{pmatrix} 0 \\ v + h \end{pmatrix} d_R + H.C. = \lambda_d \overline{Q}_L \ \phi \, d_R$$

Can be seen as giving mass to down type fermions…

To give mass to up type fermions, need to use a slightly different coupling :

$$\phi^C = i\sigma_2 \phi^* \qquad \lambda_u Q_L \ \phi^C \ \overline{u}_R = \lambda_u (\overline{u}_L, \overline{d}_L) \begin{pmatrix} v + h \\ 0 \end{pmatrix} d_R + H.C.$$

One doublet of complex scalar fields is sufficient to accommodate mass terms for gauge bosons and fermions !

The experimental crowning glory of the model

1974 - Discovery of the c quark

1975 - Discovery of the tau lepton

1977 - Discovery of the b quark

1979 - Discovery of the gluon

1983 - Discovery of the W and Z bosons

1990 - Determination of the number of light neutrino families

1991 - Precise tests of the internal coherence of the theory and top mass prediction

1993 - Top quark discovery

$$\rho = 1$$

The experimental crowning glory of the model

1997 - Neutrino Oscillations

1998 – tau neutrino discovery

1975 - CP violation in B's

The Standard Model is experimentally crowned, except…

Where is the expected massive physical state ?

Custodial symmetry

Turning again to the chiral symmetry which is also a symmetry of the Higgs sector :

$$SU(2)_L \otimes SU(2)_R \to SU(2)_V$$

It is very interesting to note that under the $SU(2)_V$ symmetry, the weak gauge bosons (W^1,W^2,W^3) transform as a triplet

Meaning that after EWSB all W^i's are mass degenerate

This directly implies that ρ=1

Under this crucial condition does any Higgs sector work for this purpose?

For N iso-multiplets :

$$\rho = \frac{\sum_{k=1}^{N} v_k^2 [I^k(I^k+1) - (I_3^k)^2]}{\sum_{k=1}^{N} 2 v_k^2 (I_3^k)^2}$$

For the condition to be fulfilled any number of doublets is fine

Higher representations need to fine tune the vevs

What we have learned

Higgs mechanism:
- Allows gauge bosons to acquire a mass
- Allows fermion masses
- Interpretation of EW interactions (not unification)
- Enables renormalizability of EW gauge theory

Legitimates $SU(2)_L xU(1)_Y$ as a gauge theory of electroweak interaction which is now known as the Standard Model

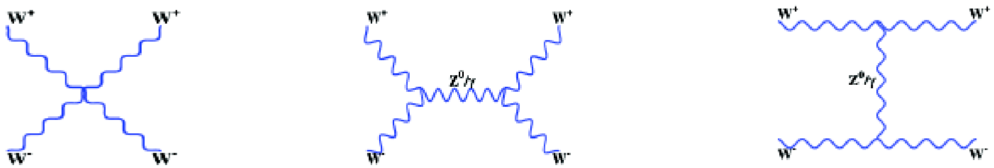
In practice : all known processes can be computed in this framework

$$\rho = 1$$

Theoretical constraints: unitarity

The cross section for the thought scattering process :

$$W^+W^- \to W^+W^-$$



Does not preserve perturbative unitarity.

Introducing a Higgs boson ensures the unitarity of this process PROVIDED that its mass be smaller than :

$\sqrt{4\pi\sqrt{2}/3G_F}$ v.i.z. approximately 1 TeV

This is not only a motivation for the Higgs mechanism but is also a strong experimental constraint on its mass… if you believe in perturbative unitarity…

If you don't the electroweak interaction should become strong at the TeV scale and one would observe non perturbative effects such as multiple W production, WW resonances… (Technicolor…)

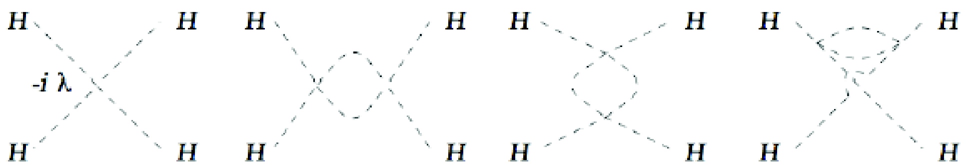
Theoretical constraints: triviality

The (non exhaustive though rather complete) evolution of the quartic coupling :

$$32\pi^2 \frac{d\lambda}{dt} = \boxed{24\lambda^2} - (3g'^2 + 9g^2 - 24y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 24y_t^4 + \cdots$$

In the case where the Higgs mass is large (large λ) : $M_H^2 = 2\lambda v^2$

The first term of the equation is dominant and due to diagrams such as :



$$\frac{d\lambda(Q^2)}{dt} = \frac{3}{4\pi^2}\lambda^2(Q^2) \longrightarrow \frac{1}{\lambda(Q^2)} = \frac{1}{\lambda(Q_0^2)} - \frac{3}{4\pi^2}\ln\left(\frac{Q^2}{Q_0^2}\right)$$

If Q can be high at will eventually lead to Landau pole

Triviality condition to avoid such pole : $1/\lambda(Q) > 0$

Then

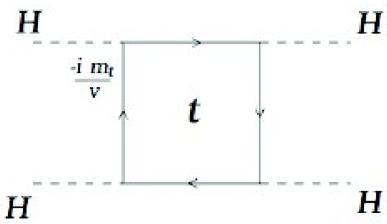
$$\boxed{M_H^2 < \frac{8\pi^2 v^2}{3\log\left(\frac{\Lambda^2}{v^2}\right)}}$$

Theoretical constraints: vacum stability

Looking closer into the limit where the Higgs boson mass is small :

$$32\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 - (3g'^2 + 9g^2 - 24y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - \boxed{24y_t^4} + \cdots$$

The last term of the equation is dominant and due to diagrams such as :



$$\sim \frac{m_t^4}{v^4}$$

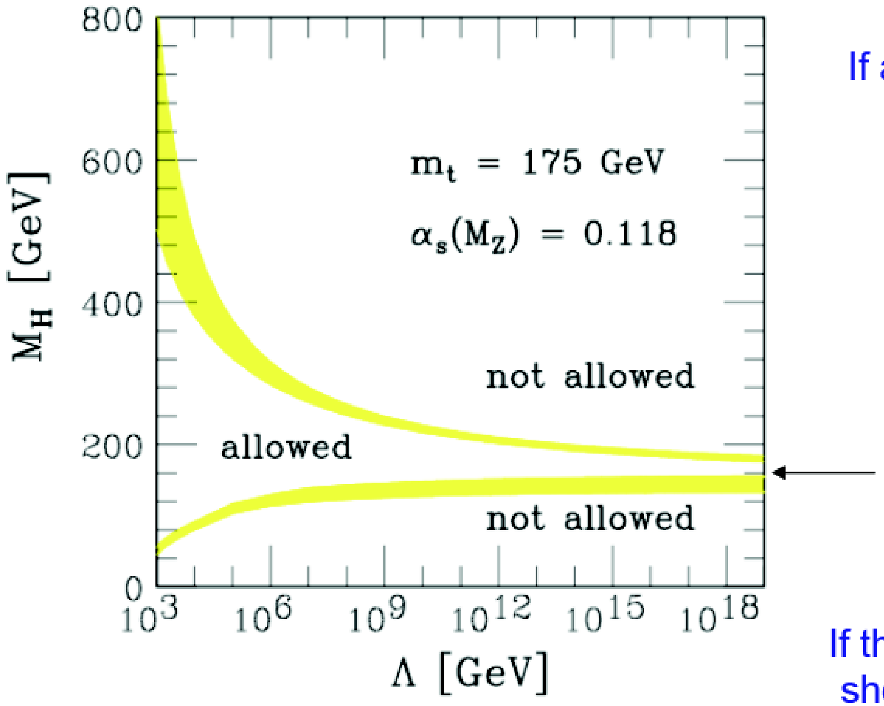
The equation is then very simply solved : $\lambda(\Lambda) = \lambda(v) - \frac{3}{4\pi^2} y_t^2 \log\left(\frac{\Lambda^2}{v^2}\right)$

Requiring that the solutions are stable (non-negative quartic coupling) :

$\lambda(\Lambda) > 0$ then $\boxed{M_H^2 > \frac{3v^2}{2\pi^2} y_t^2 \log\left(\frac{\Lambda^2}{v^2}\right)}$

Theoretical constraints: summary

More acurate analysis yields :



What do we learn from these constraints?

If a Higgs boson exists it should be found at LHC!

If it is very heavy new physics should occur at a low scale

If the SM is valid at any scale the Higgs mass is quite precisely predicted

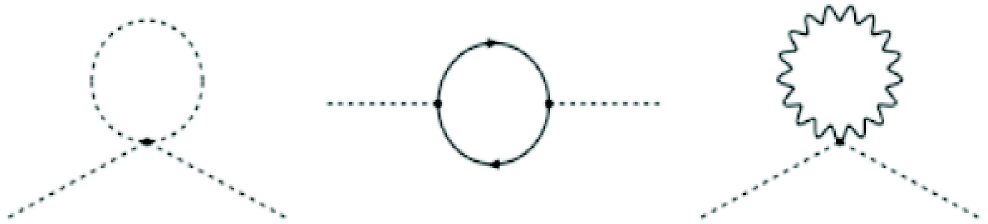
If the Higgs is rather light new physics should also be at a rather low energy

However it does not motivate the very existence of a Higgs boson…

The hierarchy problem

The Higgs potential is fully renormalizable, but…

Loop corrections to the Higgs boson mass…



…are quadratically divergent : $\Delta m^2 \propto \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \sim \frac{\Lambda^2}{16\pi^2}$

If the scale at which the standard model breaks down is large, the Higgs natural mass should be of the order of the cut-off. e.g. the Planck scale

$m = m_0 + \Delta m + \ldots$ Higher orders

…but if the Higgs boson exists it should have a low mass!

This can be achieved by fine tuning our theory… Inelegant…

Supersymmetry

The Hierarchy problem is not only a problem of esthetics : If the difference is imposed at tree level, the radiative corrections will still mix the scales and destabilize the theory.

One may note that :

$$\Delta m_H^2 \sim \frac{|\lambda_f|^2}{16\pi^2}(-2\Lambda^2 + 6m_f^2 \ln\frac{\Lambda}{m_f} + ...)$$ → Contribution of fermions

$$\Delta m_H^2 \sim \frac{\lambda_s}{16\pi^2}(\Lambda^2 + 2m_f^2 \ln\frac{\Lambda}{m_s} + ...)$$ → Contribution of scalars

Therefore in a theory where for each fermion there are two scalar fields with $\lambda_s = |\lambda_f|^2$

(which is fulfilled if the scalars have the same couplings as the fermions)

quadratic divergencies will cancel

The field content of the standard model is not sufficient to fulfill this condition

A solution is given by supersymmetry where each fermionic degree of freedom has a symmetrical bosonic correspondence

Supersymmetry

In supersymmetry the quadratic divergences naturally disappear but…

Immediately a problem occurs : Supersymmetry imposes $m_{boson} = m_{fermion}$

Supersymmetry must be broken!

But in the case of SUSY a SSB mechanism is far more complex than for the EWSB and no satisfactory SSB solution exists at this time…

…However an explicit breaking “by hand” is possible provided that it is softly done in order to preserves the SUSY good UV behavior…

$$\Delta m_H^2 \propto m_{soft}^2 (\ln \frac{\Lambda}{m_{soft}} + ...)$$

Interestingly similar relation to that of the general fine tuning one

Implies that the m_{soft} should not exceed a few TeV

What we have learned

1.- A Higgs boson is highly desirable for the unitarity of the theory and should have a mass lower than about 1 TeV

2.- If it exists the running of the quartic coupling yields interesting bounds on its mass (triviality and vacuum stability)

3.- The existence of a Higgs boson is a key to investigate theories beyond the standard model (fine tuning)

4.- It highly motivates supersymmetry

5.- It even gives indication on the mass scale of SUSY particles

The Higgs mechanism is yielding way more than what it was initially introduced for

Experimental indirect constraints

The standard model has 3 free parameters not counting the Higgs mass and the fermion masses and couplings.

Particularly useful set is :

1.- The fine structure constant : $\alpha = 1/137.035999679(94)$ | 10^{-9}

Determined at low energy by electron anomalous magnetic moment and quantum Hall effect

2.- The Fermi constant : $G_F = 1.166367(5) \times 10^{-5}\ \mathrm{GeV}^{-2}$ | 10^{-5}

Determined from muon lifetime

3.- The Z mass : $M_Z = 91.1876 \pm 0.0021\ \mathrm{GeV}$ | 10^{-5}

Measured from the Z lineshape scan at LEP

Experimental constraints

Taking the hypothesis of a Minimal Standard Model, the radiative corrections to numerous observables can be computed in order to assess the impact of certain particles e.g. the Higgs boson

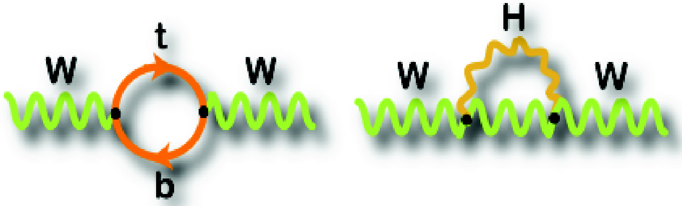
From the measurement of these observables a constraint is derived

For example the corrections to the Fermi coupling constant can be written as :

$$G_F = \frac{\pi \alpha_{QED}}{\sqrt{2} m_W^2 (1 - m_W^2 / m_Z^2)} (1 + \Delta r)$$

With :

$$\begin{cases} \Delta r_t \propto m_t^2 \\ \Delta r_H \propto \log(m_H / m_W) \end{cases}$$



Essential ingredients top, W and Z masses and α_{QED}

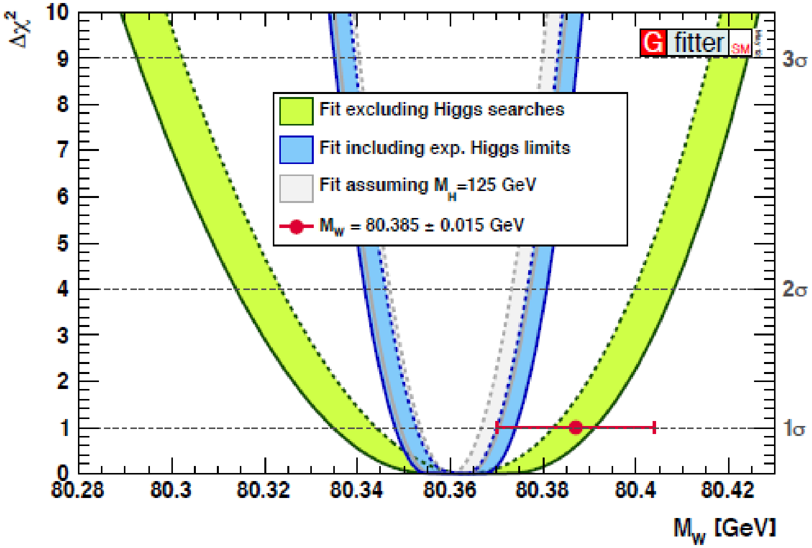
The complete data

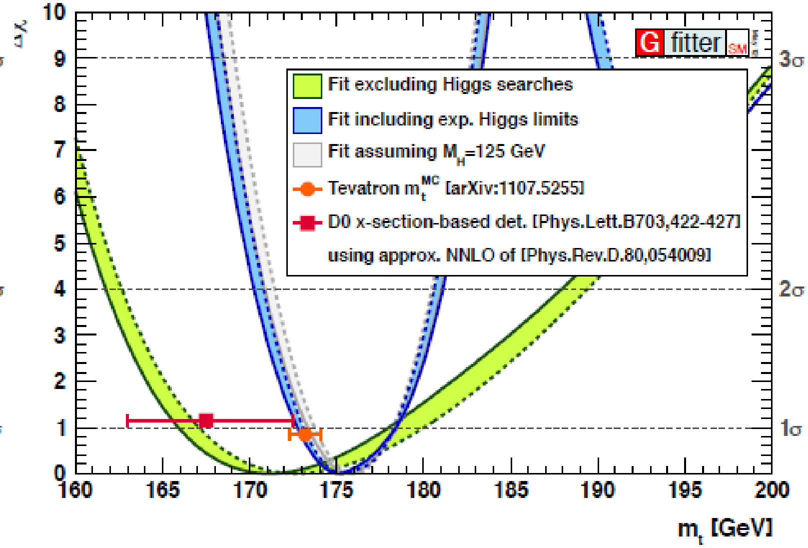
Parameter	Input value	Free in fit	Results from global EW fits: *Standard fit*	*Complete fit*	*Complete fit w/o exp. input in line*
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1874 ± 0.0021	91.1878 ± 0.0021	$91.1951\,^{+0.0136}_{-0.0112}$
Γ_Z [GeV]	2.4952 ± 0.0023	–	2.4958 ± 0.0015	2.4955 ± 0.0014	2.4952 ± 0.0016
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	–	41.478 ± 0.014	$41.477\,^{+0.016}_{-0.013}$	41.470 ± 0.015
R^0_ℓ	20.767 ± 0.025	–	20.743 ± 0.018	20.741 ± 0.017	$20.717\,^{+0.027}_{-0.008}$
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	–	0.01637 ± 0.0002	$0.01627\,^{+0.0002}_{-0.0001}$	$0.01620\,^{+0.0002}_{-0.0001}$
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	–	$0.1477^{+0.0009}_{-0.0008}$	$0.1473^{+0.0008}_{-0.0006}$	–
A_c	0.670 ± 0.027	–	$0.6682\,^{+0.00042}_{-0.00035}$	$0.6680\,^{+0.00037}_{-0.00028}$	$0.6680\,^{+0.00034}_{-0.00030}$
A_b	0.923 ± 0.020	–	$0.93468\,^{+0.00008}_{-0.00007}$	$0.93463\,^{+0.00007}_{-0.00005}$	0.93466 ± 0.00005
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	–	$0.0740\,^{+0.0005}_{-0.0004}$	$0.0738\,^{+0.0005}_{-0.0003}$	0.0738 ± 0.0004
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	–	$0.1036\,^{+0.0007}_{-0.0006}$	$0.1032\,^{+0.0006}_{-0.0005}$	$0.1037\,^{+0.0003}_{-0.0005}$
R^0_c	0.1721 ± 0.0030	–	0.17223 ± 0.00006	0.17223 ± 0.00006	0.17223 ± 0.00006
R^0_b	0.21629 ± 0.00066	–	0.21474 ± 0.00003	0.21474 ± 0.00003	0.21474 ± 0.00003
$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	–	$0.23144\,^{+0.00010}_{-0.00013}$	$0.23150\,^{+0.00008}_{-0.00011}$	$0.23145\,^{+0.00012}_{-0.00006}$
M_H [GeV] $^{(\circ)}$	95% CL limits	yes	$94^{+25[+59]}_{-22[-41]}$	–	$94^{+25[+59]}_{-22[-41]}$
M_W [GeV]	80.385 ± 0.015	–	$80.380\,^{+0.011}_{-0.012}$	$80.370\,^{+0.006}_{-0.007}$	$80.360\,^{+0.014}_{-0.012}$
Γ_W [GeV]	2.085 ± 0.042	–	2.092 ± 0.001	2.092 ± 0.001	2.092 ± 0.001
$\overline{m}_c$ [GeV]	$1.27\,^{+0.07}_{-0.11}$	yes	$1.27\,^{+0.07}_{-0.11}$	$1.27\,^{+0.07}_{-0.11}$	–
$\overline{m}_b$ [GeV]	$4.20\,^{+0.17}_{-0.07}$	yes	$4.20\,^{+0.17}_{-0.07}$	$4.20\,^{+0.17}_{-0.07}$	–
m_t [GeV]	173.2 ± 0.9	yes	173.2 ± 0.9	173.4 ± 0.8	$175.1\,^{+3.3}_{-2.4}$
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ $^{(\dagger\triangle)}$	2757 ± 10	yes	2757 ± 11	2756 ± 11	2728^{+51}_{-50}
$\alpha_s(M_Z^2)$	–	yes	$0.1192\,^{+0.0028}_{-0.0027}$	0.1191 ± 0.0028	0.1191 ± 0.0028
$\delta_{\rm th} M_W$ [MeV]	$[-4, 4]_{\rm theo}$	yes	4	4	–
$\delta_{\rm th} \sin^2\theta^\ell_{\rm eff}$ $^{(\dagger)}$	$[-4.7, 4.7]_{\rm theo}$	yes	4.7	1.5	–

$^{(\star)}$Average of LEP ($A_\ell = 0.1465 \pm 0.0033$) and SLD ($A_\ell = 0.1513 \pm 0.0021$) measurements. The *complete fit* w/o the LEP (SLD) measurement gives $A_\ell = 0.1474\,^{+0.0006}_{-0.0007}$ ($A_\ell = 0.1469 \pm 0.0006$). $^{(\circ)}$In brackets the 2σ. $^{(\dagger)}$In units of 10^{-5}. $^{(\triangle)}$Rescaled due to α_s dependency.

- Numerous observables O(40)
- Numerous experiments (with different systematics)
- Within experiments numerous analyses (with different systematics)
- Various theoretical inputs

W and top quark mass measurement





Precision of ~0.02%

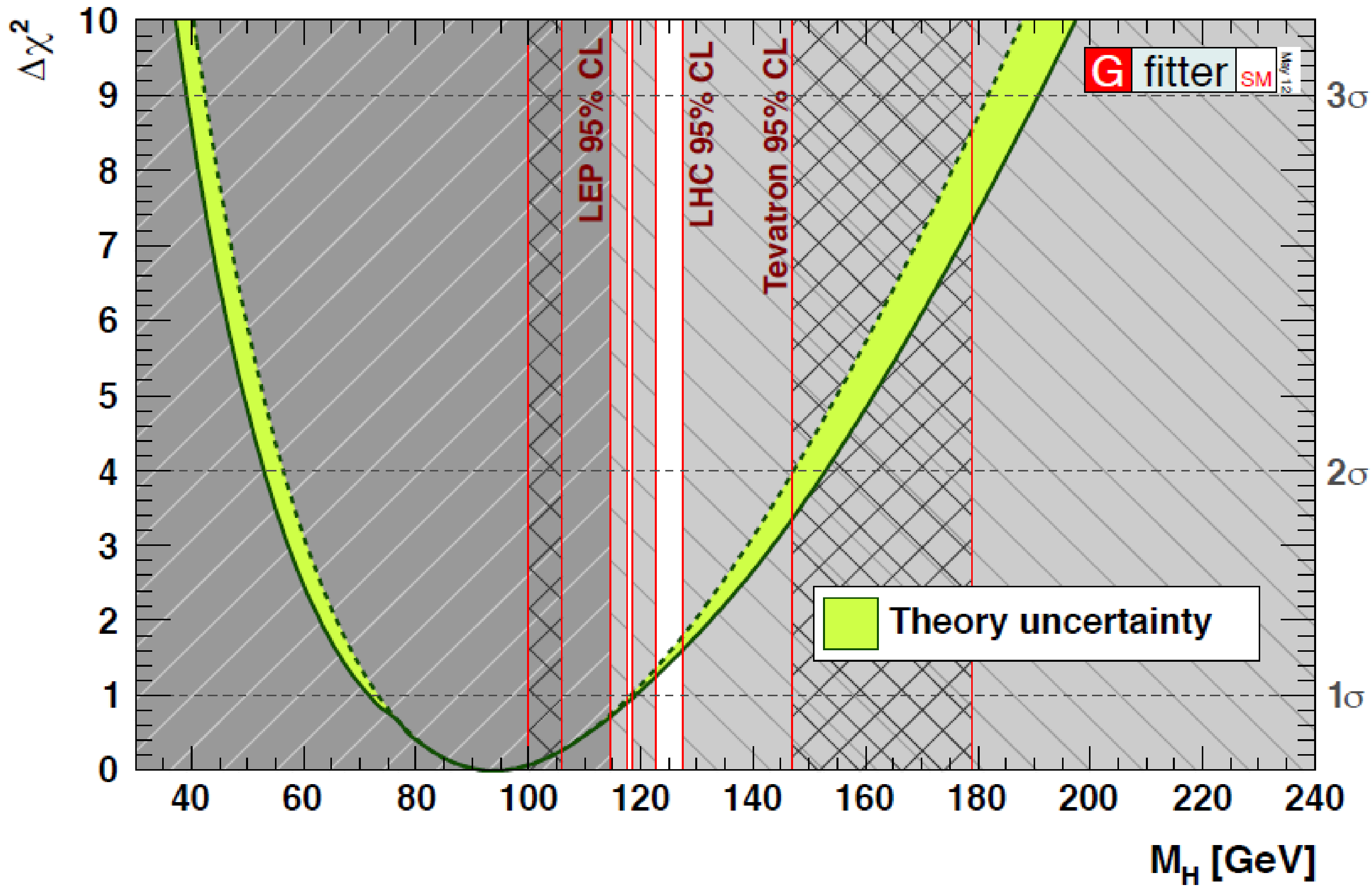
- TeVatron reached ~15 MeV
- LHC should reach ~15 MeV or better

Precision of ~0.8%

-TeVatron is aiming at ~ 0.9 GeV

- Not so clear that LHC will be able to do much better.

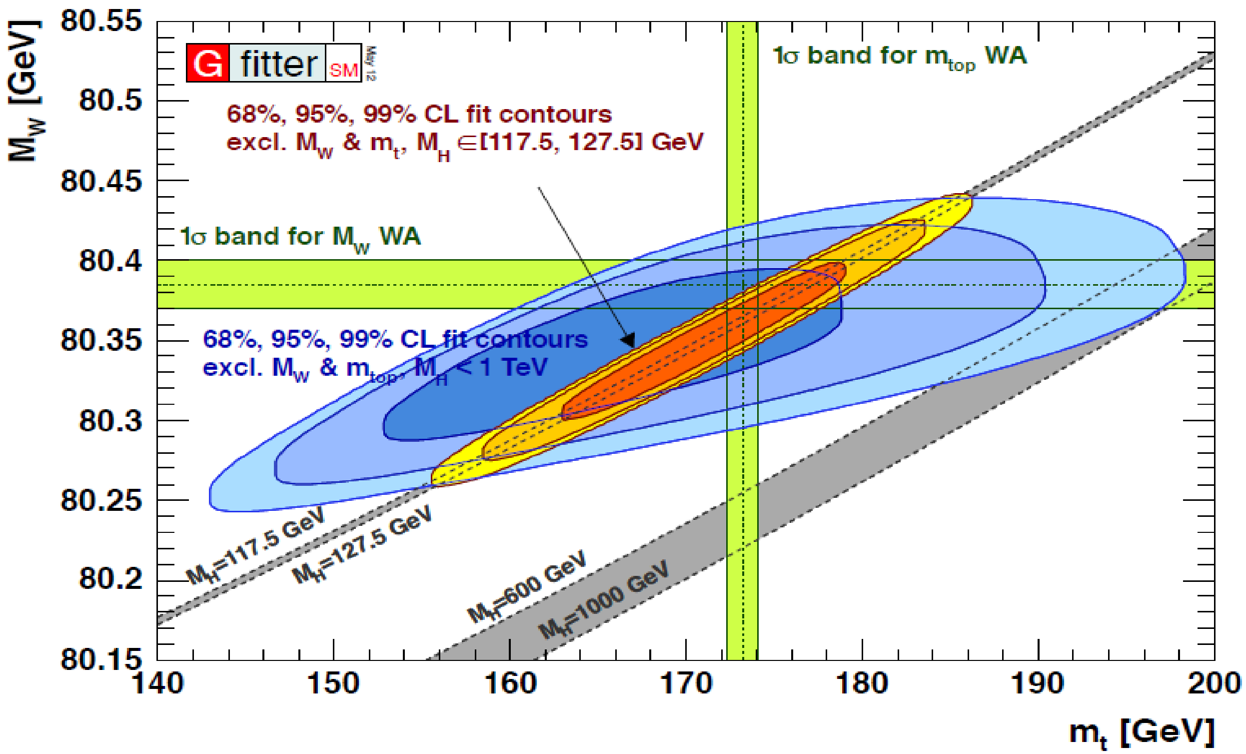
Indirect measurement of the Higgs bosons mass



M_H [GeV] $^{(\circ)}$	95% CL limits	yes	$94^{+25[+59]}_{-22[-41]}$	–	$94^{+25[+59]}_{-22[-41]}$

Indirect measurement of the Higgs bosons mass

Indirect Measurement of Higgs Boson Mass



M_H [GeV] $^{(\circ)}$	95% CL limits	yes	$94^{+25[+59]}_{-22[-41]}$	–	$94^{+25[+59]}_{-22[-41]}$